Topology

258. R. F. Arens and J. L. Kelley: *Characterization of the space of continuous functions.*

If B is a Banach space then either of the following sets of conditions is necessary and sufficient that B be the space of continuous real-valued functions over some compact Hausdorff space. I: (a) If E is the unit sphere of B then E has at least one extreme point. (b) If C is a maximal, convex subset of the surface S of E, then C and $-C$ span, that is, if $f\in E$, then $f=tf_1+(t-1)f_2$, $0\leqq t\leqq 1$, f_1, $f_2\in C$. (c) If a set $\mathcal{A}$ of maximal convex subsets of S has a void set theoretic intersection, then one may find directed sets C_μ, C_ν in $\mathcal{A}$, such that for each $f\in E$, dist $(f, C_\mu)+$ dist $(f, C_\nu)\rightarrow 2$. II: (a) If Σ is the unit sphere in B^*, then there is an f in B such that for any extreme point ξ of Σ, $|\xi(f)|=1$. (b) If $\mathcal{A}$ is any collection of extreme points of Σ, and if, in the topology for B^* determined by the elements of B, $\mathcal{A}$ has no pair of diametric limit points, then there is an f in B such that, for $\xi\in\mathcal{A}$, $\xi(f)=1$. (Received May 27, 1946.)

259. Felix Bernstein: *A new four-color problem.*

Let S be a partial set of the set of all points of the lattice whose coordinates are all the positive and negative integers, and T the complementary set. Two points of S, A and B, are called neighbors if the circle over the diameter AB in its interior (or in its interior and on its periphery) does not contain a point of the set T. It is asked how many colors are needed, coloring all points of S so that no two neighbors have the same color. The answer for $T\equiv 0$ obviously is four. It is suggested that four is the general answer. It seems impossible to prove the theorem by complete induction from n to $n+1$ unless the definition of neighborhood is much weakened. In order to clarify this further, the inequality $(1+x)^n>nx^{n-1}+(n(n-1)/2)x^{n-2}+\cdots+1$ is discussed as an example of a theorem whose direct proof by complete induction seems not possible while the more inclusive binomial theorem itself from which this inequality follows can be derived by complete induction. By analogy the four-color theorem may be a simple consequence of a more inclusive theorem which can be proved by complete induction. (Received May 17, 1946.)

260. W. H. Gottschalk and G. A. Hedlund: *Recursive properties of transformation groups.*

Let the mapping f: $X\times T\rightarrow X$ define a transformation group, where X is a topological space and T is an additive abelian locally compact topological group. Let there be distinguished in T certain nonvacuous sets, called *admissible*, which satisfy this condition: If A is an admissible set and if B is a set in T such that for some compact set C in T we have $a\in A$ implies $B\cap(a+C)\neq\emptyset$, then B is an admissible set. If $x\in X$ and if S is a set in T, then x is said to be *recursive* with respect to S provided that if U is a neighborhood of x, then there exists an admissible set A such that $A\subset S$ and $f(x, A)\subset U$. Let G be a subgroup of T such that $T=F+G$ for some compact set F in T. Define H to be the set of all elements t of T such that $x\in\bar{f}(x, t+G)$. It is proved that if H is a group and if x is recursive with respect to T, then x is recursive with respect to G. Twelve results concerning almost periodic, recurrent and strongly recurrent points follow immediately. (Received April 9, 1946.)

261. R. H. Sorgenfrey: *On the topological product of paracompact spaces.*

A space S is defined by Jean Dieudonné (J. Math. Pures Appl. vol. 23 (1944) pp. 65–76) to be *paracompact* provided that every covering of S by open sets has a neighborhood-finite refinement which covers S. In this note a negative answer is given to the question: If S is paracompact, is $S \times S$ paracompact? The example given also provides a simple instance of a normal Hausdorff space S such that $S \times S$ is not normal. (Received April 16, 1946.)