a function of bounded variation on $(0, \infty)$, for which the moment constants $b_n = \int_0^\infty t^n d\beta(t)$ exist, with $b_0 \neq 0$. (Received May 10, 1945.)

125. Otto Szász: *On the absolute convergence of trigonometric series.*

Generalizing a theorem of Fatou on trigonometric series with monotonically decreasing coefficients, the author proves the following theorem: If $\rho_n > 0$, $\rho_{n+1} \leqq c\rho_n$, $c$ a constant, and if the trigonometric series $\sum \rho_n \cos nx$ is absolutely convergent at one point $x_0$, then $\sum \rho_n < \infty$. The same is true for the sine series if in addition $x_0 \not\equiv 0 \pmod{\pi}$. The proof is quite short and elementary. The author extends this result to series of the type $\sum \rho_n \cos \lambda_n x$, $\sum \rho_n \sin \lambda_n x$, where $0 < \lambda_1 < \lambda_2 < \cdots$. (Received May 10, 1945.)

126. W. J. Trjitzinsky: *Integral equations in problems of representation of functions of a complex variable.*

This work relates to the representation of functions of a complex variable, more general than analytic, in terms of "Cauchy double integrals." (Received May 16, 1945.)

## Applied Mathematics

127. H. E. Salzer: *Formulas for direct and inverse interpolation of a complex function tabulated along equidistant circular arcs.*

When an analytic function $f(z)$ may be approximated by a complex polynomial of degree $n-1$ passing through the values of the function at $n$ points, according to the Lagrange-Hermite interpolation formula, it often happens that those $n$ points are situated along the arc of a circle (equally spaced) and it is required to obtain $f(z)$ for $z$ off the circle but near the arguments. An important case is when $f(z)$ is tabulated in polar form (including tabulation along the vertices of any regular polygon). The formulas that were obtained will facilitate direct interpolation when $f(z)$ is known at three, four, or five points. They furnish the real and imaginary parts of $L_k^{(n)}(P)$, where $f(z) \sim \sum L_k^{(n)}(P) f(z_k)$, as functions of $P^m = p_m + iq_m$ and $\theta$. Here $P = (z - z_0)/h$, $h$ being the distance between successive points $z_k$, and $\theta$ denotes the angle between successive chords joining the points $z_k$. For extensive use for a fixed $\theta$, one can readily obtain $L_k^{(n)}(P)$ in the form $\sum C_k^{(n)}(p_m + iq_m)$. A method for inverse interpolation is given, employing the coefficients of the polynomials $L_k^{(n)}(P)$ in an expansion derived in the author's *A new formula for inverse interpolation*, Bull. Amer. Math. Soc. vol. 50 (1944) pp. 513–516. (Received May 18, 1945.)

128. H. E. Salzer: *Table of coefficients for double quadrature without differences, for integrating second order differential equations.*

On the basis of a double quadrature of the Lagrange interpolation formula, a table of coefficients has been computed to determine a function at equally spaced points (to within an arbitrary $Ax+B$), when its second derivative is known at those points. The coefficients cover the cases where the second derivative may be approximated by a polynomial ranging from the second to tenth degrees (that is, from three-point through eleven-point formulas), and are given exactly. Their chief value will occur in the numerical solution of ordinary linear differential equations of the second order, which can always be reduced to the form $y'' + g(x)y = h(x)$. They can also be employed to integrate the more general equation $y'' + \phi(x, y) = 0$. In every case it is necessary to begin with a few values of $y''$ which can always be found by the usual methods.

There are indications that these coefficients can be used to extend the solution of a second order partial differential equation of the form $u_{xx}=u_t+\psi(u, x, t)$, provided that it is known at a rectangular array of points in the $x$, $t$-plane and at two other points in the next row of values of $t$. (Received April 30, 1945.)

## Geometry

### 129. P. O. Bell: *Metric properties of a class of quadratic differential forms.*

In the present paper a new invariant quadratic differential form $\Omega$ is geometrically defined for a general pair of surfaces $S$, $S'$ whose corresponding points $x$, $x'$ determine the metric normal to $S$ at $x$. The ratio of the form $\Omega$ to the first fundamental form $ds^2$ of $S$, in which $\Omega$ and $ds^2$ are defined for a common arc element of $S$ at $x$, is found to be independent of the direction of the element if and only if the surface $S'$ is the locus of the center of mean curvature of $S$; the ratio thus determined is the Gaussian curvature $K$ of $S$ at $x$. It is proved that the form $\Omega$ for an arbitrary arc element is identical with the form $Kds^2$ for either "conjugate" element if and only if the surface $S'$ is the plane net at infinity. The principal directions at $x$ of the tensor whose components are the coefficients of the form $\Omega$ are the classical principal directions of $S$ at $x$ for an arbitrary choice of $S'$. Finally, the net of lines of mean-curvature of $S$ and the mean-conjugate net of $S$ are characterized as integral nets of equations of the form $\Omega=0$, for suitable selections of $S'$. The author employs dual systems of linear equations of the first order with the use of a tensor notation. (Received May 19, 1945.)

### 130. L. M. Blumenthal: *Characterization of $\phi$-spherical subsets and pseudo sets.*

The class of $\phi$-spherical spaces is defined by four metric postulates involving an arbitrary function $\phi$. The class contains, for example, those spaces derived from the surface of the ordinary sphere in euclidean $(n+1)$-space by making it metric with respect to geodesic (shorter arc) distance and with respect to euclidean (chord) distance. The paper develops the metric geometry of this class of spaces and obtains the metric characterizations of the subsets and pseudo sets. (The paper is to appear (in Spanish) in Revista de Universidad Nacional de Tucumán, Ser. A. vol. 5 under the title *La caracterización métrica de espacios $\phi$-esféricos*.) (Received May 18, 1945.)

### 131. L. M. Blumenthal: *Metric study of elliptic spaces.*

Among the properties of elliptic spaces which make inapplicable the procedures usually employed in a metric study are (1) the "unusual" character of the locus of points equidistant from two points, (2) the lack of free movability in the large, (3) the necessity of distinguishing between "contained in" and "congruently contained in," (4) the notions of dependence and independence of subsets, if defined in the ordinary way, are not metrically invariant, and (5) the abnormal behavior with respect to equilateral subsets. The writer presents a new approach in which the metrically invariant notions of *relative independence* (*dependence*) and *class independence* (*dependence*) play fundamental roles. In terms of these notions the elliptic line is metrically defined. The extension to plane, and so on, is then possible in conventional manner. Necessary and sufficient conditions (of a *quasi-metric* nature) in order that congruent subsets of an elliptic space be superposable are obtained. (Received May 19, 1945.)

### 132. S. S. Chern: *Characteristic classes of Hermitian manifolds.* I.