tinuous mapping from $S^*$ into Euclidean $xyz$ space. Then $T$ determines a (not necessarily simple) closed surface $S$. Define an index-function $n(x, y, z)$ as follows: if the point $(x, y, z)$ lies on $S$, then $n = 0$; if $(x, y, z)$ does not lie on $S$, then $n$ is equal to the topological index of the point $(x, y, z)$ with respect to $S$. Then $n(x, y, z)$ vanishes outside of a sufficiently large sphere $K$. Define $V(S)$, the volume enclosed by $S$, as the integral of $|n(x, y, z)|$ in $K$ if this integral exists, and let $V(S) = \infty$ otherwise. The purpose of the paper is to establish the isoperimetric inequality $V(S)^2 \leq A(S)^1/36\pi$, where $A(S)$ is the Lebesgue area of $S$, as a generalization of previous results of Tonelli and Bonnesen. (Received May 29, 1946.)

**Statistics and Probability**

255. Z. W. Birnbaum: Tshebysheff inequality for two dimensions.

For independent random variables $X, Y$ with expectations zero and variances $\sigma_x^2, \sigma_y^2$ the trivial inequality $P(X^2 + Y^2 \geq T^2) \leq (\sigma_x^2 + \sigma_y^2)/T^2$ is replaced by a sharp inequality. (Received April 5, 1946.)


A particle moves along a straight line in steps $\Delta$, the duration of each step being $\tau$. The probabilities that the particle at $k\Delta$ will move to the right or left are $(1/2)(1-k/R)$ and $(1/2)(1+k/R)$ respectively. $R$ and $k$ are integers and $|k| \leq R$. M. C. Wang and G. E. Uhlenbeck in their paper On the theory of Brownian motion. II (Review of Modern Physics vol. 17 (1945) pp. 323-342) discuss this random walk problem and state several unsolved problems. In answer to some of the questions raised the following results are obtained: Let $(1-z)^R/(1+z)^R = \sum C_{-i}^{2R} z^i$ be a sum (j an integer), then the probability $P(n, m|s)$ that a particle starting from $n\Delta$ will come to $m\Delta$ after time $t = sr$ is equal to $2^{-R}(-1)^R \sum (f/R) C_{-i}^{2R} C_{-i+1}^{2R}$, where the summation is extended over all $j$ such that $|j| \leq R$. Also, if $R$ is even the probability $P'(n, 0|s)$ that the particle starting from $n\Delta$ will come to 0 at $t = sr$ for the first time is calculated. For $n = 0$ this gives a solution of the so-called recurrence time problem first studied on simpler models by Smoluchowski. Through a limiting process in which $\tau \to 0$, $R \to \infty$, $R^2/2\tau \to D$, $l/R \to \epsilon$, $n\Delta \to X_0$, $mA \to X$, $sr \to t$, one is led to fundamental distributions concerning the velocity of a free Brownian particle. In particular, $P(n, m|s)$ approaches the well known Ornstein-Uhlenbeck distribution. (Received May 23, 1946.)

257. Howard Levene: A test of randomness in two dimensions.

A square of side $N$ is divided into $N^2$ unit cells, and each cell takes on the characteristics $A$ or $B$ with probabilities $p$ and $q = 1 - p$ respectively, independently of the other cells. A cell is an "upper left corner" if it is $A$ and the cell above and cell to the left are not $A$. Let $V_1$ be the total number of upper left corners and let $V_2, V_3, V_4$ be the number of similarly defined upper right, lower right, and lower left corners respectively. Let $V = (V_1 + V_2 + V_3 + V_4)/4$. It is proved that $V$ is normally distributed in the limit with $E(V) = p(2q + p)$ and $\sigma^2(V) = N^2pq^2 - 10p + 22q^2 - 13pq)/2$. The conditional limit distribution of $V$, when $p$ is estimated from the data, and the limit distribution of a related quadratic form are also obtained. These statistics are in a sense a generalization of the run statistics used for testing randomness in one dimension. (Received May 28, 1946.)