1932, pp. 408, 409) and L. Bers and A. Gelbart (Quarterly of Applied Mathematics vol. 1 (1943) pp. 168–188; Bull. Amer. Math. Soc. vol. 50 (1944) p. 56). For all real $p \geqq 1$, $\phi(x, y)$ is interpreted as the potential of a $(p+2)$-dimensional axially symmetric flow with a stream function $\psi(x, y)$ given by $\psi_x = -y^p\phi_y$, $\psi_y = y^p\phi_x$. It is shown that the fundamental solution $\phi_0(x, y)$ of (*), with a singularity at (x_0, y_0), can be expressed in terms of Bessel functions, and that the corresponding $\psi_0(x, y)$ is many-valued, except for $y_0 = 0$. An application of the divergence theorem yields the period of ψ_0 and leads to the determination of the various limits of the integral $\int_0^\infty \exp(-|x|s)J_q(as)J_{q+1}(ys)ds$ for $x \to 0$ and $y \to b$, where q denotes a non-negative real number. Corresponding results can be formulated in terms of generalized Laplace integrals. (Received February 28, 1946.)

172. J. E. Wilkins: *A note on the general summability of functions.*

There is a result of Titchmarsh (*Introduction to the theory of Fourier integrals*, Oxford, 1937) which gives sufficient conditions for the relation $\int K(x, y, \delta)f(y)dy = f(x) + o(1)$ as $\delta \to 0$. In case $K(x, y, \delta)$ has the form $\{\Gamma(\lambda+1)/2\pi^{1/2}\Gamma(\lambda+1/2)\} \cos^{2\lambda}(1/2)(y-x)$, where $\lambda = 1/\delta$, then Natanson (*On some estimations connected with singular integral of C. de la Vallée Poussin*, C. R. (Doklady) Acad. Sci. URSS. vol. 45 (1944) pp. 274–277) has shown that the remainder term $o(1)$ may be written as $\delta f''(x) + o(\delta)$. The author proposes to provide an extensive generalization of Titchmarsh's theorem to give sufficient conditions for the existence of an asymptotic expansion for the integral of $K(x, y, \delta)f(y)$. The author will also apply the general theory thus developed to several interesting kernels $K(x, y, \delta)$, and in particular will obtain the asymptotic expansion of which Natanson gave the first two terms. (Received March 21, 1946.)

173. H. J. Zimmerberg: *Two general classes of definite integral systems.*

In this paper the notions of definite integral systems considered by Reid (Trans. Amer. Math. Soc. vol. 33 (1931) pp. 475–485), Wilkins (Duke Math. J. vol. 11 (1944) pp. 155–166) and the author (Bull. Amer. Math. Soc. Abstract 52-3-77) are extended to integral systems written in matrix form $y(x) = \lambda\{A(x)y(a) + B(x)y(b) + \int_a^b K(x, t)y(t)dt\}$, where the $n \times n$ matrix $K(x, t) \equiv H(x, t)S(t)$ and the $n \times 2n$ matrix $\|A(x)B(x)\| \equiv \|H(x, a)H(x, b)\| \mathcal{G}$, $\mathcal{G}$ denoting a $2n \times 2n$ constant matrix. These integral systems include the system of integral equations to which a system of first-order linear definite differential equations containing the characteristic parameter linearly in the two-point boundary conditions is equivalent. It is also shown that an integral system of the above form is equivalent to a system of $3n$ homogeneous equations of Fredholm type. (Received February 27, 1946.)

Applied Mathematics

174. Nathaniel Coburn: *Pressure-volume relations and the geometry of the net of characteristics in two-dimensional supersonic flows.*

In a previous paper (Quarterly of Applied Mathematics vol. 3 (1945) pp. 106–116), the author showed that for the Karman-Tsien pressure-volume relation in the two-dimensional supersonic flow of a perfect, irrotational, compressible fluid, the net of characteristics (Mach lines) consists of a Tschebyscheff (fish) net. In the present paper, it is shown that any pressure-volume relation of the type $p = Av^{\alpha}$ (where A, $\alpha \neq 1$ are arbitrary constants) leads to one condition on the three metric coefficients of the net of characteristics. A second condition is furnished by the vanishing of the Riemann-Christoffel tensor for the plane. Various properties of the nets which satisfy these two conditions are studied. Thus, it is shown that the class of nets common to all pressure-volume relations of the above type consists of generalized Tschebyscheff nets. (Received February 2, 1946.)

175. W. F. Eberlein: *Wave equation in a stratified medium.*

The author considers the one-dimensional wave equation $U''(z) + [A + z + f(z)] \cdot U''(z) = 0$ under the boundary conditions $U(0) = 0$ and $U(z) \cdot \exp(i\omega t)$ an "outgoing wave" for large positive z. In the cases of interest $\lim_{z \to +\infty} f(z) = 0$. A Laplace transform method of evaluating the characteristic values A is obtained that is applicable to the case $f(z) = \sum_{j=1}^{n} p_j(z) \exp(-\mu_j z)$, where $\mu_j > 0$ and $p_j(z)$ is a polynomial in z. (Received March 22, 1946.)

176. Wilfred Kaplan: *Qualitative analysis of physical systems.*

The classical formulation of the problem of analysis of qualitative properties of physical systems defined by differential equations $dx_i/dt = X_i(x_1, \cdots, x_n)$ $(i = 1, 2, \cdots, n)$ is criticized for the reason that the results may be extremely sensitive to small changes in the X_i, which can at best be approximately known from physical theory and observation. It is proposed that the inaccuracies of measurement be incorporated into the problem and that a mathematical description of a physical system be considered allowable for qualitative analysis only when the results are independent of the degree of inaccuracy (within prescribed limits). It is shown that this leads in a natural way to replacement of the above differential equations by a flow on the edges of an n-dimensional complex. Applications are made to relaxation oscillations and other problems. (Received March 16, 1946.)

177. Ida Roettinger: *On certain finite integral transformations.*

Finite integral transformations of the type $T\{F(x)\} = \int_0^{\pi} F(x)\phi(kx)dx = f(k)$, where the numbers k belong to a set $K = \{k\}$ of real numbers, are considered. Several properties of these transformations are obtained and specialized to the case where $\phi(kx)$, $k \in \{K\}$, are the characteristic functions of the Sturm-Liouville problem $y''(x) + k^2 y(x) = 0$, $L_i(y) \equiv a_{i1}y(0) + a_{i2}y'(0) + a_{i3}y(\pi) + a_{i4}y'(\pi) = 0$, $i = 1, 2$, $0 \leqq x \leqq \pi$, and particular cases thereof. The constants a_{ij} are assumed to be such as to guarantee real characteristic values. These special transformations are useful in obtaining quickly a formal solution of the following type of boundary value problems: $\sum_{n=0}^{m} A_{2n} d^{2n}F(x)/dx^{2n} = G(x)$, $0 \leqq x \leqq \pi$, where $G(x)$, A_{2n} and the quantities $L_i\{F^{(2n-2s-2)}\}$, $s = 1, 2, \cdots, n-1$, $i = 1, 2$, are assigned. The constants a_{ij} are allowed to be different for each s. The transformations are also useful in boundary value problems in partial differential equations, where one or more variables behave like x in the above problem. (See also Bull. Amer. Math. Soc. Abstracts 50-5-155 and 51-1-32.) (Received March 22, 1946.)

178. H. E. Salzer: *Coefficients for facilitating the use of the Gaussian quadrature formula.*

The n-point Gaussian quadrature formula, which requires the value of the function at x_i, the ith zero of the Legendre polynomial $P_n(x)$, involves an integrand at irregularly separated points. Thus the integrand is not amenable to an ordinary difference test for checking purposes. The nearest substitute is a divided difference test which is rather tedious when performed in the usual manner. Since the $(n-1)$th divided difference of $f(x_i)$, $i=1, \cdots, n$, can be written as $\sum_{i=1}^{n} C_i^n f(x_i)$ where C_i^n depend only upon $x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n$, C_i^n will be the same for each n (after the interval of integration is transformed to the usual $[-1, 1]$), and independent of $f(x)$. The coefficients C_i^n are given to $8D$ for $n=3, 4, 5$; to $7D$ for $n=6, 7, 8$; to $6D$ for $n=9, 10$. Limitations on the effectiveness of this check are due to (A) its inconclusive character because it is based upon a single divided difference and (B) the impossibility of a complete difference check whenever the Gaussian formula is employed with efficiency that appreciably exceeds that of the usual Cotes formula. (Received March 13, 1946.)

179. Seymour Sherman: *Stability calculations and time lag.*

Methods of Cauchy and Sturm have been applied to locate the zeros of transcendental expressions occurring in calculations for the stability of a one degree of freedom mechanical system with a retarded damping term. This is an example of what H. Bateman (*The control of an elastic fluid*, Bull. Amer. Math. Soc. vol. 51 (1945) especially pp. 618–626) called the transcendental problem. Previous analyses of this problem (F. Reinhardt, *Parallelbetreib von Synchronengeneratoren mit Kraftsmaschinenregeln konstanter Verzögerungszeit*, Wissenschaftlich Veröffentlichingen aus dem Siemens Werke vol. 18 (1939) pp. 24–44 and N. Minorsky, *Control problems*, Journal of the Franklin Institute, vol. 232 (1941) especially pp. 524–529) have not been consistent. Results here agree with Reinhardt at the point of the inconsistency, the possibility of stable oscillations in a system with time lag. The results are also in qualitative agreement with N. Minorsky (*Self-excited oscillations in dynamical systems possessing retarded actions*, Journal of Applied Mechanics vol. 64 (1942) pp. A65–71), as regards the fact that if the damping coefficient is larger than the absolute value of the retarded damping coefficient, then the oscillation is stable. (Received March 25, 1946.)

180. Andrew Sobczyk: *Stabilization of carrier-frequency servomechanisms.*

A servomechanism is any power amplifying device which employs a feedback arrangement, whereby the error, or difference of output and input quantities, controls a motor which drives the output in such a manner as to maintain the error as nearly zero as possible. The performance of such a device, as regards relative stability, power amplification, and fidelity of the output as a copy of the independent input, is greatly improved by adding to the control signal which is proportional to the error, a component proportional to the derivative of the error. In this paper, by means of a calculation of the steady-state effect of a linear transfer characteristic on the suppressed-carrier modulated error signal (compare Gardner-Barnes, *Transients in linear systems*, vol. 1, p. 248), the Nyquist-Bode stability criterion is extended to apply to carrier-frequency systems. A proportional-derivative effect on the modulation envelope is accomplished approximately by a band-rejection type of electrical network, which is resonant at carrier frequency. The effects on stability and fidelity of band width, and of the networks being off-tune, are shown graphically by so-called gain-phase margin diagrams, which may easily be constructed for various situations. New design formulae for the stabilizing networks, and the tolerances required on their components, in terms of stability and fidelity, are derived. (Received March 21, 1946.)