dendron with respect to its elements, and (3) *if g and h are elements of G and H respectively, the common part of g and h exists and is totally disconnected. Then W contains a point at which G is hereditarily non-equicontinuous.*

PROOF. Obtain g_e, AB, C, ρ, and g as in Theorem 7. Of every countable sequence of different elements of G having a subset of g as a limiting set, all but a finite number separate g from g_e. Hence G is hereditarily non-equicontinuous at C.

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DIMENSIONAL TYPES

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Let H and S be topological spaces. We say that H is of *dimensional type* S (symbol: D_S) if for each closed set X and mapping $f:X\rightarrow S$ there exists an extension $\bar{f}:H\rightarrow S$.

It is clear that (from a result due to Hurewicz [**1**, p. 83]) when H is separable metric and S is an n-sphere, then H can be of dimensional type S if and only if dim $H\leqq n$. For simplicity we write D_n for D_S when S is an n-sphere. It is, of course, possible to define dim H as the least integer n for which H is of type D_n even when H is not separable metric. But this seems to be open to objection except in certain cases (cf. (d) below).

It is at once clear that we have:

(a) *If H is of type D_S then so also is any closed subset.*

(b) *If the closed sets H_1 and H_2 are of type D_S then so also is the set H_1+H_2.*

As a matter of notation we may suppose that $H=H_1+H_2$. Let $f:X\rightarrow S$. Several cases may arise of which we shall consider only the

Presented to the Society, September 17, 1945; received by the editors June 13, 1945.

one in which $X \cdot H_1 \cdot H_2$ is not empty. By (a) the mapping $g = f | X \cdot H_1 \cdot H_2$ may be extended to a mapping $\bar{g}: H_1 \cdot H_2 \rightarrow S$. Let $g_i = \bar{g}$ on $H_1 \cdot H_2$ and $g_i = f$ on $X \cdot H_i$, $i = 1, 2$. Then $g_i: (H_1 \cdot H_2 + X \cdot H_i) \rightarrow S$ and so by assumption may be extended to a mapping $\bar{f}_i: H_i \rightarrow S$. Since $\bar{f}_1 = \bar{f}_2$ on $H_1 \cdot H_2$ we may combine these mappings into a transformation $\bar{f}$ carrying H into S.

With the aid of (b) and additional hypotheses we can extend (b) to

(c) *Let H be a normal space and S an absolute neighborhood retract. If $H = H_1 + H_2 + \cdots$ where each H_i is closed and of dimensional type S then H is of dimensional type S.*

From (b) there is no loss of generality in supposing that $H_1 \subset H_2 \subset H_3 \cdots$. Let f map the closed set X into S. We may assume that X meets H_1. Let $f = f_1$ so that (since S is an ANR) there is an extension $\bar{f}_1$ of f_1 mapping a neighborhood U_1 of X into S. From the normality of H it follows that there exists a neighborhood V_1 of X whose closure is contained in U_1. Let $X_1 = \bar{V}_1$ and set $g_1 = \bar{f}_1 | H_1 \cdot X_1$. By assumption g_1 admits an extension $\bar{g}_1: H_1 \rightarrow S$. Let $f_2 = \bar{g}_1$ on H_1 and $f_2 = \bar{f}_1$ on X_1 so that f_2 is a mapping of $H_1 + X_1$ into S. We may extend f_2 to $\bar{f}_2: U_2 \rightarrow S$, where U_2 is a neighborhood of $H_1 + X_1$. Let V_2 be neighborhood of $H_1 + X_1$ for which $X_2 = \bar{V}_1 \subset U_2$. Suppose that n exceeds 2 and assume that $\bar{f}_n: U_n \rightarrow S$ is an extension of $f_r: (H_{n-1} + \bar{V}_{n-1}) \rightarrow S$ and that $X_n = \bar{V}_n \subset U_n$ where V_n is a neighborhood of $H_{n-1} + \bar{V}_{n-1}$. Let $g_n = \bar{f}_n | H_n \cdot X_n$ and then denote by $\bar{g}_n$ an extension of g_n to H_n. Define f_{n+1} as $\bar{g}_n$ on H_n and put $f_{n+1} = \bar{f}_n | X_n$ so that f_{n+1} maps $H_n + X_n$ into S. Extend f_{n+1} to a transformation $\bar{f}_{n+1}$ of U_{n+1} into S where U_{n+1} is an open set containing $H_n + X_n$. Let V_{n+1} be a neighborhood of this set with $X_{n+1} = \bar{V}_{n+1} \subset U_n$.

It follows that:

(i) $V_1 \subset V_2 \subset V_3 \subset \cdots$.

(ii) $\bar{f}_{n+1}$ is an extension of $\bar{f}_n$ defined on V_{n+1}, $\bar{f}_1$ being an extension of f.

(iii) $H = \sum_{n=1}^{\infty} V_n$.

If $x \in H$ then, for some n, $x \in V_n$ and we may write $\bar{f}(x) = \bar{f}_n(x) = \bar{f}_{n+1}(x) = \cdots$. That $\bar{f}$ is continuous follows from the fact that the sets V_n *are open.*

Other generalizations of the classical sum-theorem have been given by P. Alexandroff, E. Čech, and other mathematicians.

To validate the definition suggested earlier it seems necessary to restrict the class of spaces to which it is intended to apply. A space H will be said to have *property* V provided that for any two closed sets X_1, X_2 there exist closed sets H_1, H_2 such that $H = H_1 + H_2$, $H_1 \cdot H_2 \cdot (X_1 + X_2) = X_1 \cdot X_2$ and $X_i \subset H_i$. This is a well known property of metric spaces but we have no reference to its formulation in the literature as an axiom.

(d) *If H is of type D_n and has property V then H is of type D_{n+1}.*

Following a line of argument used by Hurewicz [**2**, p. 144] let f map the closed set X into S_{n+1}. We may assume that S_{n+1} is given by the equation $y_1^2+\cdots+y_{n+2}^2=1$ and let A_1, A_2 denote the subsets of S_{n+1} given by $(y_{n+2}\geqq 0)$, $(y_{n+2}\leqq 0)$. Then A_1 and A_2 are $(n+1)$-cells which meet in the $S_n:(y_1^2+\cdots+y_{n+1}^2=1)\cdot(y_{n+2}=0)$. We may clearly suppose that $f(X)$ meets both A_1 and A_2 so that $X_i=f^{-1}(A_i\cdot f(X))$ is not empty. Since H has property V we have $H=H_1+H_2$ where $X_i\subset H_i=\overline{H}_i$, and $H_1\cdot H_2\cdot X=X_1\cdot X_2$. We consider only the case where this latter set is not vacuous. Denote by g the mapping f restricted to $X_1\cdot X_2$ so that g has values lying in S_n. By (a), $H_1\cdot H_2$ is of type D_n and so we may extend g to a mapping $\bar{g}:H_1\cdot H_2\to S_n$. Then (by Tietze's extension theorem) we may extend $\bar{g}$ to a mapping g_i of H_i into A_i. Since $g_1=g_2$ on $H_1\cdot H_2$ we combine these mappings to secure an extension $\bar{f}$ of f taking H into S_{n+1}.

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