
It is easily verified that under simple conditions on $f(x)$ the formula

$$f(x) - \int_0^x \frac{1}{k} L_{k,t}(f) \, dt$$

is an identity, where $L_{k,t}(f) = (\frac{1}{k^{k+1}} [\frac{1}{k^{k+1}}]^{-1} \cdot \int_0^t \frac{1}{k} L_{k,t}(f))$. If $k \to \infty$, $f(x) = \lim_{t \to \infty} g(t)$, where $g(t) = \lim_{k \to \infty} L_{k,t}(f)$. Obtaining conditions for which this passage to the limit is valid is tantamount to establishing a representation theory for the Laplace integral. The final results coincide with Widder's theory, but new light is shed on the central role played by the operator $L_{k,t}(f)$. (Received January 20, 1945.)

66. P. R. Rider: *A new use for tables of the incomplete beta function.*

The solution of the problem of minimizing a definite integral having an integrand of the form $(1 + y'^2)^m / y''$ has been given. This paper points out how tables of the incomplete beta function can be used to advantage in carrying out the solution in a numerical case. (Received January 22, 1945.)

**Applied Mathematics**


The differential equations of the bending theory, and hence of the membrane theory of shells of revolution, are derived as consequences of the equations of three-dimensional elasticity and certain additional assumptions. Less restrictive than customary boundary conditions at the apex of closed domes are proposed. By means of stress functions satisfying a simple ordinary differential equation, solutions of the stress equations are surveyed, and it is shown that the new boundary conditions can be satisfied for a large class of surfaces for which the old could not. Displacement functions are introduced which reduce the integration of the displacement equations to the integration of a fairly simple ordinary differential equation. Numerous exact solutions of the differential equations are given explicitly, with the aid of the stress and displacement functions, and some numerical examples are given. (Received January 10, 1945.)

**Geometry**

68. S. B. Jackson: *The four-vertex theorem for surfaces of constant curvature.*

In an earlier paper by the writer (S. B. Jackson, *The four-vertex theorem for spherical curves*, Amer. J. Math. vol. 62 (1940) pp. 795–812) it was shown that on any simple closed spherical curve of class $C'''$, not a circle, there are at least four geodesic vertices, that is, extrema of the geodesic curvature. The aim of the present paper is to extend this result to any surface of constant curvature. Specifically, it is shown that every simple closed curve of class $C''$, not a geodesic circle, in a simply connected region of a surface of constant curvature has at least four geodesic vertices. The technique of the paper is to map the given region of the surface onto the plane in such a way that the geodesic vertices of the given curve go into the vertices of the corresponding plane curve. The theorem then follows from the known facts about the vertices of plane curves. It is shown by example that the restriction that the curve be in a simply connected region of the surface is essential. Finally, it is proved that the theorem...