ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial num- of the abstract.

Algebra and Theory of Numbers

1. H. W. Becker: *The composite umbra theorem.*

Let $U, V, W, \cdots$ be different umbrae, and let $[\ ]$ confine a polynomial umbra. It is well known that $[U+V]_n=(U+V)^n$, based on the generator equation, $e^{[U+V]}=e^U\cdot e^V\cdot[UV]_n$, needs analogous definition. $e^{[UV]}=e^{UV}=(e^{eU-1})^{(V)}=e^{U\epsilon^*}(V)$, where $(V)_n$ is a Jordan factorial, and $[U\epsilon]_n=\phi_n(U)$ is the exponential polynomial of E. T. Bell (Ann. of Math. vol. 35 (1934) p. 263). Then $[UV]_n=[U\epsilon^*(V)]_n=\phi_n(U^*V)$, where * means that every term in U of weight m is multiplied by $(V)_m$. This is the composite umbra theorem. Such asymmetric composition is in general commutative, associative and distributive only for scalars, or for umbra iterates and inverses (calculated from $UU=U^{(2)}$, $UU^{(-1)}=1$, and so on). These decompositions greatly simplify, if $U_0=U_1=1$. Or they may be generalized, to an umbra form $[fU]_n$, where f is any function of any number of umbrae. Where the U are scalars, $[fU]_n$ reduces to $(fU)^n$, conveniently verifying the theorem and its consequences. The extension to any number of factors, $[UVW\cdots]$, is in close parallelism with the iterated exponential integers of E. T. Bell (Ann. of Math. vol. 39 (1938) p. 539), the classic instance. (Received October 28, 1944.)

2. H. W. Becker: *The hyper-umbra theorem.*

An umbra U is the representative of a series $U_0, \cdots, U_n, \cdots$. The umbra of an umbra, and so on, to m dimensions, or blanks, is called a hyper-umbra, and written ${}^mU={}^mU\{,\cdots,\}$. The fundamental umbra is ϵ, of generator $e^{e^t-1}=e^t\epsilon$. Its property $(\epsilon)=1$, where $(\ )$ is a Jordan factorial, underlies the new operational transformation $e^E=eE^\epsilon$ in the finite difference calculus. Application to an umbra yields $e^{E_0}U\{0\}=eU\{\epsilon\}$. The classic instance is Dombinski's theorem, in the form $e^E0^r=e\epsilon_r$. The operation may be iterated, along each dimension of a hyper-umbra. Denote by ${}^me$ a continued exponential of the mth order. Then ${}^me^{E_0m}U\{0,\cdots,0\}={}^me^{mU}=e^{m\cdot m}U\{\epsilon,\cdots,\epsilon\}$. This is the hyper-umbra theorem. Where ${}^mU=Y$ is the cubic array whose typical cell is $(\overline{UZ}+X)^n$, this gives ${}^3e^Y=\exp\exp\exp Y=e^e\cdot e^{e^U}$. Where ${}^mU=W$ is the square array of cells $\overline{UZ}_n=(U+\cdots+U)^n$ to Z U's, $e^{e^W}=e^{e^U}$. This is remarkable, in that the part is equipotent to the whole. If $U=1=$ the identity umbra, then $W=M=$ the table of all integer powers. Thus the power matrix is equipotent to unity. The theorem generalizes to ${}^me^{TE_0m}U\{0,\cdots,0\}={}^me^{mU\cdot T}=e^{mT^*}\{\epsilon T,\cdots,\epsilon T\}$, where * denotes scalar or subscript multiplication according as T is ordinary or umbral. (Received October 28, 1944.)

3. R. H. Bruck: *Quasigroup theory*. IV. *Associatral series*. Preliminary report.

For a loop G, finite or with suitable chain conditions for normal subloops, define the *associator* $A = A(G)$ to be the maximal normal subloop of G contained in the three associators of G (see abstract 50-5-107). If H is any normal subloop of G define (i) $U = U(H)$ to be the unique normal subloop of G such that $U/H = A(G/H)$, and (ii) $L = L(H)$ to be the minimal normal subloop of G such that $U(L) \supset H$. Associatral series, and in particular the upper and lower associatral series, may be defined analogously to central series. If any associatral series extends from 1 to G so do the upper and lower series, and the latter have equal length a, the associatral class of G. The *associatizer* $L(G)$ may be used for a theory of associatral solubility analogous to ordinary (central) solubility. (Received November 6, 1944.)

4. C. J. Everett: *The basis theorem for vector spaces over rings.*

A vector space of n basis elements over a ring with unit has the property that every proper subspace has a basis of at most n elements if and only if the ring has no zero divisors and is a right-principal-ideal ring. This perfects Theorem F of the author's *Vector spaces over rings*, Bull. Amer. Math. Soc. vol. 48 (1942) pp. 312–316. (Received October 26, 1944.)

5. C. J. Everett and S. M. Ulam: *Projective algebra*. I.

For a subset A of a direct product (X, Y) of two sets, define $x(A)$ as the set of all (x, y_0) for which there exists a y such that $(x, y) \in A$, and $y(A)$ similarly, where (x_0, y_0) is a fixed point of (X, Y). Define the direct product $A \times B$ for $A \subset (X, y_0)$, $B \subset (x_0, Y)$ in the usual way. If a boolean algebra $\mathfrak{B}$ of subsets of $(X, Y) = I$ is closed under projection and product, it is called a *projective algebra of* (X, Y) and has properties 1. $x(A \cup B) = x(A) \cup x(B)$; 2. $xy(I) = (x_0, y_0) = yx(I)$ is an atom in $\mathfrak{B}$; 3. $x(A) = 0$ if and only if $A = 0$; 4. $xx(A) = x(A)$; 5. $x(A \times B) = A$, $y(A \times B) = B$, and $A \times B$ contains all sets with these projections; 6. $(X, y_0) \times (x_0, y_0) = (X, y_0)$; 7. direct product is distributive with respect to union; with similar properties for y in 1, 3, 4, 6. An abstract boolean algebra with mappings $x(A)$, $y(A)$, and product satisfying these properties as postulates is called a *projective algebra*. It is proved that every such algebra is embeddable in a complete ordered projective algebra, and that every projective algebra of *all* subsets of a set is representable as a projective algebra of some (X, Y). (Received October 26, 1944.)

6. N. J. Fine: *Congruence properties of the elementary symmetric functions.*

Define $u_k(n)$ as the elementary symmetric function of order k in n independent variables. Let $P_n(p, k, a)$ be the probability that $u_k(n)$ be congruent to a (mod p), where p is any prime. It is proved that $\lim P_n(p, k, a) = P(p, k, a)$ exists. If s is the number of ones in the dyadic expansion of k, then $P(2, k, 1) = 1/2^s$. Simple recursion formulas are given for $P(3, k, a)$. The following general theorems are proved: (i) $P(p, k, a) = 1/p$ for $k = rp^t$, with $0 < r < p$, all p and t. (ii) $P(p, k, 0) > 1/p$ for $k = (p-1)p^t + R$, with $0 < R < p^t$. (iii) $P(p, k, 0) > C(p) > 0$ for all positive k. It is conjectured that equidistribution holds only if $k = rp^t$; that $P(p, k, 0)$ is not less than $1/p$ for all p and positive k; that $P(p, kp, a) = P(p, k, a)$; finally, that $\lim\sup P(p, k, 0) = 1$. (Received October 26, 1944.)

7. A. P. Hillman: *On derivatives of differential polynomials.*

Proofs of the following results are given. Let F be a differential polynomial effectively involving the unknowns $y_1, \cdots, y_n$. Then F holds no (essential) component of the manifold of its derivative F_1. Every component of the rth derivative F_r of F is the general solution of a differential polynomial of order at least r in at least one of the unknowns. For r sufficiently large, F_r is algebraically irreducible, its manifold is irreducible, and the perfect ideal it generates is prime. (Received November 30, 1944.)

8. Nathan Jacobson: *A problem on algebraic algebras.*

The question considered in this paper is the following one: Let $\mathfrak{A}$ be an algebra over a field Φ such that (1) every element of $\mathfrak{A}$ satisfies an equation of degree not greater than N with coefficients in Φ and (2) $\mathfrak{A}$ has a finite number of generators. Then is $\mathfrak{A}$ necessarily an algebra with a finite basis? This is a special case of a problem recently proposed by Kurosh and it is analogous to the well known Burnside problem in the theory of groups. A partial solution of the problem is obtained in this paper. It is shown that the question can be answered in the affirmative for algebras that are semi-simple in the sense that they contain no nil ideals not equal to 0. This makes it possible to reduce the problem to the more special one in which (1) is replaced by (1′) every element of $\mathfrak{A}$ satisfies the equation $a^N = 0$. It is shown also that an affirmative answer to a special case of Burnside's problem on groups would imply an affirmative answer to our problem for algebras over a field of prime characteristic. (Received October 13, 1944.)

9. Irving Kaplansky: *The commutativity of generalized Boolean rings.*

Stone's observation that a ring in which $x^2 = x$ is necessarily commutative is extended to more general rings, and in particular to the p-rings of McCoy and Montgomery (Duke Math. J. vol. 3 (1937) pp. 455–459). It is shown that a ring of characteristic p in which $x^n = x$ is commutative if $n = p^r$ with r a power of 2; and regardless of the characteristic, it is commutative if the regular polygon of $n-1$ sides is constructible by ruler and compass. The lowest case where commutativity remains in doubt is $n = 8$. (Received November 30, 1944.)

10. Fred Kiokemeister: *The Asano postulates for orders in a linear algebra.*

Let g be a domain of integrity with unique factorization of ideals into products of prime ideals, and let P be the quotient field of g. An investigation of the g-submodules of the linear algebra A over P leads to the theorem that if R is an order in A, the maximal chain condition and the modified minimal chain condition hold for regular ideals in R. The Asano postulates for an arithmetic are shown to hold in A. (Received October 25, 1944.)

11. Fred Kiokemeister and G. W. Whitehead: *A coset theory for left loops.*

A left loop is a multiplicative system Q with two-sided identity such that the equation $ax = b$ has a unique solution for every $a, b \in Q$. A subset H of Q is an admissible left subloop if it is closed under multiplication, the solution of the equation $ax = b$

is in H whenever $a, b \in H$, and $x(yH) = (xy)H$ for every $x, y \in Q$. Under these circumstances Q has an expansion in left cosets of H, and the system Q/H of left cosets is made into a left loop under a suitable definition of multiplication. The group $\mathfrak{L}$ spanned by the left multiplications (that is, the permutations $L_x(a) = ax$) of Q is introduced, and an isomorphism of Q with $\mathfrak{L}/\mathfrak{L}_0$ (where $\mathfrak{L}_0$ is the subgroup of $\mathfrak{L}$ consisting of all permutations keeping the identity E fixed) is established. It is shown that the admissible left subloops of Q are in 1:1 correspondence with the subgroups $\mathfrak{M} \supset \mathfrak{L}_0$ of $\mathfrak{L}$, and isomorphisms $H \cong \mathfrak{M}/\mathfrak{L}_0$, $Q/H \cong \mathfrak{L}/\mathfrak{M}$ are established. An extension theory is developed: given left loops H and K, a construction is given for all left loops Q such that $Q/H = K$. Necessary and sufficient conditions are given (when H is a group) that Q shall be a group, and specialization of H to be normal yields the Schreier extension theory. (Received October 20, 1944.)

12. Seymour Sherman: *Complex polynomials and polygonal domains.*

Theorems of Sturm, Routh, and Hurwitz have been generalized so as to provide a finite numerical algorithm for finding the number of such roots of a polynomial with complex coefficients as lie on a generalized polygon or linear transformation thereof. By this means a finite procedure is given for determining the number of roots of a polynomial lying in a quadrant, half-plane, circle, or circular sector. Such problems have proved of interest recently in connection with airplane flutter (S. Sherman, Jane DiPaola, and H. Frissel, *Routh's discriminant, flutter, and ground resonance*, abstract 50-7-190) and econometric business cycle analysis (P. A. Samuelson, *Conditions that the roots of a polynomial be less than unity in absolute value*, Annals of Mathematical Statistics vol. 12 (1941)). (Received October 17, 1944.)

Analysis

13. E. F. Beckenbach: *A Looman-Menchoff theorem for Newtonian vectors.*

It is shown that if the vector function $X(x, y, z)$ is continuous in the finite domain D, if except at most at the points of a denumerable set of points in D, $X(x, y, z)$ is totally differentiable in the planes parallel to the coordinate planes, and if the curl and divergence of $X(x, y, z)$ vanish almost everywhere in D, the $X(x, y, z)$ has continuous partial derivatives of all orders. (Received October 28, 1944.)

14. R. E. Fullerton: *Linear operators with range in a space of differentiable functions.*

The Banach space $C^n(0, 1)$ is defined to be the space of functions possessing n continuous derivations over the interval (0, 1) with norm $\|f\| = \text{l.u.b.}_{0 \leq t \leq 1} \text{l.u.b.}_{k \leq n} |f^{(k)}(t)|$. If $Tx = f$ is a bounded linear operator from a Banach space X to $C^n(0, 1)$, Tx is representable in the form $\bar{x}_t x$ where $\bar{x}_t$ is a function defined from (0, 1) to the space $\bar{x}$ conjugate to X. In this paper, necessary and sufficient conditions that $\bar{x}_t$ represent such an operator are found. Both bounded and completely continuous operators are investigated. Particular attention is devoted to representations of operators from sequence spaces and Lebesgue spaces to the space $C^n(0, 1)$. In all cases the expression for the norm of the operator is obtained in terms of the function $\bar{x}_t$. (Received October 20, 1944.)