ing conics of a plane curve when referred to a general reference frame. The method employed makes possible a unification of types of osculants which are seemingly quite diverse. (Received July 19, 1945.)

181. H. P. Pettit: *On the generation of certain algebraic surfaces.*

A surface of order $2mn$ is the locus of the curve of intersection of two cones in which the intersections of a plane of a pencil with a base surface of order m and a base surface of order n are projected from two fixed points. These fixed or base points are mn-fold points on the generated surface, the tangent cones consisting, respectively, of m cones of order n and n cones of order m. The surface contains a plane n-ic as an m-fold curve and a plane m-ic as an n-fold curve. For $m=n=1$ the method is the ordinary projective generation of the ruled quadric. For a particular choice of the base points relative to the base surfaces certain degeneracies take place in the generated surface. In a plane through the base points, the process produces the method of generating plane curves which was discussed by the author in *The projective description of some higher plane curves*, Tôhoku Math. J. vol. 27 (1926). (Received May 26, 1945.)

Logic and Foundations

182. Garrett Birkhoff: *Universal algebra.*

An unpublished result of Bruce Crabtree is extended to show that, if A is any algebra with finitary operations, and G is any subset and S any subalgebra of A, then there is a maximal subalgebra T satisfying $G \cap T \leqq S$. If the lattice of subalgebras of A is distributive, then it must satisfy $X \cap \bigcup Y_\alpha = \bigcup (X \cap Y_\alpha)$. Hence not every complete lattice is the lattice of all subalgebras of a suitable universal algebra. (Received July 5, 1945.)

183. R. M. Robinson: *Finite sequences of classes.*

This note discusses the definition of a finite sequence of classes, in an axiomatic set theory in which "sets" and "classes" are distinguished, only sets being allowable as elements. (Received July 23, 1945.)

Statistics and Probability

184. Isaac Opatowski: *Direct and reverse transitions in Markoff chains.*

The author considers stochastic processes consisting of successive transitions between $n+1$ states $\{i\}_0^n$ according to the law $dP_i/dt = k_iP_{i-1} - k_{i+1}P_i + g_iP_{i+1} - g_{i-1}P_i$, $P_0(0)=1$, $P_i(0)=0$ for $i \geqq 1$, where $P_i(t)$ is the probability that the system be in the state i at the time t if it is at the time $t=0$ in the state 0. The constants k_i and g_i represent respectively the "intensities" of the direct and reverse transitions $(i-1 \to i)$ $(i+1 \to i)$. $k_1 > 0$, $g_1 \geqq 0$ for any i, except $k_0 = g_n = 0$. It is shown that if $k_{n+1} = g_{n-1} = 0$, and consequently $\sum_{i=0}^{i=n} P_i = 1$, the process is equivalent, as far as the probability $P_n(t)$ is concerned, to a new process of the same type, between the same number of states consisting, however, of direct transitions only with intensities $\{\bar{k}_i\}_1^n$. The main part of the proof consists in showing that $\{-\bar{k}_i\}_1^n$, which are the poles of the Laplace transform of dP_n/dt, are all real and negative. They are roots of the determinant $\|a_{i,j}\|_n$ where $a_{i,i} = x + k_{i+1} + g_{i-1}$; $a_{i,i+1} = g_i$, $a_{i,i-1} = k_i$ with all the other $a_{i,j}$'s zero. The

paper is a part of an article to be published in the December 1945 issue of the Bulletin of Mathematical Biophysics. (Received July 26, 1945.)

TOPOLOGY

185. R. H. Bing: *Collections filling up a simple plane web.*

It is shown that the compact continuum W is a simple plane web provided there exist an upper semicontinuous collection of mutually exclusive continua filling up W and another such collection H filling up W such that if g and h are elements of G and H respectively, then the common part of g and h exists and is totally disconnected. It is proved that if W has a bounded complementary domain and each of the collections G and H is a dendron with respect to its elements, then each is a non-equicontinuous collection. (Received June 25, 1945.)

186. R. H. Bing: *Generalizations of two theorems of Janiszewski.*

Janiszewski proved that if H and K are continua neither of which cuts the plane, then the sum of H and K cuts the plane only if the common part of H and K is not connected. The present paper gives generalizations of this result by considering more general sets than continua. It is shown that if H and K are two sets of which neither cuts the point A from the point B, $H \cdot K$ is connected, $H-H \cdot K$ is compact and $H-H \cdot K$ and $K-H \cdot K$ are mutually separated, then $H+K$ does not cut A from B. Also, if H and K are connected sets one of which is compact, H is countinuumwise connected and $\overline{H} \cdot K+H \cdot \overline{K}$ is the sum of two mutually separated sets each containing a point of H, then $H+K$ cuts the plane. (Received July 23, 1945.)

187. J. C. Oxtoby: *Invariant measures in groups which are not locally compact.*

In any complete separable metric group which is dense in itself it is possible to construct a left-invariant measure, defined for all Borel sets and zero for points; but such a measure cannot be locally finite, nor can every compact set have finite measure, unless the group is locally compact. This result is proved independently of Haar's theorem, and the measure constructed may have properties quite unlike Haar's measure. Other constructions, based on the idea of extending a Haar measure from a subgroup, or introducing a new topology, are considered and their limitations discussed. The results throw light on Weil's converse of Haar's theorem by giving examples of measures that fail to satisfy Weil's postulates, and by showing that there is a class of groups in which no Borel measure can satisfy them. (Received July 23, 1945.)

188. Moses Richardson: *On weakly ordered systems.* Preliminary report.

A *weakly ordered system* is a system of elements $a, b, \cdots$ with a binary relation $\succ$ such that (1) $a \succ b$ implies $a \neq b$, and (2) $a \succ b$ implies b not greater than a. Transitivity is not assumed. Such a system can be represented by an oriented 1-complex or linear graph in an obvious way. J. von Neumann and O. Morgenstern (*Theory of games and economic behavior*, Princeton, 1944) prove the existence of "solutions" for the case of a strictly acyclic system, and give a method of constructing such solutions. The main result of the present note is the existence and construction, by a