$\Delta_h^2 f(x)=f(x+h)+f(x-h)-2f(x)=o(h)$ uniformly in x. If $\Delta_h^2 f=O(h)$, $f\in\Lambda^*$. If $f\in$Lip 1, then $f\in\Lambda^*$. The converse is false, since there are $f\in\Lambda^*$ nowhere differentiable. The modulus of continuity of an $f\in\Lambda^*$ is $O(\delta\log\delta)$. The class Λ^* is sometimes more natural than Lip 1. (i) A necessary and sufficient condition that the best approximation $E_n[f]=O(n^{-k})$ $(k=1, 2, \cdots)$ is $f^{(k-1)}\in\Lambda^*$. (ii) If $f\in\Lambda^*$, so does the conjugate function $\bar{f}$. (iii) Let f^α, f_α denote the αth derivative and integral of f $(0<\alpha<1)$. If $f\in\Lambda^*$, then $f^\alpha\in$Lip $(1-\alpha)$. If $f\in$Lip α then $f_{1-\alpha}\in\Lambda^*$. (iv) A necessary and sufficient condition that a harmonic function $f(r, x)$, $0\leq r<1$, be the Poisson integral of an $f\in\Lambda^*$ is $\partial^2 f(r, x)/\partial x^2=O\{1/(1-r)\}$. If $\Delta_h^2 f(x)=o(h)$ for each x, f is *smooth* $(f\in\lambda)$. A $g(x)$ defined over a set E is said to satisfy condition D, if $g(x)$ takes all intermediate values. (v) If $f\in\lambda$, $f'(x)$ exists in a dense set (Rajchman) and satisfies condition D. (vi) The sum of a trigonometric series with coefficients $o(1/n)$ satisfies condition D. (vii) If $g(x)$ is continuous, $\tilde{g}(x)$ satisfies condition D. (Received October 28, 1944.)

Applied Mathematics

24. C. H. Dix, C. Y. Fu, Mrs. E. W. McLemore: *The cubic Rayleigh wave equation.*

Consider a plane compressional wave incident on the free plane surface of a semi-infinite elastic medium. The incident and reflected compressional amplitudes are respectively A and B. Then $B/A=N(i, s)/D(i, s)$ where $s=\lambda/\mu$, $ND=16(s+1)w^3-8(3s+4)w^2+8(s+2)w-(s+2)$ and $w=\sin^2 r_1$, r_1=reflection angle of shear wave. Zeros of ND show two or no i's (also zeros of N) corresponding to no reflected compressional wave. The third zero is a zero of D and gives the reciprocal of the important solution of the Rayleigh wave cubic. The cubic curves all pass through one fixed point whose coordinates are $w=1.0957$, $ND=-1.83927$. For a fixed s, ND has the same value when $w=0$ that it has when $w=1$. If s corresponds to a small Poisson ratio then i for the larger zero of N will be very close to $90°$, giving no reflection of compressional type whereas for $i=90°$ all reflected energy is compressional. There is a discontinuity when $s=0$ where $B/A=+1$ while $B/A=-1$ for s greater than 0. (Received October 27, 1944.)

25. H. W. Eves: *A geometrical note on the isocenter.*

Consider a central projection of plane p on plane p', L being the center of projection, and adopt the convention that angular directions on p (or p') are positive if they are counterclockwise when p (or p') is viewed from L. A point on p is called a positive isocenter on p if all angles on p having the point for vertex are invariant under the projection, and a point on p is called a negative isocenter if all angles on p having the point for vertex project into equal but oppositely directed angles on p'. It is shown that a tilted photograph possesses one and only one positive isocenter and one and only one negative isocenter. These points are geometrically located on the picture. Now the positive isocenter has long been known in photogrammetry, but the existence of the negative isocenter does not seem to have been noticed before. With the combined aid of these two isocenters a simple graphical procedure is developed for rectifying a tilted photograph. The mapping process can then be continued by the method of radial plotting. (Received October 19, 1944.)

26. H. W. Eves: *Analytical and graphical rectification of a tilted photograph.*

An aerial photograph fails to be a perfect map of the ground photographed because of unavoidable tilt in the photograph and because of the presence of relief on the ground. To convert the photograph into a true map of the ground, then, it must be corrected for both tilt and relief. This correction program usually follows the order of first correcting for tilt, and then correcting that result for relief. The present paper is believed to be a new analytical and graphical treatment of the first, and more difficult, part of the program, that of correcting a photograph for map displacements induced by tilt. The graphical method devised is particularly applicable to high oblique photographs, and seems superior to the perspective grid method of the Canadian Government in that it is faster and does not require that the terrain be flat. Some linkage motions are indicated for carrying out the graphical solution on high obliques. The analytical results obtained are employed in discussing the curves which appear on parallax correction graphs. (Received November 25, 1944.)

27. Walter Kohn: *Contour integration in the theory of the spherical pendulum and the heavy symmetrical top.*

This paper is concerned with new developments of the method of complex integration as applied to the motions of the spherical pendulum and the heavy symmetrical top. Let $\phi(t)$ be the angle of azimuth at time t and Φ the advance in azimuth corresponding to the passage from the lowest of the highest level of motion. By decomposing the expression for $\phi'(t)$, the Puiseux inequality for the spherical pendulum, namely $\phi > \pi/2$, is derived in a new and very simple manner. Next, an adaption of Weinstein's vertical line of integration (Amer. Math. Monthly vol. 49 (1942) p. 521) yields a new result which includes the Puiseux inequality. Similar methods are applied to the different types of motion of a heavy symmetrical top, and bounds for Φ are determined in each case. These bounds are proved to be actually the greatest lower and least upper bounds respectively. It is found that all important characteristics of the motion of the spherical pendulum, such as those of Halphen, Puiseux and the level-inequality characteristics, are shared by the heavy top whose precession is retrograde. The spherical pendulum is a limiting case of such a top. (Received October 11, 1944.)

28. A. N. Lowan and H. E. Salzer: *Formulas for inverse interpolation within a square grid in the complex plane.*

The results obtained in the article by H. E. Salzer, *A new formula for inverse interpolation*, Bull. Amer. Math. Soc. vol. 50 (1944) pp. 513–516, are applicable to the problem of inverse interpolation of complex functions which are tabulated at equidistant intervals along any straight line in the complex plane, provided that h stands for the complex interval and p, given by (5), is a complex number from which is obtained the unknown argument as a_0+ph. But when the function is tabulated over a square grid, just as in the case of direct interpolation (see A. N. Lowan and H. E. Salzer, *Coefficients for interpolation within a square grid in the complex plane*, Journal of Mathematics and Physics, Massachusetts Institute of Technology, vol. 23 (1944) pp. 156–166), a closer approximation is obtained by choosing the points along the corners of a square. For the cases where it suffices to employ quadratic and cubic direct interpolation with the points chosen as in the latter article, formula (5) in the former article is still applicable, where for the quadratic case $r=2(f_p-f_0)/\Delta$, $s=[(-1+i)f_{-1}-2if_0+(1+i)f_1]/\Delta$, where $\Delta=(1-i)f_{-1}+2(i-1)f_0+(1-i)f_1$, $t=u=v=w=0$, and for the cubic case $r=2(f_p-f_0)/\Delta$, $s=[(-3+i)f_{-1}-4if_0+(3+i)f_1+2if_2]/\Delta$, $t=[(1-i)f_{-1}+(1+i)f_0+(-1+i)f_1+(-1-i)f_2]/\Delta$, where $\Delta=2f_{-1}+(-3+3i)f_0-2if_1+(1-i)f_2$, $u=v=w=0$. (Received October 20, 1944.)

29. F. J. Murray: *Gears, cams and differentials.*

The problem of obtaining, mechanically, the value of a function, $F(x_1, \cdots, x_m)$, without integrating or differentiating, is considered. In the machine the $x_1, \cdots, x_m$ enter as rotations of corresponding shafts and the value of F appears as a rotation. A set of gears yields an output which is a fixed multiple of its imput. The output of a differential is the average of its two imputs. A device which yields a prescribed function of one variable will be called a cam. F can be mechanized by n cams and various gears and differentials, if there exist constants, $\alpha_{i,k}, \beta_{i,j}, \gamma_k, \delta_i, i, j=1, \cdots, n$; $k=1, \cdots, m$, and n functions $J_i(t)$, such that $F=\sum_{k=1}^{m}\gamma_k x_k+\sum_{j=1}^{n}\delta_j J_j(I_j)$ for the set of I_i's defined by the system of equations $I_i=\sum_{k=1}^{m}\alpha_{i,k}x_k+\sum_{j=1}^{n}\beta_{i,j}J_j(I_j)$, $i=1, \cdots, n$. The I_i's are the imputs of the various cams. A system of partial differential equations on F which constitute necessary conditions for this are derived and sufficient conditions are obtained from a part of these when certain determinants are not zero. The difficulties of this theory arise in showing that these determinants are not identically zero. This, however, is done. It is also shown that there are analytic functions of two or more variables which cannot be mechanized by gears, cams, and differentials. (Received November 13, 1944.)

30. F. J. Murray: *On solving a set of n linear equations.*

Consider a system of n linear equations in n unknowns, $\sum_k a_{ik}x_k=b_i$. If we form the function $F_i(x_1, \cdots, x_n)=\sum_i(b_i-\sum_k a_{i,k}x_k)^2$ then the gradient of F at $(0, \cdots, 0)$ lies along the direction in which x_k is negatively proportional to $\sum_i b_i a_{i,k}$. Consequently the vectors $(y_1, \cdots, y_n)$ with $y_k=\lambda\sum_i b_i a_{i,k}$ lie along a path which is initially one of quickest descent for F. If λ is chosen so as to make F a minimum along this path, an approximation for the solutions of the given system of equations is obtained. This can be used as a step of an iterative process for solving the equations. The error of the nth iterate depends on the dispersion of the reciprocals of the characteristic roots of the matrix, $(\sum_j a_{i,j}a_{j,k})_{i,k=1,\cdots,n}$. The gradient notion can also be used to design machines with very desirable characteristics for solving such systems. (Received October 23, 1944.)

31. Isaac Opatowski: *A botanical application of Jensen's inequality.*

The paper is a mathematical elaboration of some ideas connected with the theory of form of plants of N. Rashevsky (Bulletin of Mathematical Biophysics vol. 5 (1943) pp. 33–47, 69–73). Its purpose is to show that the function of branches of a tree, which is usually explained on exclusively biological bases, may be also interpreted from a mechanical viewpoint, as some kind of tendency of the tree to shape itself in such a manner as to increase its own strength. The theory is based on some experimental relations established by the U. S. Forest Products Laboratory, and consists of a comparison of a tree (T) of n branches of length L_i $(i=1, \cdots, n)$ with a hypothetical tree (H) of one branch only which from a purely biological viewpoint is identical with (T), except for a different shape. The ratio of the maximum bending stress in the branches of (T) to a similar stress in (H) is $(\sum_i L_i^3)^5/(\sum_i L_i^5)^3$, which is greater than 1, by Jensen's inequality. The paper is a part of an article to be published in the December 1944 issue of the Bulletin of Mathematical Biophysics. (Received October 18, 1944.)

32. Ida Roettinger: *An operational approach to the solution of boundary value problems by generalized Fourier series.*

The idea of G. Doetsch (Math. Ann. vol. 112 (1935) pp. 52–68) to consider the set of Fourier coefficients of a function as the finite Fourier transformation of this function is applied to the set of coefficients in the expansion of a function according to characteristic functions of certain Sturm-Liouville problems. Results known for the sin nx- and cos nx-transformations (see H. Kniess, Math. Zeit. vol. 44 (1938) pp. 266–291; see also the author's abstract 50-5-155) are extended to these more general transformations. The main idea in this generalization is the introduction of almost periodic functions in the form of "almost periodic extensions" of a function defined on a finite interval. Some applications of these transformations to boundary value problems are given. In particular the solution of the problem of an elastically supported string is expressed in closed form by means of an almost periodic extension; this form of solution gives an intrinsic characterization of the motion of the string. (Received October 17, 1944.)

33. W. H. Roever: *Second derivatives of the potential function of the earth's weight field of force.*

The author shows that, in considering the statical and dynamical phenomena which take place in the earth's weight field of force, both the first and the second partial derivatives of the potential function of this field of force come into play. The moment actuating the Eötvös torsion balance involves some of these derivatives, and these are thus experimentally determined and found to considerably exceed in value those corresponding to the generally accepted potential function used in geodesy. Those second derivatives which occur in the curvature of the lines of force are of the order of magnitude of ω^2, where ω is the angular velocity of the earth's rotation. However, those which occur in the curvatures of the level surfaces are considerably larger and come into play in exterior ballistics if the convergence of the verticals is considered. The differential equations of motion of a trajectory are also derived using for the potential functions of the weight field an analytic approximation involving both the first and second derivatives of this function. In setting up the frame of reference in exterior ballistics it is shown that ω^2 *cannot* be neglected. (Received November 24, 1944.)

34. H. E. Salzer: *Inverse interpolation for eight-, nine-, ten-, and eleven-point direct interpolation.*

In the paper *A new formula for inverse interpolation*, Bull. Amer. Math. Soc. vol. 50 (1944) pp. 513–516, the author obtained all the terms involving the first six powers of $(f_p - f_0)$ in the expansion for p in terms of f_p and the tabular values corresponding to the cases where three- to seven-point direct interpolation was required. In the present paper that same formula for p is extended to include all the terms involving the first ten powers of $(f_p - f_0)$ and, as before, quantities are defined in terms of f_p and the tabular entries to provide for inverse interpolation when eight-, nine-, ten-, or eleven-point formulas are required for direct interpolation. Although the greater part of the expression for p will hardly ever be needed in most practical problems, due to the rapidity of convergence and smallness of many of its terms, its full use can provide unusual accuracy in solving equations (both real and complex) up to the tenth degree when the values of the polynomials are tabulated near the root at equal intervals. (Received November 14, 1944.)

35. J. J. Stoker: *Nonlinear theory of curved elastic sheets.*

The membrane theory of thin elastic shells, which is based on the assumption that bending stresses can be neglected, is relatively simple from the mathematical point of view since the stresses can be determined independently of the strains. However, it is not possible then, in general, to satisfy boundary conditions which refer to displacements—such as, for example, the condition of a fixed edge. The present paper presents a theory of thin shells which neglects bending stresses but which, nevertheless, makes it possible to satisfy various types of boundary conditions which are reasonable from a physical point of view, such as that of a fixed edge. This is accomplished by taking into account certain of the quadratic terms in the expressions for the strains as functions of the displacements, in a manner analogous to that employed in deriving the von Kármán equations for bending of thin plates. (Received October 4, 1944.)

Ergodic Theory

36. P. R. Halmos: *On an incompressible transformation.*

E. Hopf has introduced a very strong notion of incompressibility for one-to-one measurable transformations on a measure space and showed that a transformation is incompressible in that sense if and only if it possesses a positive finite invariant integral. Recently Hurewicz has shown that under the assumption of a much weaker notion of incompressibility a very elegant generalization of Birkhoff's ergodic theorem is valid. The purpose of this note is to point out the following two facts: (1) If a transformation does possess an invariant integral, the Hurewicz theorem can be made to follow from known results for measure preserving transformations. (This fact is not immediately obvious only because of the rather peculiar formation of the Hurewicz means.) (2) There exists a one-to-one measurable transformation on a measure space which has the weak but not the strong property of incompressibility. (This fact in addition to answering a question explicitly raised by Hopf also serves to show that Hurewicz's theorem is indeed an extension of Birkhoff's.) (Received October 16, 1944.)

Geometry

37. Felix Bernstein: *The swastika and the Sicilian triskelon from the standpoint of "higher geometry."*

From the standpoint of higher geometry as defined by Felix Klein, the swastika and the triskelon are interpreted by the crystallographic groups of the plane, which are known groups composed of translations and rotations, with an angle of rotation of $360/n$ degrees, where n is restricted to the values 2, 3, 4, 6. The known fundamental domains (F.D.) of these groups with one center of symmetry are altered here into F.D. with two centers of symmetry. The broken line of greatest length connecting the two centers is called an arm and the smallest region whose boundary consists of arms only is called a blitz according to its shape. A blitz and its images produced by the operations of the group in the case $n=4$ fill the whole plane with swastikas, in the case $n=6$ with a like set of triskelons. With the aid of a properly generalized swastika it is possible, in an analogous manner, to fill euclidean space. By certain alterations of given F.D., a proof of the Pythagorean Theorem is obtained. (Received December 1, 1944.)

38. John DeCicco: *Survey of polygenic functions.*

The author presents a general outline of the theory of the first and second deriva-