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A. MALCEW

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UNIVERSITY OF ILLINOIS

A SUBSTITUTE FOR THE AXIOM OF CHOICE

A. D. WALLACE

The following result appeared in the 1914 edition of Hausdorff's *Mengenlehre*, p. 140:

(A) *Any partially ordered system contains a maximal simply ordered subsystem.*

This theorem is well known to be equivalent to the axiom of choice (though there does not seem to be a proof of this fact in the literature) and it has been suggested as an alternative for this axiom. The purpose of this note (which is purely methodological) is to propose a simpler but equivalent formulation of (A) as a substitute for the Zermelo axiom. The simplicity lies in the fact that we make no assumptions concerning the relation R which replaces partial order.

Let Q be a set and R an *arbitrary* binary relation on Q . A subset of Q will be termed *R -simple* if for any pair of its elements, a and b , we have either aRb or bRa . The version of (A) we propose is:

(B) *Any R -simple subset of Q is contained in a maximal R -simple subset of Q .*

It is clear that (B) implies (A). Conversely, let Q_0 be an R -simple subset of Q . Let P be the partially ordered (by inclusion) system composed of all R -simple subsets of Q which contain Q_0 . Then by (A) there is a maximal simply ordered subsystem P_0 of P . The union of all the sets in P_0 is the desired maximal R -simple subset.

THE UNIVERSITY OF PENNSYLVANIA

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