In a recent paper (A. N. Lowan and H. E. Salzer, *Table of coefficients for numerical integration without differences*, Journal of Mathematics and Physics vol. 24 (1945) pp. 1–21) quantities $B_i^{(n)}(p)$ are tabulated to 10 decimal places, for continuous numerical integration (that is, integration to various points within an interval of tabulation) by a Lagrangian formula which uses the tabular entries only. The present note indicates how those same quantities $B_i^{(n)}(p)$ can be employed as they stand, for continuous numerical integration using differences, in formulas obtained by integrating the interpolation formulas of Gregory-Newton, Newton-Gauss (two forms), Everett, and Steffensen. (Received August 6, 1945.)

## Geometry

### 233. H. S. M. Coxeter: *Quaternions and reflections.*

Every quaternion $x=x_0+x_1i+x_2j+x_3k$ determines a point $P_x=(x_0, x_1, x_2, x_3)$ in Euclidean 4-space, and every quaternion $a$ of unit norm determines a hyperplane $a_0x_0+a_1x_1+a_2x_2+a_3x_3=0$. The reflection in that hyperplane is found to be the transformation $x\rightarrow -a\bar{x}a$. This leads easily to the classical expression $x\rightarrow axb$ for the general displacement preserving the origin. If $p$ and $q$ are pure quaternions of unit norm, the transformation $x\rightarrow(\cos\alpha+p\sin\alpha)x(\cos\beta+q\sin\beta)$ represents the double rotation through angles $\alpha\pm\beta$ about the two completely orthogonal planes $P_0P_{p\mp q}P_{1\pm pq}$. (Received October 1, 1945.)

### 234. H. S. M. Coxeter: *The order of the symmetry group of the general regular hyper-solid.*

Schlafli defined $\{p, q, r\}$ as the regular four-dimensional polytope bounded by $\{p, q\}$'s, $r$ at each edge; for example, $\{4, 3, 3\}$ is the hyper-cube. The order, $g$, of the symmetry group is found to be given by $16h/g=6/j_{p,q}+6/j_{q,r}+1/p+1/r-2$, where $\cos^2\pi/h$ is the greater root of the equation $x^2-(\cos^2\pi/p+\cos^2\pi/q+\cos^2\pi/r)x+\cos^2\pi/p\cos^2\pi/r=0$, and $j_{p,q}=[(2p+2q+7pq)/(2p+2q-pq)]^{1/2}+1$; for example, for the hyper-cube $\{4, 3, 3\}$, $128/g=6/8+6/6+1/4+1/3-2=1/3$. (Received October 1, 1945.)

### 235. H. S. M. Coxeter: *The Petrie polygon of a regular solid.*

Schlafli defined $\{p, q\}$ as the regular solid bounded by $p$-gons, $q$ at each vertex; for example, $\{4, 3\}$ is the cube. The Petrie polygon of $\{p, q\}$ is a skew $h$-gon such that every two consecutive sides, but no three, belong to a face of the solid; for example, the Petrie polygon of the cube is a skew hexagon. It is found that $h=(g+1)^{1/2}-1$, where $g$ is the order of the symmetry group (that is, four times the number of edges). Since $4/g=1/p+1/q-1/2$, we deduce an expression for $h$ in terms of $p$ and $q$. Moreover, the solid has $3h/2$ planes of symmetry. (Received October 1, 1945.)

### 236. M. M. Day: *Note on the billiard ball problem.*

In his book *Dynamical systems*, G. D. Birkhoff proves by the rather deep Poincaré ring theorem the fact that for each convex cornerless billiard table and each integer $n$ there is a closed path around the table of precisely $n$ sides such that a billiard ball will follow this path around and around if it satisfies the usual reflection law that the angle of incidence equals the angle of reflection whenever the ball hits a side. An elementary proof of this result is given by the following two statements: (1) Any $n$-sided convex polygon of stationary length inscribed in a convex cornerless curve satisfies the reflection law. (2) For each $n \geqq 2$ there is an $n$-sided convex polygon of maximal length inscribed in such a curve. (Received August 9, 1945.)

237. M. M. Day: *Polygons circumscribed about closed convex curves.*

An earlier abstract (50-5-132) announced a proof without use of fixed point theorems of the following result. If $C$ is a symmetric closed convex curve, there exists a parallelogram $P$ circumscribed about $C$ so that the midpoint of each side of $C$ is on $P$. Using very elementary minimal area methods this result is extended to polygons of an arbitrary number of sides, and to polyhedra about convex bodies in Euclidean space of any finite dimension. In higher dimensions the phrase "midpoint of each side" is replaced by "centroid of each face." (Received August 9, 1945.)

238. H. P. Pettit: *The tangents at certain multiple points on a curve* $C_{2mn}$.

In a paper published in the Tôhoku Mathematical Journal in 1927, the author discussed the construction of a curve $C_{2mn}$ of order $2mn$ by the use of two pencils of lines with centers $A_1$, $A_3$, related by means of two base curves $C_n$, $C_m$ of orders $n$ and $m$, respectively, and an auxiliary pencil with center $A_2$. It was shown that the line $A_1A_2$ meets $C_m$ in $m$ $n$-fold points of $C_{2mn}$. In the present paper, it is shown that certain projective relationships yield the following method of determining the tangents at such an $n$-fold point: Project the points in which $A_1A_2$ meets $C_n$ from the point of intersection of $A_1A_3$ with the tangent to $C_m$ at the point in question, thus determining $n$ points on $A_2A_3$. These points are projected from the $n$-fold point in the desired tangents. (Received August 6, 1945.)

239. H. P. Pettit: *The tangents at certain ordinary points on a curve* $C_{2mn}$.

As shown by the author in a previous paper published in the Tôhoku Mathematical Journal in 1927, a curve of order $2mn$ is generated by means of two pencils of lines related by means of two base curves of orders $m$ and $n$ respectively and an auxiliary pencil. The generated curve was shown to pass through all common points of the base curves. This paper discusses a method of constructing the tangents to the generated curve at these common points. There is shown to be a projective relationship between the tangents to the base curves at the common point and the tangent to the generated curve, which results in the following. If $A_1$, $A_2$, $A_3$ are the vertices of the first, auxiliary, and second pencils, and $P$ is the common point of the base curves $C_1$, $C_3$, determine $K_1$ in which the tangent to $C_3$ at $P$ meets $A_2A_3$ and the point $K_3$ in which the tangent to $C_1$ at $P$ meets $A_1A_2$. Draw the line $K_1K_3$ meeting $A_1A_3$ in $T$. The line $PT$ is the required tangent to the generated curve. (Received May 26, 1945.)

## Logic and Foundations

240. N. D. Nelson: *Recursive functions and intuitionistic number theory.* Preliminary report.

It is shown that the interpretation by the intuitionistic truth notion of realizability of Kleene (Bull. Amer. Math. Soc. abstract 48-1-85) satisfies certain formal systems of intuitionistic number theory. Further results are obtained which complete reasoning outlined by Kleene (loc. cit. and Trans. Amer. Math. Soc. vol. 53 (1943) pp. 41–73,