propositions concerned. "Compositional implication" and "compositional truth" are then explained. In II, the theory of quantification introduces logical pronouns (the "variable arguments" of some writers), and matrices (called by many "propositional forms"). Seven deductive operations are listed and used. In III is emphasized the contrast between object and symbol, and the nature of description. Only one kind of existence (material existence in the case of physical objects) is accepted; the question is reduced to whether a pronoun has an application. A brief argument is presented for the elimination, in logic, of all substantives other than pronouns. In IV, attributes are compared with and contrasted to the more readily handled classes. The *pure matrices* in the theory of classes employ only the notions of conjunction, negation, quantification and pertinence (using the "ϵ" of membership). Applications, even in the abstract field of mathematics, which bring in other signs, not themselves defined in logical terms, involve *impure* matrices. A brief survey of the construction of the number system out of pure logical concepts follows. The notion of pertinence adopted is such that "$z \epsilon y$" has the sense: "z is a number of y, if y is a class, and $z=y$, if y is not a class." A virtual theory of classes and relations, while using familiar phraseology, succeeds in eliminating the ontological presuppositions by offering acceptable defined equivalents. The chapter closes with a description of Gödel's Theorem.

Save possibly that this book may give the idea that all important problems in logical theory have been disposed of, the confident tone and clear, enthusiastic and well organized statements commend the book to all thoughtful readers—who read Portuguese.

ALBERT A. BENNETT

Rings with minimum condition. By Emil Artin, Cecil J. Nesbitt and Robert M. Thrall. University of Michigan Press, 1944. 7+123 pp. $1.50.

The major portion of this book is devoted to a new exposition of the classical theory of associative and distributive rings satisfying the minimum condition for left ideals. The authors envisage in their treatment further developments of the theory. They announce the basic observation that the theory of rings proper should not be separated from the corresponding theory of representations. In connection with this thesis it is worthwhile mentioning that the proof of Wedderburn's structure theorem for simple rings involves the theory of representations.

The plan of the authors thus required a careful study of vector spaces admitting rings for left (right) operators (Chapters I and II). The discussion of the implications of the minimum condition contains R. Brauer's result that the minimum condition for left ideals implies the maximality of the radical (Chapter III). The development of the theory of semi-simple and simple rings puts due emphasis on the existence of a nonzero idempotent element in a non-nilpotent left ideal (Chapters IV and V). To make the proof of Wedderburn's theorem on simple rings more lucid the connection between homomorphisms, matrices, and rings of homomorphisms is given emphasis. The following theorem (p. 38) foreshadows the important concept of analytic linear transformation. "Let R be a simple ring with the minimum right ideal eR. Suppose that $\alpha_1, \cdots, \alpha_m$ is a basis of eR over its division algebra of homomorphisms eRe. Then there exists for any elements $\beta_1, \cdots, \beta_m$ in eR a unique element ξ in R such that $\beta_i = \alpha_i\xi$, $1 \leqq i \leqq m$."

A linear function $L(x)$, $x \in R$, of a simple ring R over its center $\mathfrak{k}$ is defined as a homomorphism of $R/\mathfrak{k}$ given by $L(x) = \sum a_\nu x a_\nu'$ where the a_ν and a_ν' lie in R. This concept furnishes an important tool for the discussion of the subrings of a simple ring and their commutator algebras (Chapter VII). In this connection the authors present anew the theory of the Kronecker or direct product of linear spaces (Chapter VI). The new definition of the Kronecker product is independent of the particular bases of the component spaces. The definition involves a careful study of the identifications which have to be made in set $\{\sum A_\nu B_\nu\}$, A_ν in a space V, B_ν in a space W, admitting a ring $\mathfrak{k}$ with unit element for right and left operators, respectively. The identifications leading to the one-sided Kronecker product $V \times_{\mathfrak{k}} W$ depend on the distributivity of the sums $\sum A_\nu B_\nu$ with respect to both the elements A_ν and B_ν. The symmetry of $V \times_{\mathfrak{k}} W$ in both factors is established readily. A simplification of the proof is achieved if only locally finite vector spaces are considered. Such spaces have the property that any finite set of vectors lies in a subspace generated by independent vectors. The new definition has the advantage of being invariant and thus useful in the theory of infinite algebras. (See, for example, Nathan Jacobson, *Structure theory of simple rings without finiteness assumptions*, Trans. Amer. Math. Soc. vol. 57 (1945) pp. 228–245.)

Another illustration of the benefits gained by the use of the representation theory combined with the new tools can be found in the theory of crossed products (Chapter VIII). The latter theory is a significant part of the theory of representations of groups and simple rings over fields. The authors present some new simpler proofs of classical results. Many of the bothersome arguments involving idempotent elements are avoided and thus the proofs on crossed products are less computational.

The book terminates with the formulation of the theory of "nonsemisimple rings with minimum condition" (Chapter VIII). The basic algebraic facts concerned with rings admitting composition series leading to the Cartan invariants are described with an indication of their importance for the theory of modular representations of finite groups. The authors have shown that the superficial sacrifice of purity, as far as methods are concerned, was worthwhile. The literature of algebra has been enriched by an inspiring book.

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