

ing conics of a plane curve when referred to a general reference frame. The method employed makes possible a unification of types of osculants which are seemingly quite diverse. (Received July 19, 1945.)

181. H. P. Pettit: *On the generation of certain algebraic surfaces.*

A surface of order  $2mn$  is the locus of the curve of intersection of two cones in which the intersections of a plane of a pencil with a base surface of order  $m$  and a base surface of order  $n$  are projected from two fixed points. These fixed or base points are  $mn$ -fold points on the generated surface, the tangent cones consisting, respectively, of  $m$  cones of order  $n$  and  $n$  cones of order  $m$ . The surface contains a plane  $n$ -ic as an  $m$ -fold curve and a plane  $m$ -ic as an  $n$ -fold curve. For  $m=n=1$  the method is the ordinary projective generation of the ruled quadric. For a particular choice of the base points relative to the base surfaces certain degeneracies take place in the generated surface. In a plane through the base points, the process produces the method of generating plane curves which was discussed by the author in *The projective description of some higher plane curves*, Tôhoku Math. J. vol. 27 (1926). (Received May 26, 1945.)

#### LOGIC AND FOUNDATIONS

182. Garrett Birkhoff: *Universal algebra.*

An unpublished result of Bruce Crabtree is extended to show that, if  $A$  is any algebra with finitary operations, and  $G$  is any subset and  $S$  any subalgebra of  $A$ , then there is a maximal subalgebra  $T$  satisfying  $G \cap T \leq S$ . If the lattice of subalgebras of  $A$  is distributive, then it must satisfy  $X \cap \bigcup Y_\alpha = \bigcup (X \cap Y_\alpha)$ . Hence not every complete lattice is the lattice of all subalgebras of a suitable universal algebra. (Received July 5, 1945.)

183. R. M. Robinson: *Finite sequences of classes.*

This note discusses the definition of a finite sequence of classes, in an axiomatic set theory in which "sets" and "classes" are distinguished, only sets being allowable as elements. (Received July 23, 1945.)

#### STATISTICS AND PROBABILITY

184. Isaac Opatowski: *Direct and reverse transitions in Markoff chains.*

The author considers stochastic processes consisting of successive transitions between  $n+1$  states  $\{i\}_0^n$  according to the law  $dP_i/dt = k_i P_{i-1} - k_{i+1} P_i + g_i P_{i+1} - g_{i-1} P_i$ ,  $P_0(0) = 1$ ,  $P_i(0) = 0$  for  $i \geq 1$ , where  $P_i(t)$  is the probability that the system be in the state  $i$  at the time  $t$  if it is at the time  $t=0$  in the state 0. The constants  $k_i$  and  $g_i$  represent respectively the "intensities" of the direct and reverse transitions ( $i-1 \rightarrow i$ ) ( $i+1 \rightarrow i$ ).  $k_1 > 0$ ,  $g_1 \geq 0$  for any  $i$ , except  $k_0 = g_n = 0$ . It is shown that if  $k_{n+1} = g_{n-1} = 0$ , and consequently  $\sum_{i=0}^{i=n} P_i = 1$ , the process is equivalent, as far as the probability  $P_n(t)$  is concerned, to a new process of the same type, between the same number of states consisting, however, of direct transitions only with intensities  $\{\bar{k}_i\}_1^n$ . The main part of the proof consists in showing that  $\{-\bar{k}_i\}_1^n$ , which are the poles of the Laplace transform of  $dP_n/dt$ , are all real and negative. They are roots of the determinant  $\|a_{i,j}\|_n$  where  $a_{i,i} = x + k_{i+1} + g_{i-1}$ ;  $a_{i,i+1} = g_i$ ,  $a_{i,i-1} = k_i$  with all the other  $a_{i,j}$ 's zero. The