introduced in an early step may be magnified to such an extent in succeeding steps that the final result is useless. Iterative methods to meet this difficulty have been reviewed very completely by Hotelling. In this paper a different approach is taken. Conditions on the norm of a matrix are determined so that a Doolittle process will not magnify errors to more than two or three decimal places. It is then pointed out that if an approximation to the inverse of the matrix is available, most problems can be rearranged so that the required norm conditions are met. A Doolittle process may then be used to any number of decimal places with assurance that errors will not accumulate to more than a limited number of decimal places. (Received July 8, 1944.)

252. Abraham Wald: *On cumulative sums of random variables.*

Let $\{z_i\}(i=1, 2, \cdots$ ad inf.) be a sequence of independent random variables each having the same distribution. Denote by Z_j the sum of the first j elements of the sequence. Let $a>0$ and $b<0$ be two constants and denote by n the smallest integer for which either $Z_n \geqq a$ or $Z_n \leqq b$. Neglecting the quantity by which Z_n may differ from a or b (this can be done if the mean value of $|z_i|$ is small), the probability that $Z_n \geqq c$ for $c=a$ and $c=b$ is derived, and the characteristic function of n is obtained. The probability distribution of n when z_i is normally distributed is derived. These results have application to various statistical problems and to problems in molecular physics dealing with the random walk of particles in the presence of absorbing barriers. (Received July 8, 1944.)

253. Abraham Wald and Jacob Wolfowitz: *Statistical tests based on permutations of the observations.*

It was pointed out by Fisher that statistical tests of exact size, based on permutations of the observations, can be carried out without assuming anything about the underlying distributions except their continuity. Scheffé has proved that, for an important class of hypotheses, these tests are the only ones with regions of exact size. Tests based on permutations of the observations have been constructed by Fisher, Pitman, Welch, and the present authors. In the present paper, the authors prove a theorem on the limiting normality of the distribution, in the universe of permutations, of a class of linear forms. Application of this theorem gives the limiting normality (in the universe of permutations, of course) of the correlation coefficient, and of a statistic introduced by Pitman to test the difference between two means. The limiting distribution of the analysis of variance statistic in the universe of permutations is also obtained. (Received July 8, 1944.)

Topology

254. Samuel Eilenberg and N. E. Steenrod: *Axiomatic approach to homology.*

The homology (and cohomology) groups are studied starting from a system of axioms, fulfilled by all the homology theories usually considered. It is shown that in the case of complexes, more generally, in the case of absolute neighborhood retracts, the axioms have a unique interpretation. A similar discussion is carried out for products. (Received July 7, 1944.)

255. Mariano Garcia: *Orbit-components and component orbits.*

Let $f(X)=X$ be a homeomorphism, where X is compact and metric. The *orbit-*

component of a point $x \in X$ is all points y of X such that, given neighborhoods of x and y, some orbit intersects both. The homeomorphism f is *uniformly pointwise almost periodic* if f is pointwise recurrent and the gaps between iterates returning to a neighborhood are uniformly bounded for all points of X. Among other results the author proves: (1) f is uniformly pointwise almost periodic if and only if the orbit-components are identical with the orbit-closures. (2) Let f be pointwise recurrent and let $G_1, G_2, \cdots$ be a sequence of orbits or component orbits (see Bull. Amer. Math. Soc. vol. 50 (1944) p. 260) with non-empty limit inferior k and limit superior L; then (a) if M is any closed invariant set in L which intersects k, every component of L intersects M; (b) L is the closure of a component orbit. (3) If f is pointwise recurrent, (a) every orbit-component is the closure of a component orbit; (b) f is uniformly pointwise almost periodic if each component of X is contained in the orbit-closure of any of its points. (Received July 6, 1944.)

256. W. H. Gottschalk: *A note on pointwise nonwandering mappings.*

Let X be a topological space, that is, a space satisfying the first three postulates of a Hausdorff space. Let $f(X) \subset X$ be a continuous mapping and let $h(X) = X$ be a homeomorphism. In the terminology of Birkhoff, it is said that $x \in X$ is *nonwandering* (n.) under f provided that to each neighborhood U of x there correspond infintely many positive integers n such that $U \cdot f^n(U) \neq \Lambda$; also, f is *pointwise nonwandering* (p.n.) provided that each point of X is n. under f. The following theorems are proved. (1) If f is p.n., then so also is f^k for every positive integer k. (2) If A and B are closed and connected sets such that $A + B = X$ and $A \cdot B = x \in X$, if $A \cdot h(A) \neq \Lambda$ and $B \cdot h(B) \neq \Lambda$, and if x is n., then x is fixed. (3) If X is connected and if h is p.n., then each cut point of X is periodic. These results allow a weakening of hypothesis from pointwise recurrence to p.n. in several theorems of Ayres and Whyburn on the behavior of cyclic elements under a homeomorphism. (Received July 5, 1944.)

257. W. H. Gottschalk: *Totally disconnected sets and almost periodic properties.*

Let X be a metric space with metric ρ, let $f(X) \subset X$ be a continuous mapping and let $h(X) = X$ be a homeomorphism. The mapping f is said to be *pointwise recurrent* (p.r.) provided that if $x \in X$, then to each $\epsilon > 0$ there corresponds a positive integer n such that $\rho(x, f^n(x)) < \epsilon$. The mapping f is said to be *uniformly pointwise almost periodic* (u.p.a.p.) provided that to each $\epsilon > 0$ there corresponds a positive integer N so that if $x \in X$, then in every set of N consecutive positive integers appears an integer n such that $\rho(x, f^n(x)) < \epsilon$. In the usage of Whyburn, it is said that h is *pointwise regularly almost periodic* (p.r.a.p.) provided that if $x \in X$, then to each $\epsilon > 0$ there corresponds a positive integer n such that $\rho(x, f^{mn}(x)) < \epsilon$ $(m = 1, 2, \cdots)$; also, h is *regularly almost periodic* (r.a.p.) provided that to each $\epsilon > 0$ there corresponds a positive integer n such that if $x \in X$, then $\rho(x, f^{mn}(x)) < \epsilon$ $(m = 1, 2, \cdots)$. The following theorems are proved. (1) If X is compact and totally disconnected and if f is p.r., then f is u.p.a.p. (2) If h is p.r.a.p., then each orbit-closure is totally disconnected. (3) If X is a compact two-dimensional topological manifold (with or without boundary) and if h is r.a.p., then h is periodic. (4) If X is compact and if h is both p.r.a.p. and u.p.a.p., then h is r.a.p. (5) If X is compact and totally disconnected and if h is p.r.a.p., then h is r.a.p. (Received July 5, 1944.)

258. S. B. Myers: *Arcs and geodesics in metric spaces.*

In a metric space M, a geodesic arc is defined to be the shortest rectifiable arc joining its end points. Menger has shown that if bounded sets are compact in M, then any two points joinable by a rectifiable arc are joinable by a geodesic. In this paper it is proved that the same conclusion holds under the weaker hypotheses of local compactness and almost completeness of M; the method involves a proof of the existence of a limiting curve in the sense of Fréchet for a family of curves of bounded length in M. Applications of these results are made to the class of *geodesic metric spaces*; that is, rectifiably connected metric spaces in which distance is identical with the greatest lower bound of arc length. This class contains all convex, complete metric spaces. The class of locally compact geodesic metric spaces contains all symmetric Finsler manifolds, in particular all Riemannian manifolds. A series of results obtained by Hopf and Rinow for Riemannian manifolds are here generalized to locally compact geodesic metric spaces; for example, in a locally compact geodesic metric space completeness is equivalent to the compactness of bounded sets. (Received June 13, 1944.)

259. A. W. Tucker: *Antipodal-point theorems proved by an elementary lemma.*

Let K be the regular simplicial n-complex determined in Euclidean $n+1$-space by $\sum|x_a|=1$, $a=1, 2, \cdots, n+1$, and let K' be any simplicial subdivision of K that has central symmetry. Then, if K' is mapped simplicially into K so that antipodal vertices go into antipodal vertices, each n-simplex of K is the image of an odd number of n-simplexes of K'. This lemma can be proved by elementary means. It implies the following antipodal-point theorems which imply and are implied by the antipodal-point theorems of Lusternik-Schnirelmann and Borsuk-Ulam (Alexandroff-Hopf, *Topologie*, pp. 487, 486). 1. If non-null vectors in Euclidean $n+1$-space are distributed continuously throughout an $n+1$-disk ($\sum x_a^2 \leqq 1$), there is some pair of antipodal boundary-points whose vectors are parallel. 2. The nerve of a covering of an n-sphere ($\sum x_a^2=1$) by $n+1$ pairs of disjoint closed sets, no one of which contains two antipodal points, is the join of $n+1$ pairs of vertices (represented by K, above). 3. A covering of an n-disk ($\sum x_i^2 \leqq 1$, $i=1, 2, \cdots, n$) by $n+1$ closed sets, no one of which contains two antipodal boundary-points, has order $n+1$. 4. If an $n-1$-sphere ($\sum x_i^2=1$) is covered by $n+1$ closed sets, no one of which contains two antipodal points, the intersection of any p ($\leqq n$) of the sets contains some point whose antipode belongs to the intersection of the remaining $n-p+1$ sets. (Received July 5, 1944.)