under T. The enumeration of such sets for arbitrary n is carried out for general symmetric and nonsymmetric correspondences. (Received February 28, 1947.)

262. R. M. Robinson: *On the decomposition of spheres.*

According to the "Banach-Tarski paradox" (Fund. Math. vol. 6 (1924) pp. 244–277), it is possible to cut a solid unit sphere into a finite number of pieces, and reassemble these by translation and rotation to form two solid unit spheres. Recently, Sierpinski (Fund. Math. vol. 33 (1945) pp. 228–234) showed that the total number of pieces may be taken as eight, three pieces being used to form one of the new spheres and five the other. In this paper, it is shown that the smallest possible total number of pieces is five; one of these pieces may be taken to consist of a single point. For the surface of the sphere, a similar result is true with four pieces. This follows from the fact that the surface of a sphere can be divided into two pieces, each of which can be subdivided into two pieces congruent to itself. More generally, the surface of a sphere may be decomposed into pieces satisfying any system of congruences, provided that it is not demanded, explicitly or implicity, that two complimentary portions of the surface be congruent. (Received March 3, 1947.)

Logic and Foundations

263. J. C. C. McKinsey and Alfred Tarski: *Some theorems about the sentential calculi of Lewis and Heyting.*

In this paper the authors prove certain theorems regarding systems of sentential calculus, by making use of results they have established elsewhere regarding closure algebras and Brouwerian algebras. Some of the results are new (in particular it is shown that there are infinitely many functions of one variable in the Heyting calculus, and various theorems about extensions of the Lewis system S4 are obtained); others have been stated without proof in the literature (in particular the authors establish some theorems due to Gödel, which enable one to translate the Heyting calculus into the Lewis system S4). (Received March 21, 1947.)

264. Ira Rosenbaum: *A method of determining the nth q-ary truth-function in m-valued logic.*

The following method is believed useful since truth-functional modes of propositional combination are very numerous in m-valued logic. There are, for example 19,683 distinct binary modes of combination in three-valued logic and m^{m^q} q-ary modes in m-valued logic. Denote these truth-functional modes of propositional combination by F_i, $i=1, 2, \cdots, m^{m^q}$. Let each function be regarded as determined by the truth-values it assigns to each combination of truth-values of its components or arguments. Let $\{n, r\}$ denote the truth-value assigned to the rth of the m^q q-ary truth-combinations by the nth q-ary truth-function, let $\iota x(\cdots)$ denote the one and only object x satisfying the condition $\cdots$, and let $[---]$ denote the integral part of $---$. Then the nth q-ary truth-function of m-valued logic is defined thus: $F_n=(\{n, 1\}, \{n, 2\}, \cdots, \{n, m^q\})$ where $\{n, 1\}=\iota x(1\leqq x\leqq m \cdot\&\cdot x\equiv n \pmod{m})$ and $\{n, k\}=\iota x(1\leqq x\leqq m \cdot\&\cdot x\equiv[(n-\{n, 1\})/m^{k-1}+1] \pmod{m})$, $1<k\leqq m^q$. The above formulae determine the nth q-ary truth-function of m-valued logic without the construction of tables with m^{m^q} columns, provide a simple nomenclature for the numerous functions of m-valued logic, and are valid also for two-valued logic. A simple process for determining n, given the values $\{n, 1\}, \cdots, \{n, m^q\}$, is also available. (Received March 6, 1947.)

265. Ira Rosenbaum: *A method of determining the number n correlated with the given truth table of an arbitrary q-ary function in m-valued logic.*

In a previous paper, 1-1 correlation formulae were obtained between the numbers $1, 2, \cdots, m^{m^q}$ and the truth-tables of the m^{m^q} distinct q-ary truth-functional modes of propositional combination of m-valued logic. In the present paper a formula is given for proceeding in the reverse direction, that is, from the given truth-table of an arbitrary q-ary function to the number n correlated with that table. The original simple procedure for going from a given table to the number n associated with it lacked analytical representation. W. V. Quine suggested that this procedure was analogous to transforming the expression for an integer in the m-ary scale of notation into one in the denary scale of notation. This suggestion led to the following result. Let the m^q truth values in the given table be $V_1, V_2, \cdots, V_{m^q}$. From each of these values subtract one to obtain the values $W_1, W_2, \cdots, W_{m^q}$. The latter values are integers x of the range $0 \leqq x \leqq m-1$ rather than, like the V's, of the range $1 \leqq x \leqq m$. Hence the sequence of W's may be regarded as representing in the m-ary scale of notation an integer n. The mode of representation is indicated by the formula $n-1 = \sum_{j=0}^{c-1} W_{c-j} \cdot m^{c-1-j}$ in which $c = m^q$. (Received February 20, 1947.)

Statistics and Probability

266. Z. W. Birnbaum: *Probabilities of sample-means for bounded random variables.*

Lower bounds are given for the probabilities $P(|\overline{X}_n| \geqq t)$, where $t \leqq a$, $\overline{X}^n = (1/n)\sum_{j=1}^{n} X_j$, and $X_1, X_2, \cdots, X_n$ is a sample of a continuous random variable with a probability density $f(X)$ such that: $f(X) = f(-X)$, $f(|X|)$ is a nonincreasing function of $|X|$, and $f(X) = 0$ for $|X| \geqq a$. (Received March 21, 1947.)

Topology

267. Felix Bernstein: *A lattice color problem.* Preliminary report.

The points of the lattice L of all points with coordinates which are integers or are centers of the elementary squares of L are considered as the regions of a color problem. In a partial set S two points A and B are called neighbors if AB does not contain another point of S and if the distance AB is equal either to 1 or to $2^{1/2}/2$ or to $2^{1/2}$. The number of colors required in order that two neighbors may be colored differently is obviously 5 or less. It is shown that 5 colors are necessary. For the proof two methods are used. The one method is based on the studying of color schemes at certain conveniently chosen boundaries in the manner introduced by G. D. Birkhoff. The other method is based on the study of the effect of the "centers" on the coloring of the total neighborhood. The efficiency of each method varies with the nature of the given configuration. (Received March 22, 1947.)

268. R. H. Bing: *A homogeneous indecomposable plane continuum.*

An example is given of a homogeneous bounded nondegenerate continuum which is not a circle. This answers the following question raised by Knaster and Kuratowski in Fund. Math. vol. 1 (1920) p. 223: If a nondegenerate bounded plane continuum is