

Consider two normal populations  $N(a_1, \sigma_1^2)$  and  $N(a_2, \sigma_2^2)$ , where  $\sigma_1/\sigma_2$  (ratio of the standard deviations) has a known value  $C$ . If the equality of the means,  $a_1 = a_2$ , is to be tested by a  $t$ -test (one-sided or symmetrical) using  $n_1$  sample values from  $N(a_1, \sigma_1^2)$  and  $n_2$  values from  $N(a_2, \sigma_2^2)$  ( $n_1 + n_2 = n$ , fixed), it is shown that this experiment is most powerful when  $n_1/n_2 = \sigma_1/\sigma_2$  (integer considerations neglected). The  $t$ -tests satisfying this condition are called balanced. Thus information is lost by not using a balanced experiment. A quantitative measure of the information lost by using given values of  $n_1$  and  $n_2$  is determined by the total sample size  $m$  ( $m_1 + m_2 = m$ ) of the balanced  $t$ -test (same significance level) having approximately the same power. Then  $n - m$  sample values are wasted by using  $(n_1, n_2)$  rather than  $(m_1, m_2)$ , that is, only  $100m/n\%$  of the information obtainable per observation is used by  $(n_1, n_2)$ . A symmetrical  $t$ -test with significance level  $2\alpha$  has the same value of  $m$  as a one-sided  $t$ -test with significance level  $\alpha$ . For one-sided  $t$ -tests with significance level  $\alpha$ :  $m \doteq 2^{-1}(B + (B^2 - 8A)^{1/2})$ , where  $B = 2 + A + K_\alpha^2/2$ ,  $A = (C+1)^2[1 - K_\alpha^2/2(n-2)] \cdot [C^2/n_1 + 1/n_2]^{-1}$ , and  $K_\alpha$  is the standardized normal deviate exceeded with probability  $\alpha$ . This approximation to  $m$  is valid for  $m \geq 5$  if  $\alpha = .05$ ,  $m \geq 6$  if  $\alpha = .025$ ,  $m \geq 7$  if  $\alpha = .01$ ,  $m \geq 8$  if  $\alpha = .005$ . (Received July 16, 1947.)

350. J. E. Walsh: *Some significance tests for the median which are valid under very general conditions*. Preliminary report.

Consider  $n$  independent values drawn from populations satisfying only: (1) Each population has a unique median. (2) The median has the same value  $\phi$  for each population. (3) Each population is symmetrical. (4) Each population is continuous. (No two of the values are necessarily drawn from the same population.) Significance tests are derived for  $\phi$ . These tests are based on order statistics of certain combinations of order statistics, each combination being either a single order statistic of the  $n$  values or one-half the sum of two order statistics. The tests are reasonably efficient if the values represent a sample from a normal population. The significance levels are of the form  $r/2^n$  ( $r = 1, \dots, 2^n - 1$ ). Each value of  $r$  can be obtained for some one-sided test. The major disadvantage of these tests is the limited number of suitable significance levels for small  $n$ . This disadvantage is partially eliminated by the development of tests having a specified significance level if the values are a sample from a normal population and a significance level bounded near this value if only (1)–(4) hold. Applications of these tests furnish generalized results for the Behrens-Fisher problem, certain large “sample” cases, quality control, slippage tests, the sign test, and situations where some of the  $n$  values are dependent. (Received July 16, 1947.)

## TOPOLOGY

351. S. S. Chern: *On the characteristic ring of a differentiable manifold*.

Let  $M$  be a differentiable manifold of dimension  $n$ , which may be finite or infinite, orientable or non-orientable, and let  $H(n, N)$  be the Grassmann manifold of all the linear spaces of dimension  $n$  through a fixed point  $O$  of a Euclidean space  $E^{n+N}$  of dimension  $n+N$ ,  $N \geq n+2$ . By imbedding  $M$  in  $E^{n+N}$  and constructing through  $O$  the linear spaces parallel to the tangent linear spaces of  $M$ , a mapping of  $M$  into  $H(n, N)$  is obtained. This mapping induces a ring homomorphism of the cohomology ring of  $H(n, N)$  into the cohomology ring of  $M$ , whose image is called the characteristic ring  $C(M)$  of  $M$ . Take as coefficient ring the ring of residue classes mod 2. Formulas

are established which give the cup product of two elements of  $C(M)$  as a linear combination of elements of  $C(M)$ . Consequences of these formulas are: (1) The Stiefel-Whitney classes  $W^k$  ( $k=1, \dots, n$ ) form a basis of the characteristic ring under ring operations; (2) Necessary conditions are obtained for  $M$  to be imbeddable in  $E^{n+1}$  or  $E^{n+2}$ ; (3) Projective spaces of even dimension  $n$ , when  $n$  is not of the form  $n=2(2^k-1)$ ,  $k \geq 1$ , cannot be imbedded in  $E^{n+2}$ . (Received June 23, 1947.)

352. S. S. Chern: *On the common zeros of a set of scalar densities on a complex analytic manifold and the automorphic forms in several complex variables.*

On a compact complex analytic manifold of dimension  $n$  a scalar density  $f$  of weight  $k$  is a geometric quantity transformed according to the law:  $f^* = f \cdot J^k$ , where  $J$  is the Jacobian. On the locus of common zeros of a set of scalar densities  $f_1, \dots, f_m$ ,  $m \leq n$ , can in general be defined a cycle, called the cycle of common zeros and denoted by  $[f_1, \dots, f_m]$ . Then when both sides have sense,  $[f_1, \dots, f_i] \cdot [f_{i+1}, \dots, f_{i+m}] = [f_1, \dots, f_{i+m}]$ , where the left-hand side denotes the intersection cycles of the two cycles in the sense of algebraic topology. The homology classes of the cycles  $[f_1, \dots, f_m]$  are identified with the characteristic classes which the author introduced in his study of Hermitian manifolds. These results can be applied to Siegel's theory of automorphic functions in several complex variables, with the conclusion that the cycle of common zeros of a set of automorphic forms with respect to a discontinuous group  $\Delta$  in Siegel's half-space depends only on the analytic structure of the fundamental domain  $F$  of  $\Delta$ , if  $F$  is compact. In particular, the number of common zeros of  $n$  automorphic forms is equal to the Euler-Poincaré characteristic of  $F$ . (Received June 23, 1947.)

353. H. S. M. Coxeter: *Configurations and maps.*

The regular maps on a torus (cf. Brahana, Amer. J. Math. vol. 48 (1926) pp. 225-240) are found to be of three types: a map of  $b^2+c^2$  quadrangles, one of  $b^2+bc+c^2$  hexagons, and one of  $2(b^2+bc+c^2)$  triangles. Alternate faces of the last map form a triple system which can sometimes be interpreted as an interesting configuration. When  $b=2$  and  $c=1$  this becomes Fano's finite projective geometry  $PG(2, 2)$ ; when  $b=3$  and  $c=0$ , the Pappus configuration. Similarly, alternate faces of the map of 24 triangles on the unorientable surface of characteristic  $-3$  (Duke Math. J. vol. 10 (1943) p. 298, Fig. 11) yield the triple system formed by the collinear sets among the nine inflexions of the general cubic curve in the complex projective plane. (Received July 26, 1947.)

354. W. H. Gottschalk: *Recursive properties of transformation groups. II.*

Let  $T$  be a locally compact topological group which acts as a transformation group on a topological space  $X$ . Let there be distinguished in  $T$  certain sets, called *admissible*, which satisfy this condition: If  $A$  is an admissible set and if  $B$  is a set in  $T$  such that  $A \subset BK$  for some compact set  $K$  in  $T$ , then  $B$  is an admissible set. A subgroup  $R$  of  $T$  is said to be *recursive* at  $x \in X$  provided that to each neighborhood  $U$  of  $x$  there corresponds an admissible set  $A$  such that  $A \subset R$  and  $xA \subset U$ . It is proved that if  $T$  is recursive at  $x \in X$  and if  $S$  is a relatively dense invariant subgroup of  $T$ , then  $S$  is recursive at  $x$ . (Received June 30, 1947.)

355. S. W. Hahn: *Universal spaces under strong homeomorphisms*. Preliminary report.

Two subsets  $X$  and  $Y$  of a Euclidean  $n$ -space  $E_n$  are said to be *strongly homeomorphic* if there exists a homeomorphism  $f$  of  $E_n$  onto itself such that  $f(X) = Y$ . A subset  $Y_k$  of  $E_n$  will be called *universal for  $k$ -dimensional subsets of  $E_n$*  if: (1)  $\dim Y_k = k$  and (2) for every  $X$  in  $E_n$  with  $\dim X = k$ ,  $X$  is strongly homeomorphic to a subset of  $Y_k$ . It is proved that for  $0 \leq k < n-1$  there is no universal  $Y_k$ . (Received April 25, 1947.)

356. Deane Montgomery: *Analytic parameters in three-dimensional groups*.

It is shown that every connected locally Euclidean three-dimensional topological group must be a Lie group. (Received May 22, 1947.)

357. Deane Montgomery: *Connected one-dimensional groups*.

Let  $G$  be a locally compact connected one-dimensional topological group whose topology is separable metric. If  $G$  is not compact then  $G$  is isomorphic to the group of real numbers under addition. (Received May 22, 1947.)

358. Paul Olum: *Pseudomanifolds and their covering spaces*.

Let  $K$  be an  $n$ -dimensional pseudomanifold and  $\bar{K}$  its universal covering space. The present paper studies the structure of  $\bar{K}$  in relation to that of  $K$ , particularly with regard to questions of orientation. This leads to an explicit determination of a certain  $n$ -dimensional cohomology group with operators in  $K$  over an arbitrary coefficient group. (This is essentially the  $n$ th cohomology group of  $K$  with local coefficients, in the sense of Steenrod, for an arbitrary system of local coefficient groups.) On the basis of this result, a classification is given of the homotopy classes of mappings of  $K$  into a topological space  $T$ , where the homotopy groups  $\pi_r(T)$  vanish for  $1 < r < n$ . (See the abstract *Homology with operators and mapping theory*, to be published in November.) The results are applied in particular to the mappings of an  $n$ -pseudomanifold into projective  $n$ -space. (Received July 25, 1947.)

359. Pierre Samuel: *Ultrafilters and compactification of uniform spaces*.

The purpose of the author is to give a purely topological (that is, not using real numbers) proof of the fact that every separated uniform space  $S$  is a subspace of a compact Hausdorff space. This compact space  $\bar{S}$  is constructed as an identification space of the Boolean space of all ultrafilters over  $S$ .  $\bar{S}$  contains the completion  $S^*$  of  $S$  as a subspace, and is identical with  $S^*$  if, and only if,  $S$  is precompact. The Čech, Wallman and Alexandroff compactifications are studied as particular cases of this construction. A locally compact space is characterized as a space for which there exists a minimal uniform structure compatible with its topology. A characterization of normal sequentially compact spaces is also given. A purely topological characterization of uniformizable spaces is given in terms of mappings into compact spaces, and its equivalence with complete regularity is proved. (Cf. Bourbaki, *Topologie générale*, Paris, 1940, chap. 2.) (Received May 12, 1947.)

360. P. A. White: *Regular transformations on generalized manifolds*

It is shown that the image of a sphere-like orientable generalized- $n$ -manifold under a  $(n-1)$ -regular transformation with the additional property that the inverse of each point is an  $lc_{n-1}$  is an orientable generalized- $n$ -manifold. Other more general theorems concerning the above transformations are also studied. (Received March 17, 1947.)

361. J. W. T. Youngs: *Homeomorphic approximations to monotone mappings.*

In this paper the author proves the following theorem. If  $m: M \rightarrow M$  (=from  $M$  onto  $M$ ) is a monotone mapping where  $M$  is a closed 2-manifold or a 2-manifold with boundary, and  $\epsilon > 0$ , then there is a homeomorphism  $h: M \rightarrow M$  such that  $\rho\{m(x), h(x)\} < \epsilon$ ,  $x \in M$ . (Received July 24, 1947.)