ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

103. Leon Alaoglu and Paul Erdös: *On highly composite and similar numbers.*

A number n is highly composite if for all $m<n$, $d(m)<d(n)$; superabundant if for all $m<n$, $\sigma(m)/m<\sigma(n)/n$; and highly abundant if for all $m<n$, $\sigma(m)<\sigma(n)$. If q^k is the highest power of the prime q dividing the highly abundant number n, it is found that with at most $c(\epsilon)\ \log_2 n/\log_3 n$ exceptions, $(1-\epsilon)\log n\ \log_2 n/q<q^k \log q <(1+\epsilon)\log n\ \log_2 n$. The superabundants satisfy a stronger inequality with no exceptions, and a similar formula is proved for the highly composite numbers. For these two latter classes the inequalities determine the prime power factors almost uniquely. Highly composite numbers were introduced by Ramanujan. Pillai's D.Sc. thesis may contain related results, but it was never published and is inaccessible. (Received February 14, 1944.)

104. S. P. Avann: *Relations between join-irreducibles and meet-irreducibles in a modular lattice.* I.

Let r and r' be the respective orders of the partially ordered sets P of join-irreducible elements and P' of meet-irreducible elements in a finite modular lattice L. It is proved that $r=r'$, if certain restrictions are made upon the quotient sublattice structure of L. A reasonable conjecture is that $r=r'$ without restriction, since P and P' are even isomorphic when L is distributive. In a complete set Q_i of projective prime quotients of L the number of minimal quotients, characterized by join-irreducible numerators, is equal to the number of maximal quotients, characterized by meet-irreducible denominators, if and only if $r_i=r_i'$ for the simple homomorphic image L_i of L corresponding to Q_i. (Received March 29, 1944.)

105. Reinhold Baer: *Groups without proper isomorphic quotient groups.*

This paper contains a discussion of groups G with the following property: If N is a normal subgroup of G, and G and G/N are isomorphic groups, then $N=1$. (Received February 2, 1944.)

106. Garrett Birkhoff: *Subdirect products in universal algebra.*

It is proved that any abstract algebra A (in a very general sense) is a subdirect union of subdirectly irreducible algebras. This theorem contains as special cases various representation theorems due to Kothe, Stone, McCoy and Montgomery, and the author. (Received March 6, 1944.)

107. R. H. Bruck: *Quasigroups with the inverse property*. I. Preliminary report.

If M is an arbitrary multiplicative system with a single-valued multiplication, the left associator of M is the set of elements a such that $a \cdot xy = ax \cdot y$ for all x, y of M. The middle and right associators are similarly defined. For isotopic systems with units the corresponding associators are isomorphic. The three associators coincide for an I. P. loop. If Q is a quasigroup and U, V are one-to-one mappings of Q upon itself, let $Q(U, V)$ be the principal isotope defined by $xoy = x^U \cdot y^V$. An intensive study is made of the following questions: (I) If Q is either an I. P. loop or a totally symmetric quasigroup, when is $Q(U, V)$ an I. P. loop? (II) If Q is a T. S. quasigroup, when is $Q(U, V)$ a T. S. quasigroup? (III) If Q is an I. P. loop, when is $Q(U, V)$ an I. P. quasigroup? The theory of Moufang loops has intimate connections with (I), and the theory of autotopisms of a loop, with (III). (Received February 28, 1944.)

108. R. H. Bruck: *Simple quasigroups*.

This note is mainly concerned with a proof of the following conjecture, due to A. A. Albert: There exist simple loops of every finite order except order four. The proof is by graphical construction of so-called hyperabelian loops of every composite order different from 4. Other constructions are also considered. A concluding section contains the proof of a theorem on arbitrary quasigroups which might be stated thus: Let Q be a quasigroup homomorphic to a quasigroup R, and let Q_0 be a loop isotopic to Q. Then there exists a loop R_0, isotopic to R, such that Q_0 is homomorphic to R_0. (Received February 28, 1944.)

109. W. B. Fite: *The degree of a linear homogeneous group*.

The problem considered in this paper is that of the relation between the degree of a linear homogeneous group and the abstract properties of the group. In the first place the theorem of Frobenius states that the degree of an irreducible group is a divisor of the order of the group; then the generalization of this by Schur states that the order of the group is divisable by the product of the degree and the order of the central. Later the author called attention to the importance of the commutators of the group in this connection by showing that the degree of an irreducible group is not less than the order of any invariant commutator. The present paper starts from this point and shows that the order of certain non-invariant commutators also has a bearing on the degree of the group whether it be reducible or irreducible. Moreover for groups of a certain category the degree is a multiple of the order of the whole commutator subgroup. (Received February 10, 1944.)

110. Alfred L. Foster and B. A. Bernstein: *A dual-symmetric definition of field*.

In another paper the groundwork was laid for a dual-symmetric treatment of commutative rings with unit—a symmetric treatment exhibiting a ring-duality theory which subsumes the duality theory of Boolean algebra. In the present paper this dual-symmetric method is specialized to the case of fields, yielding a definition of field in terms of two groups which play perfectly symmetrical roles. (Received March 24, 1944.)

111. R. D. James: *Power series expansions for doubly periodic functions of the second kind.* Preliminary report.

It is well known that results on the representation of integers as sums of squares can be obtained by comparing coefficients in the expansions in power series and trigonometric series of the Jacobian elliptic functions (E. T. Bell, J. Reine Angew. Math. vol. 163 (1930) pp. 65–70). Trigonometric series expansions for the doubly periodic functions of the second kind $\vartheta_1' \vartheta_a(x+y)/\vartheta_b(x)\vartheta_c(y)$ are also well known (E. T. Bell, Trans. Amer. Math. Soc. vol. 22 (1921) pp. 198–219) but the corresponding double power series expansions do not seem to have been considered. These expansions are obtained in the present paper. The coefficients turn out to be polynomials in the usual ϑ constants ϑ_2, ϑ_3, and an additional one $\vartheta_0''/\vartheta_0$. In a later paper the arithmetical applications will be considered. (Received March 27, 1944.)

112. S. A. Jennings: *A note on chain conditions in nilpotent rings and groups.*

Let R be a group and let Ω be any set of operators of R which contains all inner automorphisms of R. We say that R is *Ω-nilpotent* if there exists in R a strictly decreasing chain of Ω-subgroups: $R=R_1 \supset R_2 \supset \cdots \supset R_M \supset R_{M+1}=1$, such that, if $a_i \in R_i$, and $\lambda \in \Omega$, then $a_i^{-r}(\lambda a_i) \in R_{i+1}$, $i=1, \cdots, M$, where in general r is any integer, but if λ is an inner automorphism r equals one. For Ω-nilpotent groups we prove that the maximal or minimal condition for Ω-subgroups implies the corresponding condition for all subgroups. Nilpotent groups, and nilpotent rings and algebras, can be considered as special types of Ω-nilpotent groups, and it follows that in any nilpotent group the maximal or minimal condition for normal subgroups implies the corresponding condition for all subgroups, while in any nilpotent ring or algebra the maximal or minimal condition for two-sided ideals implies the corresponding condition for modules. (Received March 24, 1944.)

113. G. K. Kalisch: *Generalized Hilbert spaces over fields with a non-Archimedean valuation.*

Let F denote an algebraically closed field complete with respect to a non-Archimedean valuation. Consider a complete normed vector space S over F which satisfies the following conditions: I. $|a|=0$ if and only if $a=0$, $|fa|=|f|\,|a|$, $|a+b| \leqq \max(|a|, |b|)$ for all $f \in F$, $a \in S$, $b \in S$. II. There is defined in S a symmetric bilinear inner product (a, b) for all $a \in S$, $b \in S$, such that (1) $|(a, b)| \leqq |a|\,|b|$ for all $a \in S$, $b \in S$; (2) if $T \subset S$ is a finite set then for every $a \in S$ linearly independent of T and orthogonal to T there exists $x \in S$ orthogonal to T such that $|(a, x)|=|a|\,|x|$, $|(x, x)|=|x|^2$; (3) if $R \subset S$ is a countably infinite set such that $(r_i, r_j)=S_{ij}$, $|r_i|=1$, then $\lim (a, r_i)=0$ for all $a \in S$. It is proved that S is isomorphic with a space of countable sequences $f=(f_i)$ of elements $f_i \in F$ such that $\lim f_i=0$, $|f|=\max(|f_i|)$, $(f, g)=\sum f_i g_i$. Closed manifolds, orthogonal manifolds, and projections are defined and it is proved that S is the direct sum of any closed manifold and its orthogonal complement. The usual theorems regarding projections and closed manifolds are found to be true. These results lead to investigations to be pursued in subsequent papers. (Received April 1, 1944.)

114. Knox Millsaps: *On powers of third and fourth order matrices.*

Explicit formulae for arbitrary powers of square matrices of the third and fourth orders are obtained by using Fantappie's definition for an analytic function of a matrix. The formulae are somewhat complicated and contain Waring functions of the first and second kinds. As a result of the aforementioned complications some approximate expressions are given for large powers. (Received April 1, 1944.)

115. Albert Newhouse: *On finite extending groups.*

In this paper it is shown that the elements of a finite extending group G of an algebra A of order n over a field F have representations in the total matrix algebra of degree n over F which are matrices whose minimum functions of degree less than or equal to n are divisors of some x^m-1. Furthermore it is shown that there exist extending groups for any algebra of order $n>2$ which cannot be regarded as permutation groups for any basis of the algebra (a problem proposed by A. A. Albert, Ann. of Math. vol. 43 (1942) pp. 685–723). (Received February 17, 1944.)

116. M. F. Smiley: *An application of lattice theory to quasigroups.*

It is shown that O. Ore's generalized Jordan-Hölder theorem (*Chains in partially ordered sets*, Bull. Amer. Math. Soc. vol. 49 (1943) pp. 558–566) applies in the theory of loops (A. A. Albert, *Quasigroups.* I, Trans. Amer. Math. Soc. vol. 54 (1943) pp. 507–519) to yield the Jordan-Hölder theorems recently proved by A. A. Albert (*Quasigroups.* II, abstract 50-1-1). The proof is based on the fact that the normal divisors of A. A. Albert are the subloops H of a loop G which commute and associate with the elements of G in the sense that $xH=Hx$, $(xy)H=x(yH)$, $(xH)y=x(Hy)$, $(Hx)y=H(xy)$ for every x, $y\in G$. (Received April 1, 1944.)

117. R. M. Thrall: *On modular representations induced by ordinary representations.*

Let G be a finite group, and R any representation of G by matrices over a field of characteristic zero. After suitable number theoretic preparation one can replace each matrix in R by its residue class, a matrix in a field of prime characteristic. The set, S, of matrices thus obtained is called an *induced representation* of G. Not all modular representations can be induced. In the present note it is proved if any induced representation is split into constituents, each corresponding to a single minimal non-nilpotent two-sided ideal of the modular group ring, Γ_p, of G, then each such constituent is an induced representation. The main step in the proof is a lemma stating that any idempotent in the center of Γ_p is "induced" by an idempotent in the center of the group ring of G formed relative to a suitable p-adic closed field. (Received March 27, 1944.)

118. Bernard Vinograde: *On properties of completely primary rings.*

A cleft ring is defined as the sum of a semisimple ring and a radical. Let L, T, and R stand for composition series of left, two-sided, and right ideals respectively. Let $C=K+N$ be a completely primary (or primitive) cleft ring with minimum condition on left ideals (ML). Then $C=\sum_i^l Ku_i$ and has a matric representation of finite degree with coefficients from K. If K has only a finite number n of isomorphisms into a proper subset of itself and if C has an LT series, then MR is implied. If $n=1$ then C has an LR series and $C=\sum Ku_i=\sum u_iK$ with $u_ik=k^{S_i}u_i$ where S_i is an automorphism. If S_i is inner then C is the K envelope of an algebra over the center of K. When C is an algebra its L and R series have equal length, so an LT series implies an LR series. More generally, for any cleft C with ML and MR, an LT series implies an LR series.

Uncleft rings are extensions of cleft rings, and the study of extensions naturally starts with inseparable fields. (Received April 1, 1944.)

119. John Williamson: *Hadamard's determinant theorem and the sum of four squares.*

A square matrix H of order n is called an Hadamard or an H-matrix (Jacques Hadamard, *Résolution d'une question relative aux déterminants*, Bull. Sci. Math. (2) vol. 17 (1893) Part I, pp. 240–246) if each element of H has the value ± 1 and if $|H|$ has the maximum possible value $n^{n/2}$. In the first part by an adaptation of methods used by R. E. A. C. Paley, *On orthogonal matrices*, Journal of Mathematics and Physics, Massachusetts Institute of Technology, vol. 12 (1933) pp. 311–320, it is shown that: (1) if there exists an H-matrix of order $m>1$, there exists an H-matrix of order $m(p^h+1)$ where p is an odd prime, and (2) there exists an H-matrix of order $N(N-1)$ where $N=2^t k_1 k_2, \cdots, k_r$ and $k_i=p_i^{h_i}+1\equiv 0 \bmod 4$, p_i an odd prime. In the second part it is shown that an H-matrix of order $4n$ exists if there exist four polynomials $A_i(x)=\sum_{j=0}^{n-1} a_{ij}x^j$, $i=1, 2, 3, 4$, satisfying the following conditions: $a_{ij}=a_{i,n-j}=\pm 1$, $\sum_{i=1}^{4}(A_i(\omega))^2=4n$ for every nth root ω of unity. Such polynomials $A_i(x)$ are determined for specific small values of n and in particular for $n=43$ thus showing the existence of an H-matrix of order 172, a result not previously known. (Received March 22, 1944.)

ANALYSIS

120. R. P. Agnew: *Summability of subsequences.*

If A is a regular (real or complex) matrix method of summability and x_n is a bounded complex sequence, then there exists a subsequence y_n of x_n such that the set L_Y of limit points of the transform Y_n of y_n includes the set L_x of limit points of the sequence x_n. (Received February 2, 1944.)

121. E. F. Beckenbach and Maxwell Reade: *Further results on mean-values and harmonic polynomials.*

In this paper the authors study the relation between the "vertex averages" used by Walsh (J. L. Walsh, Bull. Amer. Math. Soc. vol. 42 (1936) pp. 923–930) and the "peripheral" and "areal averages" used by Beckenbach and Reade (E. F. Beckenbach and Maxwell Reade, Trans. Amer. Math. Soc. vol. 53 (1943) pp. 230–238). From the relation noted it follows that some of the results of Walsh are equivalent to those obtained by Beckenbach and Reade, and moreover, by following the methods outlined by the latter authors, it is possible to extend Walsh's results to more general "vertex averages." (Received March 27, 1944.)

122. E. F. Beckenbach and Maxwell Reade: *Regular solids and harmonic polynomials.*

Suppose D is a domain containing a regular solid V_0 and ϕ is a class of functions $f(x, y, z)$ defined and continuous on D. It is assumed that if V is similar and parallel to V_0 then the value of $F(x, y, z)$ at the center of V is the mean of values of $f(x, y, z)$ at the vertices. The class ϕ is shown to consist of certain harmonic polynomials. For the five regular solids these classes are given in terms of three spherical harmonics and their partial derivatives. The solution of the problem, suggested by J. L. Walsh (Bull. Amer. Math. Soc. vol. 42 (1936) pp. 923–930), of determining the class of