NOTE ON THE MERSENNE NUMBERS M_{157} AND M_{167}

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The information here offered supplements two short papers written respectively by Charles B. Barker[1] and the present author.[2] The delay in presenting this material was due primarily to the miscarriage of a letter dated February 2, 1945, to Barker from the author which was not delivered to the addressee nor returned to the sender's address as given on the envelope.

With reference to the smaller Mersenne number Barker wrote (January 23, 1945): "I also completed the proof for M_{157}—still ignorant of your result—and I found precisely the same value for the last residue that you determined." This agreement should establish beyond reasonable doubt the composite character of M_{157}. The significance of this verification is somewhat enhanced by the facts that Barker performed direct long division while my potential divisions were effected by multiplication by the reciprocal of the modulus M_{157}, and that the first computation was done on a ten-bank electrically driven machine and the second on a similar eight-bank machine.

In the case of M_{167} the value of the 166th residue as finally obtained by me on December 2, 1944 was 59077 89471 97183 05021 04043 18653 76339 69475 17591 49076. Unfortunately the two investigators used different Lucasian sequences for r_{166}. Barker employed 3, 7, 47, 2207, $\cdots$ and the writer used 4, 14, 194, 37634, $\cdots$. This fully explains the lack of agreement between the datum given by Barker[1] and the residue shown above. Nevertheless since both values are nonzero they agree qualitatively as to the composite character of M_{167}.

Incidentally Barker has just informed me that he used other checks than the one given in the Bulletin, such as the auxiliary moduli 10^5+1 and 10^7+1. Finally the character of the Mersenne number M_{229} is now under investigation by the author on the basis of the sequence 4, 14, 194, $\cdots$.

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[1] Charles B. Barker, *Proof that the Mersenne number M_{167} is composite*, Bull. Amer. Math. Soc. vol. 51 (1945) p. 389.

[2] H. S. Uhler, *First proof that the Mersenne number M_{157} is composite*, Proc. Nat. Acad. Sci. U.S.A. vol. 30 (1944) pp. 314–316.