equations, $-\sum \nabla_\nu(|\psi|^2 \nabla_\nu S) + (\partial/\partial t)|\psi|^2 = 0$, a continuity equation with density $|\psi|^2$ and streaming velocity $-\nabla_\nu S$, and $(\bar{\sigma}^2/2|\psi|)\nabla^2|\psi| + \partial S/\partial t - c^2 - U - \sum(\nabla_\nu S)^2/2 = 0$, a Hamilton-Jacobi equation or Bernouilli equation with an assumed pressure function $(\bar{\sigma}^2/2|\psi|)\nabla^2|\psi|$. An analysis of the motion of stars in a stellar system, treated as a hydrodynamical problem, shows that the above assumption about pressure function is plausible and fits the observed facts. Discrete effects (like quantum effects) cannot show up in this astronomical theory because observed hydrodynamical quantities in a stellar system are to be understood as averages taken over volumes containing many statistically independent elements (stars or star clusters), and they have to be confronted with a ψ function which is a superposition of many stationary ψ. Because of its mathematical simplicity this theory provides for an approach to problems such as transient solutions of the wave equation, selfconsistent steady and transient fields (that is, Poisson equation and hydrodynamical equations combined). (Received August 1, 1945.)

171. Arturo Rosenbluth and Norbert Wiener: *Mathematics of fibrillation and flutter in the heart.*

The known facts about the continuation and refractory period of a muscle fiber are used to explain the phenomena of flutter and fibrillation in the vertebra heart. A geometrical discussion is given of the flutter problem while the fibrillation problem is reduced to a statistical form. (Received July 20, 1945.)

172. H. E. Salzer: *Table of coefficients for repeated integration with differences.*

For functions tabulated at a uniform interval, formulas for k-fold integration, using advancing or backward differences, are obtained by integrating the Gregory-Newton advancing-difference interpolation formula or the Newton backward-difference formula. The quantities $G_n^{(k)} \equiv (1/n!) \times \int_0^1 \cdots \int_0^p \int_0^p p(p-1) \cdots (p-n+1)(dp)^k$ and $H_n^{(k)} \equiv (1/n!) \times \int_0^1 \cdots \int_0^p \int_0^p p(p+1) \cdots (p+n-1)(dp)^k$ are the coefficients of the nth advancing and backward differences respectively in the formulas $\int_{x_0}^{x_1} \cdots \int_{x_0}^{x} \int_{x_0}^{x} f(x)(dx)^k = h^k[f(x_0)/k! + \sum_{n=1}^m G_n^{(k)} \nabla^n f(x_0)] + Rm = h^k[f(x_0)/k! + \sum_{n=1}^m H_n^{(k)} \nabla^n f(x_0)] + R_m'$, where $x_1 - x_0 = h =$ the tabular interval. Previous tables (A. N. Lowan, H. E. Salzer, Journal of Mathematics and Physics vol. 22 (1943) pp. 49–50, and W. E. Milne, Amer. Math. Monthly vol. 40 (1933) pp. 322–327) furnish exact values for $k=1$, $n=1$ (1) 20 and $k=2$, $n=1$ (1) 7. The present table has exact values for $k=2$, $n=1$ (1) 20 and decimal values for $k=2$ (1) 6, $n=1$ (1) $22-k$. The quantities $G_n^{(2)}$ and $H_n^{(2)}$ are expressed in several ways as functions of $B_\nu^{(n)}(x)$, Bernoulli polynomials of order n and degree ν, where $t_n e^{xt}/(e^t-1)^n = \sum_{\nu=0}^{\infty} t^\nu B_\nu^{(n)}(x)/\nu!$, and were checked in terms of previously tabulated values of $B_\nu^{(n)}(x)$. Also a simple recursion formula for $G_n^{(k)}$ in terms of $G_n^{(k-1)}$ and $G_{n+1}^{(k-1)}$ (and similarly for $H_n^{(k)}$), valid for $k \geqq 2$, was used for computation when $k>2$. (Received July 7, 1945.)

Geometry

173. P. O. Bell: *Power series expansions for the equations of a variety in hyperspace.*

For the study of the local properties of an arbitrary variety V_m in a linear space S_n a moving reference frame $F(x_0, x_1, \cdots, x_n)$ may be selected whose vertex x_0 is a generic point of V_m. The general projective homogeneous coordinates x_j^ν of the points

x_j, $j=0, 1, 2, \cdots, n$, are solutions of a system of $m(n+1)$ partial differential equations of the form $\partial x_i/\partial u^\alpha = C_{i\alpha h}x^h$, $\alpha=1, 2, \cdots, m$, in which h is a dummy index. Local nonhomogeneous coordinates z^i with respect to F of a point X of V_m (near to x_0) are defined by the vector relation $X=z^h x_h$, in which $z^0=1$. The present paper is concerned with the calculation of power series expansions for $n-m$ coordinates z^r, $r=m+1, m+2, \cdots, n$, in terms of the remaining m coordinates of X. On differentiating with respect to u^α the coordinates z^r and their power series representations with undetermined coefficients and replacing the derivatives of z^i by their values obtained from the conditions of immovability of the point X, the author obtains two power series for each derivative of z^r. On equating corresponding coefficients of these series, the author derives recursion-relations among the required coefficients. The ease with which these relations may be applied is exemplified by new determinations of certain power series expansions. (Received May 19, 1945.)

174. N. A. Court: *On a skew quartic associated with a tetrahedron.*

Associate with a tetrahedron (T) the three orthogonal hyperboloids of one sheet which have for their axes the three pairs of opposite edges of (T). The three surfaces belong to the same pencil of quadrics. The hyperboloid (H) determined by the altitudes of (T) also belongs to this pencil. The vertices of (T) and the feet of its altitudes lie on the skew quartic (C_4) common to the surfaces of the pencil of quadrics. The skew quartic which is thus associated with (T) is the locus of points from which the vertices of (T) are projected by an orthocentric group of lines. The tangents to (C_4) at the vertices of (T) are the orthocentric lines of the respective trihedral angles of (T). The osculating planes of (C_4) at the vertices of (T) are the tangent planes at those points to the hyperboloid (H). If the tetrahedron (T) is isosceles, the skew quartic associated with (T) is also the skew quartic associated with the twin tetrahedron of (T): the common circumcenter of the two tetrahedrons is a center of symmetry of the skew quartic. (Received July 25, 1945.)

175. John DeCicco: *Equilong maps of the ∞^3 circles.*

The author obtains theorems in equilong geometry analogous to those considered by Kasner and DeCicco in the paper *Families of curves conformally equivalent to circles* (Trans. Amer. Math. Soc. vol. 49 (1941) pp. 378–391). Upon applying any equilong transformation to the totality of ∞^3 circles, there results a set of ∞^3 curves, which is termed an ω family. The magnilong group is the totality of all line transformations in the plane carrying every ω family into an ω family. A general, affinilong, or magnilong transformation carries at most $2\infty^2$, ∞^2, or ∞^1 curves of a given ω family into curves of the same family. The focal locus of the osculating parabolas of the ∞^1 curves of an ω family which pass through a lineal element E is a cissoid with the cusp at E. The envelope of the directrices of the osculating parabolas is a parabola with vertex at the point P of E and the line E as the axis. The envelope of the ∞^1 circles defining the focal cissoids constructed along a given line L consists of two straight lines symmetrical to L. Finally the envelope of the ∞^1 directorial parabolas constructed along a given line L consists of two straight lines symmetrical to L. (Received June 25, 1945.)

176. Edward Kasner: *Neo-Pythagorean triangles.*

In contrast to right or Pythagorean triangles, defined by $a^2+b^2-c^2=0$, the author introduces the class of triangles defined by $a^2+b^2+c^2=0$, which he terms neo-

Pythagorean. One property of these imaginary or complex triangles is $\cos A \cos B \cos C = -1$. It is shown that the medians are proportional to the opposite sides, the ratio being $3^{1/2}i/2$. The only other triangles with such a proportionality are the equilateral, with the ratio $3^{1/2}/2$. Many other properties are found. (Received June 21, 1945.)

177. Edward Kasner: *Null hexagons.*

The author defines a null hexagon as one in which each of the six sides is of length zero; the slopes are alternately $+i$ and $-i$. There are three main diagonals and six minor diagonals, all distinct from zero. All the interrelations of the nine diagonals are found. In particular, the square of a main diagonal is equal to the sum of the squares of two minor diagonals. Certain real covariant diagrams are found. (Received June 25, 1945.)

178. Edward Kasner and John DeCicco: *Multi-isothermal systems.*

The authors extend to euclidean space R_{2n} of $2n$ dimensions the results already considered in the paper *Bi-isothermal systems* (Bull. Amer. Math. Soc. vol. 51 (1945) pp. 169–174). A multi-isothermal system of ∞^{2n-1} curves in R_{2n} is any system which is pseudo-conformally equivalent to a parallel pencil of ∞^{2n-1} straight lines. Any such system consists of ∞^{2n-2} isothermal families of ∞^1 curves, each such family lying upon a conformal surface. A multi-isothermal system of ∞^1 hypersurfaces S_{2n-1} in R_{2n} is any system which is pseudo-conformally equivalent to a parallel pencil of ∞^1 hyperplanes. Any such system may be defined by placing a multiharmonic function equal to an arbitrary constant. The pseudo-angle between any multi-isothermal system of hypersurfaces and any multi-isothermal system of curves is a multiharmonic function. If a surface is intersected by every multi-isothermal system of hypersurfaces in an isothermal system of curves, then the surface is conformal. Finally if a given system of ∞^1 hypersurfaces is intersected by every conformal surface in an isothermal system of curves, then the given system of ∞^1 hypersurfaces is multi-isothermal. (Received June 25, 1945.)

179. J. W. Lasley: *On the classification of collineations in the plane.*

This paper reduces nonsingular collineations of the plane into itself to a common canonical form, that of Jordan. The basis of the classification is a pair of absolute invariants: the multiplicity of the roots of the characteristic equation and the rank of the characteristic matrix arising from the multiple root. Considerations of incidence of double elements is limited to the proof of a single theorem. In adjusting the reference frame to meet the needs of the several cases a full use of duality is made. The effect of the adjustments of the reference frame upon the analytic representation of the collineations is kept in the foreground. The essential geometric features of the collineation are permitted to emerge unhampered by needless algebraic complications. The essential unity present in the diverse patterns appears in the Jordan canonical form, by means of which one is able readily to distinguish one class of collineations from another. (Received July 19, 1945.)

180. J. W. Lasley: *On the equations of certain osculants.*

Abel Transon in 1841 provided by means of his theory of aberrancy a visualization of the derivatives of the third and fourth orders. Since then Cesàro, Wilczynski, the Mukhopadhyayas and others have done much toward welding isolated bits of osculant theory into a cohesive whole. This paper obtains the equations of all osculating conics of a plane curve when referred to a general reference frame. The method employed makes possible a unification of types of osculants which are seemingly quite diverse. (Received July 19, 1945.)

181. H. P. Pettit: *On the generation of certain algebraic surfaces.*

A surface of order $2mn$ is the locus of the curve of intersection of two cones in which the intersections of a plane of a pencil with a base surface of order m and a base surface of order n are projected from two fixed points. These fixed or base points are mn-fold points on the generated surface, the tangent cones consisting, respectively, of m cones of order n and n cones of order m. The surface contains a plane n-ic as an m-fold curve and a plane m-ic as an n-fold curve. For $m=n=1$ the method is the ordinary projective generation of the ruled quadric. For a particular choice of the base points relative to the base surfaces certain degeneracies take place in the generated surface. In a plane through the base points, the process produces the method of generating plane curves which was discussed by the author in *The projective description of some higher plane curves*, Tôhoku Math. J. vol. 27 (1926). (Received May 26, 1945.)

Logic and Foundations

182. Garrett Birkhoff: *Universal algebra.*

An unpublished result of Bruce Crabtree is extended to show that, if A is any algebra with finitary operations, and G is any subset and S any subalgebra of A, then there is a maximal subalgebra T satisfying $G \cap T \leqq S$. If the lattice of subalgebras of A is distributive, then it must satisfy $X \cap \bigcup Y_\alpha = \bigcup (X \cap Y_\alpha)$. Hence not every complete lattice is the lattice of all subalgebras of a suitable universal algebra. (Received July 5, 1945.)

183. R. M. Robinson: *Finite sequences of classes.*

This note discusses the definition of a finite sequence of classes, in an axiomatic set theory in which "sets" and "classes" are distinguished, only sets being allowable as elements. (Received July 23, 1945.)

Statistics and Probability

184. Isaac Opatowski: *Direct and reverse transitions in Markoff chains.*

The author considers stochastic processes consisting of successive transitions between $n+1$ states $\{i\}_0^n$ according to the law $dP_i/dt = k_iP_{i-1} - k_{i+1}P_i + g_iP_{i+1} - g_{i-1}P_i$, $P_0(0)=1$, $P_i(0)=0$ for $i \geqq 1$, where $P_i(t)$ is the probability that the system be in the state i at the time t if it is at the time $t=0$ in the state 0. The constants k_i and g_i represent respectively the "intensities" of the direct and reverse transitions $(i-1 \to i)$ $(i+1 \to i)$. $k_1 > 0$, $g_1 \geqq 0$ for any i, except $k_0 = g_n = 0$. It is shown that if $k_{n+1} = g_{n-1} = 0$, and consequently $\sum_{i=0}^{i=n} P_i = 1$, the process is equivalent, as far as the probability $P_n(t)$ is concerned, to a new process of the same type, between the same number of states consisting, however, of direct transitions only with intensities $\{\bar{k}_i\}_1^n$. The main part of the proof consists in showing that $\{-\bar{k}_i\}_1^n$, which are the poles of the Laplace transform of dP_n/dt, are all real and negative. They are roots of the determinant $\|a_{i,j}\|_n$ where $a_{i,i} = x + k_{i+1} + g_{i-1}$; $a_{i,i+1} = g_i$, $a_{i,i-1} = k_i$ with all the other $a_{i,j}$'s zero. The