with the unbiased estimate of least variance. Thus the classical estimates of the mean and the variance are justified from a new point of view, and also computable estimates of all higher moments are presented. It is interesting to note that for n greater than 3 neither the sample nth moment about the sample mean nor any constant multiple thereof is an unbiased estimate of the nth moment about the mean. (Received October 6, 1945.)

44. Isaac Opatowski: *Markoff chains and Tchebychev polynomials.*

Let the possible states be $0, 1, \cdots, n+1$ and the only transitions possible during any time dt $(i-1\to i)$ for $1\leqq i\leqq n+1$ and $(i+1\to i)$ for $0\leqq i\leqq n-1$. Let the conditional probabilities for these transitions be respectively $k_i dt+o(dt)$ and $g_i(dt)+o(dt)$, where $k_i=k$ for $1\leqq i\leqq n$, $k_{n+1}=k$ or 0, $g_i=g$ or 0, k and g being two positives constants. The probability $P(t)$ of the existence of the state n at a time t if the state 0 existed at $t=0$ is in the general case a convolution of particular functions $P(t)$ corresponding to the following cases: (i) $k_{n+1}=0$, $g_i=g$ $(i\leqq n-1)$; (ii) $k_{n+1}=0$, $g_i=g$ $(i\leqq n-2)$, $g_{n-1}=0$; (iii) $k_{n+1}=k$, $g_i=g$ $(i\leqq n-1)$. In (i), $p(s)=\int_0^\infty e^{-st}P(t)dt=(k/g)^{n/2}/[sU_n(x)]$, where $U_n(x)$ is the Tchebychev polynomial of second kind and x is a linear function of s. The roots of U_n give an explicit expression of $P(t)$ as a linear combination of n exponentials whose coefficients are calculated in a form convenient for computations. In cases (ii) and (iii), $[p(s)]^{-1}$ is a linear combination of two U_i's and the roots of $[p(s)]^{-1}$ are located within narrow ranges, which makes the calculation of $P(t)$ possible within any accuracy desired. These chain processes occur in some biophysical phenomena and the paper will appear in Proc. Nat. Acad. Sci. U.S.A. under a slightly different title. (Received October 11, 1945.)

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45. Lipman Bers and Abe Gelbart: *A remark on the Lebesgue-Sperner covering theorem.*

A new and elementary proof is given of a somewhat stronger form of the well known Lebesgue-Sperner covering theorem (Math. Ann. vol. 70 p. 166; Abh. Math. Sem. Hamburgischen Univ. vol. 6 p. 265). Some corollaries are discussed. (Received October 19, 1945.)

46. R. H. Bing: *Solution of a problem of J. R. Kline.*

It is shown that a locally connected, compact, metric continuum S is topologically equivalent to the surface of a sphere provided no pair of points separates S but every simple closed curve separates S. On the assumption that an arc separates S, a simple closed curve is constructed that does not separate S. (Received October 10, 1945.)

47. O. G. Harrold: *The* ULC *property of certain open sets.* I. *Euclidean domains.*

Let M be a compact continuum which separates Euclidean 3-space. If M is deformation-free into a complementary domain A and $p^1(M)=0$, then the fundamental group of A vanishes. By means of this: if M^* is a compact continuum separating 3-space which is deformation-free into a complementary domain A, then A is ULC. If, in addition, $p^1(M^*)=0$ and A is bounded, this implies $\bar{A}$ is a singular 3-cell by a result of S. Eilenberg and R. L. Wilder (Amer. J. Math. vol. 64 (1942) pp. 613–622).

This yields an affirmative solution to a problem of that paper for $n=3$. (Received October 19, 1945.)

48. F. B. Jones: *Concerning the separability of certain locally connected metric spaces.*

The main purpose of this paper is to settle in the affirmative the question: Is a metric space satisfying Axioms 0–4 of R. L. Moore's *Foundations* completely (perfectly) separable? In other words: Must every locally connected complete metric space which contains no cut point and in which the Jordan curve theorem holds true be separable? The answer is obtained rather easily after establishing the following more general theorem: Let S denote a locally connected complete metric space such that no subset of S containing exactly two points cuts S. If S contains no skew curve of type 1, then S is separable. (A skew curve of type 1 is a set formed by the sum of three triods such that no point except an end point of one of them belongs to more than one of them but each such end point belongs to all three of them.) (Received October 15, 1945.)

49. J. L. Kelley and Everett Pitcher: *Applications of natural homomorphism sequences.*

The systematic framework of natural homomorphism sequences introduced by Hurewicz (Bull. Amer. Math. Soc. Abstract 47-7-329) is elaborated and several applications are made. For instance the theory is used to introduce a new set of invariants of a class of continuous maps which includes simplicial maps (compare Bull. Amer. Math. Soc. Abstract 46-5-216 by one of the authors). It is likewise used to prove the duality theorem of Alexander-Pontrjagin. The argument used shows in particular that if X is a compact space whose $(r-1)$- and r-dimensional homology groups vanish and Y is a closed subset, then the $(r-1)$-dimensional homology group of Y is a function of the space $X-Y$. The theory is applied also to obtain principal results about characteristic groups at critical levels in the critical point theory of Morse. (Received October 20, 1945.)

50. S. B. Myers: *Equicontinuous sets of mappings.*

Let X^Y be the set of all continuous mappings of a connected, separable, first countable T_1-space Y into a locally compact, complete metric space X. The author proves that the following three conditions on a subset T of X^Y are equivalent. (1) T is closed in X^Y under the compact-open topology (see R. H. Fox, Bull. Amer. Math. Soc. vol. 51 (1945) p. 429), T is equicontinuous, $T(y)$ is compact for at least one $y\in Y$. (2) T is compact under the compact-open topology. (3) There exists an admissible topology on T under which T is compact (an admissible topology is one in which $t(y)$ is simultaneously continuous in y and t for all $y\in Y$ and all $t\in T$). (1) and (3) are equivalent without the first countability assumption on Y. Now let H be the group (under superposition) of all homeomorphisms of a connected, locally compact, complete metric space X onto itself. Then it is shown that an abstract group can be topologized to become an effective, compact, transformation group on X (see Montgomery and Zippin, Duke Math. J. vol. 4 (1938) p. 363) if and only if it is isomorphic to an equicontinuous subgroup G of H, such that $G(x)$ is compact for at least one $x\in X$ and G is closed in H under the compact-open topology. (Received October 18, 1945.)

51. R. H. Sorgenfrey: *Concerning continua irreducible about n points.*

In this note it is shown that for a (compact) continuum M to be irreducible about n points it is necessary and sufficient that for every proper decomposition of M into $n+1$ subcontinua the sum of some n of these fails to be connected. (Received October 5, 1945.)

52. F. A. Valentine: *Set properties determined by conditions on linear sections.*

Let R_n be an n-dimensional Euclidean space. A linear section of a set $S \subset R_n$ with an $(n-1)$-dimensional hyperplane L_{n-1} is the set $S \cdot L_{n-1}$. The following theorems are the principal ones proved: (1) Let $S \subset R_n$ $(n \geqq 2)$. If each $(n-1)$-dimensional linear section of S is connected and closed, then S is closed; (2) Let $S \subset R_n$ $(n \geqq 3)$. Suppose that relative to each $(n-1)$-dimensional hyperplane L_{n-1}, the set $S \cdot L_{n-1}$ is an open one with a connected complement. Then S is open; (3) If each $(n-1)$-dimensional linear section of $S \subset R_n$ $(n \geqq 2)$ is bounded and connected, then S is bounded and connected. The following theorem is one which is related to those of Liberman: (4) Let $S \subset R_n$ $(n \geqq 3)$. If each two-dimensional plane section $S \cdot L_2$ is a continuum (compact) with a connected complement relative to L_2, then S is convex. This last theorem can be generalized so that S need not be bounded. It should be observed that in the above theorems no hypotheses are placed on S itself. Hypotheses are placed only on linear sections. (Received October 7, 1945.)