

a curve  $C_\lambda$ , the general transformation of Čech, the curves of Darboux and Segre, and a class of generalized pangeodesics. (Received September 9, 1946.)

391. Walter Prenowitz: *Characterization of the lattice of convex sets of a descriptive geometry.*

$L_1$ , the lattice of convex sets of a descriptive (ordered linear) geometry, and  $L_2$ , that of the linear spaces of a projective geometry, are characterized simultaneously. The geometries are of arbitrary dimension greater than 1. A new lattice concept *linear dependence of a point on a sequence of points* is introduced and used to define *closed element* as one which contains, with any sequence of points, all linearly dependent points. A generalization of modularity involving "closed element" is a common property of  $L_1$  and  $L_2$ . Garrett Birkhoff's characterization of  $L_2$  in the finite-dimensional case is deducible from the results. The simultaneous treatment of  $L_1$  and  $L_2$  is based on axiomatizations of descriptive and projective geometry in terms of point and "order" which differ only in a single postulate. (Received September 28, 1946.)

392. Alice T. Schafer: *The neighborhood of an undulation point on a space curve.*

This paper employs the methods of projective differential geometry to study the neighborhood of an undulation point on an analytic space curve. By a suitable choice of the projective coordinate system, canonical power-series expansions representing the curve in the neighborhood of the undulation point are deduced. These expansions are then used to study properties of the curve in the neighborhood of the point, with particular emphasis placed on osculants of the curve, projections of the curve onto the faces of the tetrahedron of reference, and sections of the tangent developable of the curve made by faces of the tetrahedron of reference. (Received September 28, 1946.)

393. Oscar Zariski: *The concept of a simple point of an abstract algebraic variety.*

Let  $V$  and  $W$  be irreducible algebraic varieties over an arbitrary ground field  $\kappa$ , of dimension respectively  $r$  and  $\rho$ ,  $W \subseteq V$ . If  $\mathfrak{m}$  is the maximal ideal in the quotient ring  $\mathfrak{o}$  of  $W$  then the ring  $\mathfrak{m}/\mathfrak{m}^2$ , regarded as a vector space over the field  $\mathfrak{o}/\mathfrak{m}$ , is of dimension not less than  $r - \rho$ . If the dimension is exactly  $r - \rho$ ,  $W$  is said to be simple for  $V$ . In the first part of the paper this general concept of a simple  $W$  is studied by purely local methods. In the second part the global theory is developed. Here the main result is a general Jacobian criterion for simple loci, which reduces to the classical one whenever  $\kappa$  is either of characteristic zero or is a perfect field of characteristic  $p \neq 0$ . This general criterion implies that the singular manifold of  $V$  is always an algebraic proper subvariety of  $V$ . An absolutely simple  $W$  is defined by the condition that it remain simple under any extension of the ground field. Criteria for an absolutely simple  $W$  are: (1) the ordinary Jacobian criterion of the classical case; (2)  $V$  is locally, at  $W$ , analytically equivalent to a linear  $S_r$ . (Received August 22, 1946.)

#### LOGIC AND FOUNDATIONS

394. E. L. Post: *Recursive unsolvability of a problem of Thue.*

Thue's problem (Skrifter utgit av Videnskapsselskapet i Kristiania 1914. I.

Matematisk-Naturvidenskabelig Klasse, no. 10) may be restated as follows. Given an arbitrary finite set of symbols, with  $A$ 's and  $B$ 's arbitrarily given strings (zeichenreihen) involving no other symbols than those in the given set,  $P$  and  $Q$  operational variables, to determine whether  $B$  is an assertion in the system with initial assertion  $A$  and operations  $P A_i Q$  produces  $P B_i Q$ ,  $P B_i Q$  produces  $P A_i Q$ ,  $i=1, 2, \dots, \mu$ . Through the intermediary of the Turing machine, a known recursively unsolvable decision problem is reduced to the decision problem of a system with initial assertion  $A'$  and operations  $P A'_i Q$  produces  $P B'_i Q$ ,  $i=1, 2, \dots, \mu'$ , having the property that the set of assertions of the system is unchanged when the system is transformed into Thue type by adding the inverse operations  $P B'_i Q$  produces  $P A'_i Q$ . The recursive unsolvability of the problem of Thue easily follows. (Received September 20, 1946.)

### STATISTICS AND PROBABILITY

#### 395. H. W. Becker: *Rooks and rhymes.*

Kaplansky and Riordan have shown that  $cR_{r-1}$ , or the number of ways of putting  $c$  non-attacking rooks on a right-angled isosceles triangle of side  $r-1$ , is the Stirling number  $\Delta^{r-c} 0^r / (r-c)!$ . This is the number of selections of  $c$  points on such a chess board, such that none have any row or column index in common, an idea incidental to various statistical problems. The point sets are well ordered, in 1-to-1 correspondence with the sequenations (rhyme schemes) and distribution cycles. Further classifications  ${}^{(a)}R_r$ ,  ${}^a R_r$ ,  ${}^o R_r$ , and  ${}_{(a)}R_r$  are formulated in terms of rhyme functions according to: row location of topmost rook; number of rooks in the principal diagonal; column vacancies; and column location of the bottom rook. A typical isomorphism is  ${}^o R_r$  is equal to the  $R_r$  with  $c$ th column empty is equal to the number of distributions of  $r+1$  men into crews, such that one man is incompatible with, and must be segregated from,  $r-c$  other men. (Received July 24, 1946.)

#### 396. Nilan Norris: *An extension of an equality among averages.*

A classic theorem of algebra states that if  $A$ ,  $G$ , and  $H$  are respectively the arithmetic, geometric, and harmonic means of two positive real numbers, then  $G^2 = AH$ . In this paper proof is given that a sufficient condition for the extension of this equality to any  $n$  positive numbers is that the logarithms of the variates be symmetrically distributed about an axis of ordinates at  $\log G$  of the  $n$  numbers. For samples and populations obeying the symmetry condition with respect to  $\log G$ , the theorem is extended to an unlimited number of averages as yielded by certain sample and integral forms of general means (generalized means value functions). (Cf. an unpublished manuscript of J. B. Canning, *A theorem concerning a certain family of averages of a certain type of frequency distribution.*) (Received August 15, 1946.)

### TOPOLOGY

#### 397. Salomon Bochner and Deane Montgomery: *Groups on analytic manifolds.*

This paper studies the nature of complex and real Lie groups acting on certain complex or real manifolds in the large. For example, it proves that the group of all complex analytic homeomorphisms of a compact complex manifold is a complex Lie group. (Received September 20, 1946.)