In order to secure a high degree of precision in the statments of the theorems and in the proofs, the author makes much use of concepts and theorems belonging to general topology. Although some brief explanations of these concepts and theorems are included in the first chapter, I believe that most readers will find this explanatory material insufficient, and will be forced to make frequent reference to other books. This task would have been lightened considerably if a greater number of precise references had been given to places where the necessary topological theory is to be found.

Aside from these matters of exposition, my chief criticism concerns the discussion of linear differential equations with periodic coefficients. The treatment, which contains no illustrative material, seems to be too brief, condensed, and general to give an adequate idea of the difficult problems which are presented by these important equations. Thus, although the entire discussion centers around the characteristic exponents, nothing whatever is said about the problem of calculating these numbers effectively. At the very least, one would have expected to see the general theory illustrated by some discussion of the familiar Mathieu equation.

Although some parts of the contents might have been dealt with advantageously in a more ample and leisurely fashion, the book remains an interesting and valuable exposition of a part of differential equation theory which has been too much neglected in American and British works. It will be exceedingly useful to people working on the theory of nonlinear dynamical systems; and it should do much toward attracting mathematicians to a fascinating field, where many further advances are urgently needed.

L. A. MacColl

Sur les bases du group symêtrique et les couples de substitutions qui engendrent un groupe rêgulier. By Sophie Piccard. (Mémoires de l'Université de Neuchâtel, vol. 19.) Paris, Vuibert, 1946. 223 pp.

This book is divided into two main parts. The first is a collection of the author's previously published results on pairs of substitutions generating the symmetric and alternating groups. It has appeared in the Polish, French, and German journals during the years 1938–1942 (see Mathematical Reviews vol. 1 (1940) p. 161, vol. 4 (1943) p. 1, vol. 7 (1946) p. 410 and also Zentralblatt für Mathematik und ihre Grenzgebiete vols. 19, 21, 22). Besides the paper of Hoyer (Math. Ann. (1895)) referred to in her bibliography, the only other work having any close bearing on that of the author, which the reviewer was able to find, is a paper by Hadwiger (Tôhoku Math. J. vol. 49

(1942) pp. 87–89). The second part of the book is entirely new work.

Two permutations S, T of the set N of symbols $(1, 2, \cdots n)$ are said to be *connected* if for every separation of N into two parts, there is either a cycle of S or of T containing symbols from both parts. S, T are *primitive* if the complex S, T is primitive, which is equivalent to saying that the group (S, T) (group generated by S and T) is primitive in the usual sense. If (S, T) is the symmetric group $\mathfrak{S}_n$ on n symbols, or the alternating group $\mathfrak{A}_n$ on n symbols, then it is shown that S, T are connected and primitive. The first 39 propositions give explicitly pairs of elements generating $\mathfrak{S}_n$ and $\mathfrak{A}_n$. Designate the substitution $(12 \cdots n)$ by S_1, $(12 \cdots m)$ $(m+1 \cdots n)$ by S_2, $(12 \cdots m_1)(m_1+1 \cdots m_2)(m_{k-1}+1 \cdots m_k)$ by S_k and cycles of order i by C_i. Then, given S_1, conditions are found on C_2, C_3, C_4, C_5, C_r,

$$C_2 \cdot C_3,\ C_2 \cdot C_5,\ C_2 \cdot C_{m_1} \cdots C_{m_r},\ C_3 \cdot C_4,\ C_3 \cdot C_5,\ C_3 \cdot C_4 \cdot C_5,\ C_2 \cdot C_3 \cdot C_5$$

so that S_1 and one of these latter substitutions generate $\mathfrak{S}_n$ or $\mathfrak{A}_n$. The parity of n frequently determines whether it is $\mathfrak{S}_n$ or $\mathfrak{A}_n$ that is generated. Given S_2, similar conditions are found on C_3, C_4, C_5 so that S_2 and one of them generate $\mathfrak{S}_n$ or $\mathfrak{A}_n$. This is also done with S_3 and C_3, C_4, C_5; with S_4 and C_4; and with S_5 and C_5. Using five of these theorems, the author proceeds to show that for each S there is a T such that S, T generate $\mathfrak{S}_n$ (similarly for $\mathfrak{A}_n$), except in the case of $\mathfrak{S}_4$, where the double transpositions are contained in the ϕ group and so cannot form a part of any basis of two elements. It is also shown for any even (odd) n that for any circular permutation on n letters there exists another circular permutation such that the two generate $\mathfrak{S}_n$ $(\mathfrak{A}_n)$.

In §V of Part I the possible bases (S, T) of $\mathfrak{S}_n$ and $\mathfrak{A}_n$ are enumerated and found to be multiples of $n!/2$ and $n!/4$ in number, respectively. In §VI a complete listing of all pairs of substitutions generating $\mathfrak{S}_n$ and $\mathfrak{A}_n$ is given for $n=3, 4, 5, 6$.

The second part of the book is an original study of the author. The problem proposed is to find conditions on two substitutions which will make the permutation group generated by them regular. A more concise definition of *connectedness* is given in this part. It is shown that if S, T generate a regular group, then they must both be regular of the same degree on the same symbols, and also connected.

The whole work rests on the important concept of "property p_r." Definition. Let

$$T = \begin{pmatrix} a_1 & a_2 \cdots & a_n \\ b_1 & b_2 \cdots & b_n \end{pmatrix}$$

be regular of degree n, and let S, regular and of the same degree, be written as a product of cycles. Then TST^{-1} can be obtained by replacing a_i by b_i in S. (Multiplication is on the right.) We shall say that T transforms a_i into b_i in S. If T transforms the elements of each cycle of S into elements of r other cycles of S, $1 \leqq r \leqq n$, then we shall say that T has property p_r with respect to S (in symbols Tp_rS).

If Tp_rS, then T transforms S^r into a power of itself, and r is a divisor of the order of S. The proof of this is involved and could be shortened (as in some other places in the book) by more direct appeal to abstract principles. If (S, T) is regular, then Tp_aS and Sp_bT for some numbers a, b. After the manner in which T transforms the elements of S has been particularized further, this section is closed with the exhibition of a pair S, T which generate a regular group of order $2mr$ such that Tp_rS, r a divisor of m.

Let Tp_1S, S regular. Necessary and sufficient conditions are found for T so that (S, T) is regular. Similar conditions are found so that (S, T) is regular and also abelian.

Let Tp_2S, S regular. Necessary and sufficient conditions are found for T, provided T is already of a certain form, so that (S, T) is regular. When S is of order m, T of order k, both S and T of degree mk, then sufficient conditions of a complicated number congruence nature are found for (S, T) to be regular.

Let Tp_mS, S regular of order mt, T of order 2, and both S and T of degree mkt. The cases for $m=2, 3, 4, 5$ are analyzed when either $TST=S^{m-1}TS^{m-1}$ or $TST=S^{-1}TS^{m-1}$. It is shown that (S, T) need not be regular under these circumstances when $m=6$.

Let Tp_mS, S regular of order m and degree m^2+m. If $T^2=1$, (S, T) is regular if and only if for every l there exist a pair of numbers (a, b), distinct for every l, and such that

$$\text{(A)} \qquad TS^lT = S^aTS^b.$$

Illustrations are given when $m=3, 4, 5, 6, 7, 8, 12$, and 16. The relations (A) are usually dependent, but unfortunately the nature of this dependence is not analyzed. If T is of order greater than 2, (S, T) is regular if and only if $T^2=S^iTS^j$, s^i, $s^j \neq 1$, and either $TS^lT=S^a$ or $TS^lT=S^aTS^b$ for every l, a and b functions of l.

Let Tp_mS, S regular of order m and of degree $mk \geqq m^2+m$. Let $T^2=1$. (S, T) is regular if and only if, when T transforms the elements of S in a certain way, identities of the form $\prod_i^k(TS^{A_i}) = \prod_i^k(S^{\mu_i}T)$ are satisfied.

The following misprints were noted: p. 86, line 33, "proposition 44" should read "proposition 43"; p. 116, line 6, ($\mu/$) should read ($\mu/2$); p. 123, line 12, "$(jm+km'+i(m'+j-1)m+Km'+j+1)$" should read "$(jm+km'+i\ (m'+j-1)m+km'+j+1)$." There are too many substitutions given with the spacing not clear to enumerate them all. We content ourselves with a general warning on this point.

A. W. JONES

An index of mathematical tables. By A. Fletcher, J. C. P. Miller, and L. Rosenhead. New York, McGraw-Hill; London, Scientific Computing Service, 1946. 8+451 pp. $16.00.

The compilation of this index of tables has been achieved by a tremendous outlay of painstaking work over a period of many years. Many tables of value have appeared in scientific and engineering articles and bringing these to light constitutes by itself a most valuable endeavor. The material is listed according to functions, there being twenty-four sections of which two, for example, are logarithms of trigonometrical functions and Gamma function psi function, Polygamma function, Beta function, Incomplete gamma and Beta functions. There are also sections devoted to primes, binomial coefficients, Bernouilli numbers, mathematical constants, and so on.

The authors have indicated with each table the number of decimals, the range of the argument and the intervals of tabulation, the facilities for interpolation (that is, whether first or second differences are given), and the authorship with date. For example, under §21.31—Incomplete elliptic integrals of the first and second kinds—we find that $F(\theta, \phi)$ and $E(\theta, \phi)$ were tabulated as follows:

12dec. $\theta = 0(1°)90°$ $\phi = 45°$ Δ^5 or Δ^6 Legendre 1816, 1826.

Similarly, under Bernouilli numbers the entries for exact values for B_n begin with: $n=1(1)5$ Bernouilli (1713) and continue with some 30 other tables.

There is a 70 page bibliography and an introduction in which the authors describe in detail the interpretation to be given to their notations and remarks.

E. R. LORCH

Tables of the Bessel functions of the first kind of orders zero, one, two, and three. (The Annals of the Computation Laboratory of Harvard University, vols. 3–4.) Cambridge, Harvard University Press; London, Oxford University Press, 1947. $20.00.

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