we obtain a result by T. Carleman (*L'integrale de Fourier*, Uppsala, 1944, p. 42). (Received October 19, 1946.)

61. J. W. T. Youngs: *Remarks on the isoperimetric inequality for closed Fréchet surfaces.*

The author treats the particular isoperimetric inequality $36\pi V^2 \leqq A^3$, where A is the Lebesgue area of a closed Fréchet surface S, and V is the volume enclosed by S. These terms are defined in a forthcoming paper by Radó, who establishes the inequality for every S. In the present paper, the purpose is twofold: first, to offer a variation of the definition of enclosed volume which generally increases the left side of the inequality; second, to investigate the consequences of equality, under both definitions of enclosed volume. The basic concept employed is that of the cyclic decomposition of a closed Fréchet surface S. It is shown, for example, that if $36\pi V^2 = A^3$ and $0 < A < \infty$, the cyclic decomposition of S reduces to precisely one closed surface. (Received November 20, 1946.)

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62. N. R. Amundson: *Unsymmetrically loaded orthotropic thin plates on elastic foundations.*

The author considers orthotropic thin plates of infinite extent on elastic foundations of two different kinds. In the first case the foundation exerts a reactive pressure proportional to the deflection. The solution of the plate equation for this case, $D(u) = au_{xxxx} + 2bu_{xxyy} + cu_{yyyy} = q(x, y) - ku(x, y)$ with suitable conditions at infinity, is $u(x, y) = (4\pi^2)^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\alpha, \beta)d\alpha d\beta \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} q(s, t)\exp(-i[\alpha(x-s)+\beta(y-t)])dsdt$ where $q(x, y)$ is the loading function and $F(\alpha, \beta) = (a\alpha^4 + 2b\alpha^2\beta^2 + c\beta^4 + k)^{-1}$. In the second case the foundation is the homogeneous istropic semi-infinite medium of classical elasticity theory (Timoshenko, *Theory of elasticity*, p. 332). The solution of the plate equation for this case, $D(u) = q(x, y) - p_s(x, y)$, where $p_s(x, y)$ is the reactive pressure of the foundation, is obtained from an equivalent integral equation, $u = k\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[q(s, t) - D(u)]rdsdt$, and is $u(x, y) = (4\pi^2)^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\alpha, \beta)d\alpha d\beta \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} q(s, t)\exp(-i[\alpha(x-s) + \beta(y-t)])dsdt$ where $r = [(x-s)^2 + (y-t)^2]^{-1/2}$ and $G(\alpha, \beta) = [(\alpha^2+\beta^2)^{1/2}(2\pi k)^{-1} + a\alpha^4 + 2b\alpha^2\beta^2 + c\beta^4]^{-1}$. These solutions are obtained by the use of the double Fourier transform. The solution of the first problem can also be obtained from the finite plate as a limiting case. Various special cases are considered. This work is connected with that of Holl, Hogg, Woinowski-Krieger, Happel, Lewe, et al. (Received October 25, 1946.)

63. J. W. Beach: *Flow of slow viscous fluid between rotating cylinders.*

A solution is obtained for the biharmonic equation in the region between eccentric cylinders when the stream function and its normal derivative are given on both cylinders. The solution is set up in terms of two functions of a complex variable and a transformation of co-ordinates is made so that the bounding cylinders become concentric. The form of the two functions is determined with the single-valued parts given as infinite series. Unknown coefficients are determined and the solution obtained. The limit of this solution as the cylinders become concentric is obtained and agrees with the known solution for concentric cylinders. (Received October 23, 1946.)

64. F. E. Bortle: *Boundary problem in partially clamped rectangular plates.*

The problem of a rectangular plate with two adjacent edges clamped and the other edges simply supported is solved by the method of superposition. In order to satisfy the edge conditions, one is lead to linear infinite systems of equations in infinitely many unknowns. It is shown that a solution of the infinite systems exists. A numerical example is carried out. (Received October 24, 1946.)

65. D. G. Bourgin. *Representations of compositions.*

A representation theory is developed in terms of transforms, analogous to the Laplace transform, for the composition of certain kernels not of the closed cycle type. (Received October 26, 1946.)

66. G. W. Brown. *The stability of feedback solutions of simultaneous linear equations.*

The feedback solution of n simultaneous equations, represented by the equation $y=Ax$, where A is a square matrix of rank n with $-1\leqq A_{ij}\leqq 1$, involves n identical high gain amplifiers, whose vector input is $y-Ax$, where x is the n-vector output of the system of amplifiers. If the amplifier gain is sufficiently high at the signal frequency the steady state vector x is a good approximation to the solution of the system $y=Ax$, essentially independent of the actual gain. The stability of such a simultaneous system is equivalent to the simultaneous stability of the n orthogonal loops, each consisting of a single amplifier identical to those mentioned above, with a feedback network of characteristic λ_i, where $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the characteristic roots of the matrix A. Sufficient conditions are imposed on the amplifier characteristics to insure stable solution of simultaneous systems whose characteristic roots all have positive real parts. Applications to least squares solutions and universal stability are discussed. In particular, the least squares solution for m equations of rank $r\leqq m$ can be obtained with universal stability, using $m+r$ amplifiers with realizable amplifier characteristics. (Received October 21, 1946.)

67. R. E. Carr: *Enantiomorphism in mathematical models of organic molecules.* Preliminary report.

Organic molecules can be described mathematically as skeletal frameworks with n available positions $x_1, x_2, \cdots, x_n$, to each of which is attached a chemical radical (figure). The available radicals (figure supply) can be described as $\Phi^{(\lambda)}$ $(\lambda=1, 2, 3, \cdots)$ where $\Phi^{(\lambda)}$ contains α_λ atoms of type α, β_λ atoms of type β, and so on. Molecules with the same sets of atoms which differ in space arrangement are called stereoisomers. Those stereoisomers which are not superimposable on their own mirror images are called enantiomorphs. A method of enumerating stereoisomers has been published by G. Pólya (Acta Math. vol. 68 (1937) pp. 145–253). Basic in his method is definition of a permissible group of interchanges of radicals between the positions on the skeletal framework. Since reflection through a mirror sometimes changes the space arrangement of the radicals as well as their positions on the skeletal framework, the method of Pólya is not directly applicable to enumeration of enantiomorphs. In this paper new skeletal frameworks and new figure supplies are abstracted from those used in enumeration of stereoisomers, making it possible for the author to apply the method of Pólya to enumeration of the enantiomorphous alcohols, alkenes, and paraffines. (Received October 8, 1946.)

68. Nathaniel Coburn: *Application of the Kármán-Tsien relation to the two-dimensional supersonic flow of fluids in jets.*

The application of the Kármán-Tsien relation to the two-dimensional, supersonic, steady, irrotational flow of a compressible fluid in a symmetric jet issuing from two straight walls inclined at an angle θ to the x-axis and bounded by two free stream lines is considered. By mapping the flow into the hodograph plane, a boundary value problem for the stream function as a solution of the wave equation is obtained. Thus, the problem reduces to the determination of the proper regions over which the wave equation may be integrated. By use of some mapping theorems which relate the hodograph plane to the plane determined by the stream function and the velocity potential, these regions are determined and the proper solutions of the wave equation are found. These solutions are then mapped into the physical plane by use of some general results previously obtained by the author (Quarterly of Applied Mathematics vol. 3 (1945). (Received October 30, 1946.)

69. H. S. M. Coxeter: *The product of three reflections.*

The product of reflections in the sides of a spherical (or plane) triangle is reduced to a rotatory reflection (or glide-reflection) consisting of the reflection in a side of the pedal triangle combined with a rotation (or translation) along that side, of an amount equal to the perimeter of the pedal triangle. In the spherical case this is essentially the dual of a result of Synge (Quarterly of Applied Mathematics vol. 4 (1946) pp. 166–176). In the plane case it is a corollary of a result of Schwarz (Ges. Math. Abhandlungen vol. 2 (1890) pp. 344–345). The analogous problem for m reflections in m dimensions is mentioned briefly. (Received October 17, 1946.)

70. Bernard Epstein: *A method for the approximate solution of the Dirichlet problem for certain types of domains.* Preliminary report.

Let a plane domain S be divisible into two nonoverlapping domains for each of which the Green's function is known, the division being performed by adding arcs to the boundary of S. The solution to the Dirichlet problem for the domain S is ex-expressed in each of the two sub-domains in terms of the prescribed boundary values, the (unknown) values of the solution along the added arcs, and the Green's functions. By expressing the mean-value property along the added arcs there is then obtained an integral equation of the second kind with singular kernel for the values of the solution along the added arcs. This equation can be solved by a method of successive approximations (not requiring the iteration of the kernel). The solution inside the two sub-domains may then be obtained by the usual formula involving the boundary values and the Green's function. The same underlying idea may be applied when the original domain is divided into three or more sub-domains. In particular the method appears promising in the case where S consists of the entire plane cut by a finite number of collinear slits. Several numerical examples are presented for this type of domain. (Received November 18, 1946.)

71. A. E. Heins: *The scattering and transmission properties of a pair of semi-infinite parallel plates.*

The excitation of a pair of semi-infinite parallel metallic plates of perfect conductivity and zero thickness by an electromagnetic plane wave may be formulated as a pair of simultaneous Wiener-Hopf integral equations for the surface current density. It is shown how these equations may be solved by two different methods. One method takes advantage of the symmetry of the structure and the one uses a method of matrix decomposition. Both techniques employ the Fourier transform in the complex domain (Paley and Wiener, *The Fourier transform in the complex domain*, Amer. Math. Soc. Colloquium Publications, vol. 19, chap. 4). From the Fourier transforms of the surface current densities, it is possible to calculate the physically interesting parameters. (Received November 19, 1946.)

72. Herbert Jehle: *Hydrodynamical self-consistent fields in stellar dynamics.*

In order to avoid mathematically too involved problems, stellar dynamics was usually confined to investigations of steady solutions of Boltzmann-Gibbs' equation $0=\partial f/\partial t=\sum_1^3\{\partial f/\partial t+v_\nu\partial f/\partial x_\nu-(\nabla_\nu\overline{U})\partial f/\partial v_\nu\}$. Of particular interest are two extensions of this problem, (1) to include time-dependent solutions with given $\overline{U}$, and (2) to investigate self-consistency, that is, a combination of the above hydrodynamical equation with the Poisson equation between $\overline{U}$ and the smoothed-out density $\rho=\int\int\int f dv_1 dv_2 dv_3$, a nonlinear problem. These problems, however, still admit reversibility in time. It is only when the statistical shaking effect of force fluctuations is introduced (causing a nonvanishing $\partial f/\partial t$ and thereby a slow diffusion superimposed on the smooth streaming in phase space) that we can expect to get the f distribution tending to develop in one direction. The case $\overline{U}=0$ is exemplified in the kinetic theory of gases by the approach towards a Maxwell-Boltzmann distribution, whereas the case $\overline{U}\neq 0$ concerns the Brownian movement in a field of force. The time-dependent and the self-consistent transient case provides some particularly interesting problems as it establishes a trend of evolution for spatial distributions $\rho(x_1, x_2, x_3, t)$. (Received November 21, 1946.)

73. Cornelius Lanczos and G. E. Forsythe: *Approximation of the eigen-vectors of a nonsymmetric matrix.* Preliminary report.

The eigen-values λ_k, the eigen-vectors $\boldsymbol{u}_k$ of a given square matrix $A=\|a_j^i\|$ $(i, j=1, 2, \cdots, n$; a_j^i complex numbers), and the eigen-vectors $\boldsymbol{u}_k'$ of the transposed matrix $A'=\|a_i^j\|$ are defined by the relations $A\boldsymbol{u}_k=\lambda_k\boldsymbol{u}_k$, $A'\boldsymbol{u}_k'=\lambda_k\boldsymbol{u}_k'$ $(k=1, 2, \cdots, n)$. Suppose $|\lambda_1|>|\lambda_2|>\cdots>|\lambda_n|$. In a certain type of flutter analysis it is necessary to approximate λ_1, λ_2, λ_3 and $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$. In the usual method (see Frazer, Duncan and Collar, *Elementary matrices*, Cambridge, 1938, pp. 140–145) λ_1, $\boldsymbol{u}_1$, $\boldsymbol{u}_1'$ are approximated by a pair of iterative processes of form $A^m\mathbf{x}$, $(A')^m\mathbf{x}$. A is then reduced to a new matrix of order $n-1$, and the entire procedure is repeated for each new λ_k, $\boldsymbol{u}_k$ $(k=2, 3)$. The present investigation shows how to approximate λ_2, $\boldsymbol{u}_2$, λ_3, $\boldsymbol{u}_3$ from the same iterations which yield λ_1, $\boldsymbol{u}_1$, $\boldsymbol{u}_1'$. The process is largely self-checking, and trials indicate it to be more accurate in several respects than the usual procedure. For $5\leqq n\leqq 8$ the elimination of matrix reductions and iterations results in a saving of from 45 to 70 per cent of the usual lengthy computation time. A separate procedure, based on the moments $\sum_k\lambda_k^m$, is useful when only λ_1, λ_2, λ_3 are desired. The latter procedure, when used to approximate the complex roots of an algebraic equation, has advantages over the customary method of Graeffe. (Received October 18, 1946.)

74. Kenneth May: *Technological change as a functional variation.*

Innovations, or more generally any qualitative changes in the productive process, may be represented as modifications in the production functions relating output to input of factors. Innovation is introduced in this manner into a static macroeconomic model and results derived in terms of the character of the functional increment. The dynamic implications of technological change are studied by introducing into Keynes' equation of effective demand a production function varying with time. The resulting differential equation provides a simplified model of the trade cycle in terms of employment, profits, wages, capital accumulation and innovation. (Received October 19, 1946.)

75. Kenneth May: *The aggregate effect of technological changes in a two-industry model.*

It has been shown that a general economic equilibrium system of one degree of freedom determines a one-industry model, $U=\psi(N)$ and $p=\psi'(N)$, where N, U and p are suitable aggregates of employment, output, and the real wage rate (Econometrica vol. 14 (1946) pp. 285–298). The function ψ depends on the functions of the general equilibrium. If the latter is Evans' two-industry system, which involves the production functions $\phi(U_1^{(1)}, N_1)$ and $\theta(U_1^{(3)}, N_3)$, $\psi\equiv\theta(U_1^{(3)}, N_3)$ where $U_1^{(3)}$ and N_3 are replaced by their values in terms of N found by solving the equations of the two-industry model. It is shown that if technological changes occur in the two industries, so that ϕ and θ are subjected to small functional changes $\delta\phi$ and $\delta\theta$, then the resulting aggregate technological change is given by $\delta\psi(N)=\delta\theta[U_1^{(3)}(N), N_3(N)]+\theta_1[U_1^{(3)}(N), N_3(N)]\delta\phi[U_1^{(1)}(N), N_1(N)]$. The method is applicable to studying the effect upon any macro-economic model of changes in the functions involved in a micro-economic model from which it can be derived. (Received November 18, 1946.)

76. W. B. Stiles. *Bending of clamped plates.*

A method of obtaining approximate solutions for small deflections of thin elastic plates with clamped edges, based on the minimization of energy, is presented in this paper. This solution was proposed by Weinstein and applied to a square plate with a uniform load by Weinstein and Rock. The method is extended here to include rectangular plates with either uniform or point loads and also to plates with mixed boundary conditions in which part of the boundary is clamped and the remainder is pinned. The approximation functions are obtained from the characteristic functions of static and vibrating membranes. Analytical results are compared with experimentally determined deflections and stresses. (Received October 21, 1946.)

77. J. L. Synge: *The general problem of antenna radiation.*

An antenna is regarded as a closed surface, some of this surface being a perfect conductor ($\boldsymbol{E}_{\text{tang}}=0$) and the rest being a "gap" ($\boldsymbol{E}_{\text{tang}}$ assigned). A Maxwellian field with time factor exp $(-ikct)$ is assumed to exist outside the antenna, and to satisfy at infinity the usual conditions of outward radiation. Application of the method of Helmholtz gives an integro-differential equation for $\boldsymbol{H}_{\text{tang}}$ on the surface of the antenna. In the case of an antenna of revolution, considerable simplification takes place, and a comparatively simple *exact* integro-differential equation for the current in the antenna is obtained. In seeking approximate solutions, two parameters are involved,

η/a (the ratio of the length of the gap to the diameter of the antenna) and η/l (the ratio of the length of the gap to the length of the antenna). The classical sinusoidal current distribution is obtained in the limiting case where η/a is large and η/l is small. A general method of successive approximations is set up, but no proof of convergence is given. (Received October 31, 1946.)

78. Gabor Szegö: *The capacity of a circular plate-condenser.*

Consider two thin circular discs of radius a with a common axis and at a distance d, $d/a=q$, charged to constant and opposite potentials, $V=\pm V_1$. If the charges are $\pm Q$, respectively, the constant $C=Q/V_1$ is called the capacity of the condenser. G. Kirchoff (*Gesammelte Abhandlungen*, p. 112) gave the following approximate formula for this important quantity: $a^{-1}C=(4q)^{-1}+(4\pi)^{-1}\log(1/q)+\alpha(q)$, $\limsup\alpha(q)\leqq(4\pi)^{-1}\cdot(\log(16\pi)-1)=K$ as $q\to 0$. Recently (Acad. des Sciences l'URSS, 1932) Ignatowsky gave the following sharper result: $\lim\alpha(q)=(4\pi)^{-1}(\log 8-1/2)=I$. The proofs are in both cases somewhat incomplete. In the present paper Kirchoff's proof is revised by using Dirchlet's principle. Moreover by means of the so-called Thomson principle a very simple proof is given for $\liminf\alpha(q)\geqq I$. Finally the case of thick plates is discussed. (Received November 23, 1946.)

79. H. L. Turrittin: *Stokes multipliers for asymptotic solutions of a certain differential equation.*

If v is a positive integer, the differential equation $d^ny/dx^n-x^vy=0$, $n\geqq 2$, has n independent solutions $y_j=x^j(1+a_{1j}x^p+a_{2j}x^{2p}+\cdots+a_{mj}x^{mp}+\cdots)$, $p=v+n$, convergent for all x. If the complex x-plane, $x=re^{i\theta}$, is divided into $2p$ sectors by the radial lines $\theta=h\pi/p$, $h=0, 1, \cdots$, Trjitzinsky (Acta Math. (1934) pp. 167–226) has shown that to each sector there corresponds n independent solutions $\tilde{y}_k\sim\xi_k^{v(1-n)/2p}\exp\xi_k\{1+b_1/\xi_k+b_2/\xi_k^2+\cdots\}$ where $\xi_k=(n/p)x^{p/n}e^{2\pi ik/n}$. These asymptotic representations are valid *uniformly* throughout the sector (edges included). Therefore there exists a nonsingular linear relationship $y_j=\sum_{k=0}^{n-1}c_{jk}\tilde{y}_k$, $j=0, 1, \cdots, n-1$. These constants c_{jk}, which change from sector to sector, are the *Stokes multipliers* that have been computed. To do so the author borrowed heavily from the Ford-Newsom-Hughes theory of asymptotic expansion (Bull. Amer. Math. Soc. vol. 51 (1945) pp. 456–461). However this theory does not yield directly the desired uniform asymptotic representation in all cases, nor even the desired form when the real part of ξ_k is negative. The F–N–H theory is extended to supply the requisite information. Scheffé (Trans. Amer. Math. Soc. vol. 40 (1936) pp. 127–154) computed two of the n multipliers corresponding to each j. (Received October 7, 1946.)

Geometry

80. L. M. Blumenthal: *Superposability in elliptic space.* II.

Let f denote a one-to-one correspondence between the points of two subsets P, Q of the elliptic space $E_{n,r}$. Two corresponding subsets A_P, B_Q of P, Q, respectively, are called f-superposable provided there exists a congruence Γ of $E_{n,r}$ with itself which gives the same correspondence between A_P and B_Q as f does. The writer defines a space to have superposability order σ provided any two subsets of the space are superposable whenever a one-to-one correspondence f between the points of the subsets exists such that each two corresponding σ-tuples are f-superposable. A principal result of this paper is that $E_{n,r}$ has minimum superposability order $n+1$. Two subsets