ing invariants of any specified order and these are then exhibited by contraction of fundamental surface tensors and their covariant derivatives appearing as coefficients of a system of partial differential equations defining the surface. The projective normal is attained by the formal elimination procedure of tensor analysis. (Received April 10, 1943.)

195. H. T. Muhly: *Independent integral bases and a characterization of regular surfaces.*

Let $\mathfrak{o}^*$ denote the ring of homogeneous coordinates, $\xi_0^*, \xi_1^*, \cdots, \xi_n^*$ associated with a normal, nonsingular model F, of a field Σ of algebraic functions of two variables, and assume that $\xi_0^*, \xi_1^*, \xi_2^*$ are selected so that they are algebraically independent and so that each element of $\mathfrak{o}^*$ depends integrally upon these three elements. Let the relative degree $[K(\xi_0^*, \xi_1^*, \cdots, \xi_n^*):K(\xi_0^*, \xi_1^*, \xi_2^*)]$ be denoted by ν. It is shown in this paper that if there exists a set of ν elements $\lambda_1^*, \lambda_2^*, \cdots, \lambda_\nu^*$ in $\mathfrak{o}^*$ which form an independent modular base for $\mathfrak{o}^*$ over the ring $K[\xi_0^*, \xi_1^*, \xi_2^*]$, then the field Σ is regular, that is, its arithmetic genus p_a coincides with its geometric genus p_g. Furthermore, it is shown that if the field Σ is regular then there exist projective models of Σ which are normal and nonsingular, and which are such that the associated ring of homogeneous coordinates has a linearly independent modular base over a suitably chosen ring of independent variables. In fact if F is any normal, nonsingular model of a regular field Σ, and if F_h is the derived normal model belonging to the character of homogeneity h, then F_h has these properties for all sufficiently large values of h. (Received May 15, 1943.)

Numerical Computation

196. A. N. Lowan and H. E. Salzer: *Coefficients in the expansion of derivatives in terms of central differences.*

The coefficients in the expansion of the first 52 derivatives in terms of mean central and central differences (in most cases up to the 42nd difference) were computed by the Mathematical Tables Project. For the first 30 derivatives the exact values of the coefficients are given in the form of ordinary fractions for the first 30 differences and for some differences beyond the 30th. For all derivatives beyond the 30th exact fractional values are given for the coefficients of differences up to orders varying between the 41st and 52nd. Finally for most of the coefficients of differences of orders varying from the 31st to the 42nd, 18 significant figures are given. All fractions are believed to be in lowest terms. The tabulated coefficients are valuable in the evaluation of analytic functions of a complex variable when known along a straight line within the region. The coefficients were checked by means of two recursion formulas, which are not mentioned in the literature. (Received April 8, 1943.)

Statistics and Probability

197. Jacob Wolfowitz: *Asymptotic distributions of ascending and descending runs.*

Let $a_1, a_2, \cdots, a_N$ be any permutation of N unequal numbers. Let there be assigned to each permutation the same probability. An element a_i $(1<i<N)$ is called a turning point if a_i is greater than or less than both a_{i-1} and a_{i+1}. Let a_j and a_{j+k} be consecutive turning points; they are said to determine a "run" of length k. The author