considering in the product set E^n sets of the form $Z_1 \times Z_i \times Z_n$, where Z_i belongs to K, the Borel field over such sets and using the operation of projection and complementation a finite number of times. Necessary and sufficient conditions are found in terms of a special mapping of E into E^n preserving a given class of sets in E^n, for the product isomorphism in case of sets A, B in the projective class. The classical theories of Borel sets and of projective sets correspond to the case where K is the sequence of rational intervals. Many results of these theories hold for the case of a general K and thus show a purely combinatorial and not topological origin. This permits the formulation of an analogous theory in the even more general case of projective algebras (abstract 49-5-151). (Received October 22, 1943.)

37. S. E. Warschawski: *On Theodorsen's method of conformal mapping of nearly circular regions.*

The paper deals with the problem of determining the mapping function of a circle onto a nearly circular region. This problem has some practical importance in the theory of airfoils. Let C be a nearly circular closed Jordan curve: $\rho = \rho(\phi)$, $0 \leqq \phi \leqq 2\pi$, (ρ, ϕ polar coordinates) where $(1/\rho)(d\rho/d\phi)$ is continuous and $1 \leqq \rho(\phi) \leqq 1+\epsilon$ for some ϵ, $0 < \epsilon < 1$. Suppose that $w = f(z)$ ($f(0) = 0$, $f'(0) > 0$) maps the circle $|z| = |re^{i\theta}| < 1$ conformally onto the interior of C. If $\arg f(e^{i\theta}) = \phi(\theta)$, the $\log (f(z)/z)$ for $z = e^{i\theta}$ may be written as $\log \rho[\phi(\theta)] + i(\phi(\theta) - \theta)$. Hence by Fatou's formula, $\phi(\theta) - \theta = (1/2\pi)\int_0^{\pi} \{\log \rho[\phi(\theta+t)] - \log \rho[\phi(\theta-t)]\} \cot (t/2)dt \equiv H[\phi(\theta)]$, and the function $\phi(\theta)$ may thus be determined by solving this integral equation. Theodorsen (National Advisory Committee for Aeronautics, Report 411, 1932) and Theodorsen and Garrick (National Advisory Committee for Aeronautics, Report 452, 1933) developed a practical method for computing a solution of this equation by successive approximations. In the present paper the theoretical basis of this method is studied. Sufficient conditions for the curve C are established under which the approximations $\phi_0(\theta) \equiv \theta$, $\phi_n(\theta) = H[\phi_{n-1}(\theta)]$ and their derivatives $\phi_n'(\theta)$ converge to $\phi(\theta)$ and $\phi'(\theta)$ respectively, and the errors $|\phi_n(\theta) - \phi(\theta)|$ and $|\phi_n'(\theta) - \phi'(\theta)|$ are estimated. (Received October 27, 1943.)

Applied Mathematics

38. L. W. Cohen and S. M. Ulam: *On the algebra of systems of vectors and some problems in kinematics.*

The properties of equivalence for systems of vectors as postulated for the mechanics of rigid bodies are studied in linear spaces. It is proved that any finite system of vectors in n-space is equivalent to a unique system of vectors collinear with the edges of an n-simplex. It is also proved that any such system is equivalent to a system of $[n/2]+1$ vectors. Similar theorems hold for infinitely many vectors and for spaces of infinitely many dimensions. The problem of topological invariants of trajectories of systems of n points with respect to arbitrary motions of the coordinate system is formulated and results are obtained for the case of three points. (Received October 22, 1943.)

39. A. H. Copeland: *The nature of turbulence.*

There are exhibited in this paper a number of flows which consist of series of disturbances distributed temporally but unfortunately not specially at random, and which satisfy the hydrodynamic equations together with appropriate boundary con-

ditions. In conformance with experimental evidence each disturbance becomes dissipated as time increases. The following motions are obtained: flow without obstructions with constant velocity at infinity, flow through a pipe, flow outside a rotating cylinder, flow past a wall, and flow between parallel walls. These solutions may be regarded as mathematical models displaying some but not all of the characteristics of physical turbulent motion. (Received October 30, 1943.)

40. Isaac Opatowski: *Isoperimetric problems in bending of cantilevers.*

The paper considers cantilevers characterized by the fact that any two cross sections are obtainable from each other by a transformation of dilatation. The surface bounding these cantilevers is representable in the parametric form: $y=F(x)f(t)$, $z=G(x)g(t)$; the beam's axis is taken as the x-axis. The load consists of the cantilever's own weight W and a concentrated force P at the free end. The problem of finding cantilevers which have an assigned type of cross section (that is, assigned expressions of f and g), assigned length and weight, and which are of uniform strength (S. Timoshenko, *Strength of materials*, 2nd ed., part 1, p. 209) and have a minimum deflection Y is of an isoperimetric type with an additional condition for F and G in the form of a Volterra integral equation. If W is neglected with respect to P the problem has no meaning because Y becomes proportional to W. However if, besides this, the condition of uniform strength is abandoned, the classical type of a simple isoperimetric problem is obtained. This is a generalization of a result given by Blasius for circular cross section (Zeitschrift für Mathematik und Physik vol. 62 (1914) pp. 182–197). (Received October 21, 1943.)

41. H. E. Salzer: *Supplementary calculation of coefficients for numerical integration with central differences.*

The coefficients M_{2s} which are the $(2s)$th Bernoulli polynomials of order $2s$ for argument equal to s, divided by $(2s)!$, were calculated from M_{22} to M_{40} to 20 significant figures, from M_{42} to M_{52} to 8 significant figures, and finally from M_{54} to M_{62} to 20 decimals. (This extends the earlier calculation which went up to M_{20}, mentioned in an abstract of this Bulletin, 49-9-224.) With these coefficients it is possible to integrate functions that are tabulated at very wide intervals, even when the size of the interval is such that the differences do not fall off. Functions tabulated to an extremely large number of decimal places can be integrated to the same or greater accuracy, since we can use many differences. The ratio of consecutive coefficients, about 1/4, practically annuls the error that accumulates in successive differences when the tabulated function has a certain accuracy. This is an advantage over the Laplace formula. These coefficients are very useful for checking by quadrature the key values of functions whose integrals are known and for computing key values of functions defined by integrals. (Received October 29, 1943.)

42. L. R. Wilcox: *Theory of traffic light networks.* I.

Let points P_{ij} with coordinates (x_i, y_j) $(i=1, 2, \cdots, m; j=1, 2, \cdots, n)$ be given, and let a matrix function $A(t)=(a_{ij}(t))$ be given, where each $a_{ij}(t)$ is a step function with values 0, 1. The combined system $S=(P_{ij}, A(t))$ is called a rectangular system of traffic lights. Motivation of the definition lies in the obvious connection with ordinary systems of traffic lights, where $a_{ij}(t)=1$ (0) means that the light of P_{ij} at the time

t shows green (red) for the horizontal line through P_{ij} and red (green) for the vertical. Various questions are asked of S, notably those concerning the possibility of non-stop travel along all horizontal and vertical paths through the point network. Those systems S admitting "through" speeds are determined and for each such S all through speeds are found. Such problems are solved by the theory of simultaneous congruences. The results are applied to the determination of the synchronization for the "best" flow of traffic. Consideration of "almost" through speeds and more general problems in traffic control is deferred to a later paper. (Received October 19, 1943.)

43. František Wolf: *On a problem of L. M. K. Boelter. Viscous fluid forced through a heated vertical pipe.*

The temperature at the wall is prescribed to be $t^\circ = \lambda \coth mz + \mu$, independent of θ. The density and viscosity are analytic functions of t. There are four differential equations of which three are of the second order. Solving Stokes' hydrodynamical equations for a cylinder, we obtain a set of integro-differential equations which can be used for successive approximations. The chief difficulty is in determining the boundary value of the pressure which, generally, is determined by an integral equation. After some preliminary changes of the differential system, the Laplace transformation can be applied. This leads to an algebraic equation for the transform of the boundary value of the pressure. A later paper will be devoted to the study of the convergence of the successive approximations and to the problem of the existence of the solution. (Received October 28, 1943.)

Ergodic Theory

44. P. R. Halmos: *In general a measure preserving transformation is mixing.*

The first proof is given of the old standing conjecture announced in the title. "In general" means of course that the exceptional set is of the first category in one of the usual natural topologies (the strong neighborhood topology) for measure preserving transformations. The principal new and quite surprising fact used in the proof is that for any almost nowhere periodic transformation T the set of all conjugates of T, that is, the set of all STS^{-1}, is everywhere dense. (Received October 26, 1943.)

Geometry

45. V. G. Grove: *The transformation of Čech.*

The purpose of this paper is to give a simple geometric construction of the general transformation of Čech. This is accomplished by first constructing a two parameter family of quadrics having second order contact with a surface and associated in a simple manner with a conjugate net on the surface. The polar plane of a point on the tangent to a curve of the net with respect to a quadric of the family is related to that point by a general transformation of Čech. Geometrical characterizations are made for several particular transformations of Čech. (Received November 19, 1943.)

46. Janet MacDonald: *Conjugate nets in asymptotic parameters.*

This paper presents some contributions to the projective differential geometry of conjugate nets in asymptotic parameters on an analytic nonruled surface in ordinary space. The equation of the bundle of quadrics each of which has contact of at least