BOOK REVIEW

Vectors and matrices. By C. C. MacDuffee. (Carus Mathematical Monographs, no. 7.) Ithaca, N. Y., Mathematical Association of America, 1943. 11+192 pp. $2.00.

Professor MacDuffee's book is a clear and careful introduction to the theory of vector spaces and matrices. It should prove extremely useful not only to the student of mathematics, but also to the ever increasing circle of other scientists who show an interest in these fields. The more advanced undergraduate student will have no difficulty reading the book. The material is given in easy, almost leisurely steps. The book sets out from familiar facts, formulating the well known aspects in a new manner, gradually approaching new ideas, and almost inadvertently the reader will have become familiarized with the more abstract ways of mathematical thinking. There is, of course, another way of reaching the same aim: to push the reader into the cold waters of mathematical abstraction right away and let him swim around as well as he can. The student who follows Professor MacDuffee's book will not experience a sudden shock of this kind. Unless he ventures out into the last chapter prematurely (and in a footnote he is given permission to do so if he desires), he will learn about the more abstract concepts such as groups with operators, endomorphisms, and rings of endomorphisms only after he has digested the more "concrete" theories of vectors and matrices.

The book starts with a treatment of linear equations. Determinants are avoided at this stage. The author feels that determinants have been vastly overrated and that most parts of the theory of linear equations can be developed much better without determinants. This is a view with which at least most algebraists will concur. Fifty years ago it was a sign of a progressive attitude in a book to use determinants; to-day it is a sign of progressive thinking to avoid determinants as far as possible.

In the second chapter, vector spaces are defined and matrices are introduced, first in a rather formal manner. The reader is made familiar with the concept of rank which is at once made useful for the theory of linear equations. Determinants can now be approached in the next chapter. The author emphasizes the fact that the idea of matrix precedes that of determinant and suggests that to discuss determinants without matrices is like having the feline grin without the Cheshire cat. Determinants are studied as polynomials $d(X)$ in the coefficients of a square matrix X, which satisfy the equation $d(X)d(Y) = d(XY)$ and which are of minimal positive degree. No completeness is attempted, but the most important properties of determinants are treated briefly.

Chapter 4 deals with matrix polynomials. The characteristic and the minimum equation of a matrix are studied. In the following chapter the union and the intersection of vector spaces are introduced. Application to the spaces spanned by the rows of matrices leads to a theory of greatest common divisors and least common multiples of matrices.

Chapter 6 forms, in a sense, the center piece of the whole book. It brings a complete and detailed account of the rational canonical form for a matrix. The normal form used is somewhat simpler than that usually given; for the proofs, ideas of M. H. Ingraham have been used. Readers who study matrices out of necessity because of their usefulness in physics, statistics, or economics will perhaps not learn much of "practical" value in this chapter. Still, let us hope that at least some of them have become sufficiently interested in the subject so that they read these pages for their own sake. The treatment of the normal form is completed in the following chapter by a discussion of elementary divisors and their significance for the question of equivalence of matrices. A short chapter deals with orthogonal and symmetric matrices. The book concludes with a brief introduction to the concept of endomorphisms of abelian groups.

This discussion of the contents will show that the author did not attempt to give a complete treatment of the subject. This would, indeed, have been an impossible task for a book of this size. The selection of the material in a case like this will, of course, depend on the personal taste of the author.

Our remarks will indicate that the book can be used very well as a textbook for certain classes, specially for students at the difficult stage of transition from undergraduate to graduate work. Some instructors may object to the lack of exercises but this would be an unjust criticism in the eyes of the reviewer. At this level of mathematical education there is no need to suggest exercises which consist essentially in substituting numerical values for the indeterminates appearing in the considerations. As regards exercises and problems of a more valuable type, the instructor will have no difficulty in suggesting them himself according to his personal tastes. Alternate proofs for statements in the text can often be found. Material omitted in the book may be given. Geometrical and other applications can be discussed.

There are a few minor misprints, but it does not seem necessary to mention them here. The proof on page 52 for the equation $dW(l) \equiv l^i$ holds only in the case that the underlying field F is algebraically closed. This assumption is not made in the book and it is superfluous. The expression $dW(l)$ represents a polynomial $f(l)$ of a certain degree i, and it is known that $f(l)f(m) = f(lm)$ for all l, m in F. Comparing the terms of degree i in m in this equation, the equation $f(l) = l^i$ is obtained at once, provided that F contains infinitely many elements. The case of a finite field F (if it is to be included) can be treated directly.

Many students of mathematics will be extremely grateful to Professor MacDuffee for his well planned and lucid exposition.

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