tinuous mapping from $S^*$ into Euclidean $xyz$ space. Then $T$ determines a (not necessarily simple) closed surface $S$. Define an index-function $n$ $(x, y, z)$ as follows: if the point $(x, y, z)$ lies on $S$, then $n=0$; if $(x, y, z)$ does not lie on $S$, then $n$ is equal to the topological index of the point $(x, y, z)$ with respect to $S$. Then $n(x, y, z)$ vanishes outside of a sufficiently large sphere $K$. Define $V(S)$, the volume enclosed by $S$, as the integral of $|n(x, y, z)|$ in $K$ if this integral exists, and let $V(S)=\infty$ otherwise. The purpose of the paper is to establish the isoperimetric inequality $V(S)^2 \leqq A(S)^3/36\pi$, where $A(S)$ is the Lebesgue area of $S$, as a generalization of previous results of Tonelli and Bonnesen. (Received May 29, 1946.)

## Statistics and Probability

### 255. Z. W. Birnbaum: *Tshebysheff inequality for two dimensions.*

For independent random variables $X$, $Y$ with expectations zero and variances $\sigma_X^2$, $\sigma_Y^2$ the trivial inequality $P(X^2+Y^2 \geqq T^2) \leqq (\sigma_X^2+\sigma_Y^2)/T^2$ is replaced by a sharp inequality. (Received April 5, 1946.)

### 256. Mark Kac: *A discussion of the Ehrenfest model.* Preliminary report.

A particle moves along a straight line in steps $\Delta$, the duration of each step being $\tau$. The probabilities that the particle at $k\Delta$ will move to the right or left are $(1/2)(1-k/R)$ and $(1/2)(1+k/R)$ respectively. $R$ and $k$ are integers and $|k| \leqq R$. M. C. Wang and G. E. Uhlenbeck in their paper *On the theory of Brownian motion.* II (Review of Modern Physics vol. 17 (1945) pp. 323–342) discuss this random walk problem and state several unsolved problems. In answer to some of the questions raised the following results are obtained: Let $(1-z)^{R-j}(1+z)^{R+j} = \sum C_k^{(j)} z^k$ ($j$ an integer), then the probability $P(n, m|s)$ that a particle starting from $n\Delta$ will come to $m\Delta$ after time $t=s\tau$ is equal to $2^{-2R}(-1)^{R+n}\sum (j/R)^s C_{R+j}^{(-n)} C_{R+m}^{(j)}$, where the summation is extended over all $j$ such that $|j| \leqq R$. Also, if $R$ is even the probability $P'(n, 0|s)$ that the particle starting from $n\Delta$ will come to 0 at $t=s\tau$ for the first time is calculated. For $n=0$ this gives a solution of the so-called recurrence time problem first studied on simpler models by Smoluchowski. Through a limiting process in which $\tau \to 0$, $\Delta \to 0$, $\Delta^2/2\tau \to D$, $1/R\tau \to \beta$, $n\Delta \to x_0$, $m\Delta \to x$, $s\tau = t$, one is led to fundamental distributions concerning the velocity of a free Brownian particle. In particular, $P(n, m|s)$ approaches the well known Ornstein-Uhlenbeck distribution. (Received May 23, 1946.)

### 257. Howard Levene: *A test of randomness in two dimensions.*

A square of side $N$ is divided into $N^2$ unit cells, and each cell takes on the characteristics $A$ or $B$ with probabilities $p$ and $q=1-p$ respectively, independently of the other cells. A cell is an "upper left corner" if it is $A$ and the cell above and cell to the left are not $A$. Let $V_1$ be the total number of upper left corners and let $V_2$, $V_3$, $V_4$ be the number of similarly defined upper right, lower right, and lower left corners respectively. Let $V=(V_1+V_2+V_3+V_4)/4$. It is proved that $V$ is normally distributed in the limit with $E(V)=p(Nq+p)^2$ and $\sigma^2(V)=N^2pq^2(2-10p+22p^2-13p^3)/2$. The conditional limit distribution of $V$, when $p$ is estimated from the data, and the limit distribution of a related quadratic form are also obtained. These statistics are in a sense a generalization of the run statistics used for testing randomness in one dimension. (Received May 28, 1946.)