A NOTE ON LOCAL CONNECTIVITY

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Two neighborhoods of a point are involved in the definition[1] of local connectivity: a space T is p-LC at a point x if every neighborhood U of x contains a neighborhood V of x such that any continuous p-sphere in V bounds a continuous $(p+1)$-cell in U. T is p-LC if it is p-LC at every point, and it is LC^n if it is p-LC for $0 \leqq p \leqq n$.

For the case $p=0$, it is well known that there is an equivalent definition: a space T is 0-LC if every point has arbitrarily small neighborhoods V such that any continuous 0-sphere in V bounds a continuous 1-cell in V. But for $p>0$, Borsuk and Mazurkiewicz have shown by an example[2] that these two definitions are not equivalent.

Hence the question arises as to the relative size of V with respect to U in the first definition. At first glance, the Borsuk-Mazurkiewicz example would seem to indicate that V must be considerably smaller than U. This, however, is not the case.

THEOREM. *If a space T is LC^n, then each point of T has arbitrarily small neighbrhoods V such that any continuous p-sphere, $0 \leqq p \leqq n$, in V bounds a continuous $(p+1)$-cell in $\overline{V}$.*

PROOF. Let U be a neighborhood of a point x of T such that any continuous 0-sphere in U bounds a continuous 1-cell in U. Let A be the class of all neighborhoods V of x such that any continuous p-sphere, $0<p\leqq n$, in V bounds a continuous $(p+1)$-cell in U. Since T is LC^n, A is not vacuous. Order the elements of A by inclusion. Since the continuous image of a sphere is a compact set, the union of the elements of any simply ordered subset of A is again an element of A. Hence, by Zorn's lemma, A contains a maximal element, V_0.

We assert that $\overline{V}_0 \supset U$. If not, let y be a point of the open set $U-\overline{V}_0$, and let W be a neighborhood of y, $W \subset U-\overline{V}_0$, such that any continuous p-sphere, $0<p\leqq n$, in W bounds a continuous $(p+1)$-cell in U. Since $p>0$, any continuous p-sphere in $V_0 \cup W$ is either in V_0 or in W, so $V_0 \cup W$ is an element of A, which contradicts the maximality of V_0.

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[1] S. Lefschetz, *Locally connected and related sets*. I, Ann. of Math. vol. 35 (1934) pp. 118–129.

[2] K. Borsuk and S. Mazurkiewicz, *Sur les rétractes absolus indécomposables*, C.R. Acad. Sci. Paris vol. 199 (1934) pp. 110–112.

Now, by the original choice of U, V_0 has the property required by the theorem.

We remark that the same proof holds, with trivial modifications, for homology local connectivity.

It is an open question whether the neighborhood V can be required to be connected.

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UNCONDITIONAL CONVERGENCE IN BANACH SPACES

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Introduction. This note investigates an apparent generalization of unconditionally convergent series $\sum x_i$ in weakly complete Banach spaces. A series of elements with x_i in E is said to be unconditionally convergent if for every variation of sign $\epsilon_l = \pm 1$, $\sum_1^m \epsilon_l x_l$ is convergent. This formulation of the definition of unconditional convergence is equivalent to that given by Orlicz [4].[1] We call $\sum x_i$ unconditionally summable if there exists a finite row Toeplitz matrix (b_{ik}) such that for every variation of sign $\sigma_i = \sum_{k=1}^{n_i} b_{ik} \sum_{l=1}^{k} \epsilon_l x_l$ converges. The fact that unconditional summability implies unconditional convergence is established in this note. Finally, applications to orthogonal functions are presented.

Preliminary lemmas. In what follows, b_{ik} will denote an arbitrary finite row Toeplitz matrix.

LEMMA 1. *If $S_n(\theta) = \sum_1^m a_l r_l(\theta)$ converges to an essentially bounded function $f(t)$, then $\left| \sum_1^m a_n r_n(\theta) \right| \leqq c$ almost everywhere. ($r_n(\theta)$ denote the Rademacher functions.)*

PROOF. This is an immediate consequence of the result that

$$\left(\int_0^1 \left(\max_{1 \leqq n \leqq m} \left| \sum_1^n a_l r_l(\theta) \right| \right)^p d\theta \right)^{1/p} \leqq C \left(\int_0^1 \left| S_m(\theta) \right|^p d\theta \right)^{1/p}, \qquad 1 \leqq p \leqq \infty, \tag{1}$$

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[1] Numbers in brackets refer to the references cited at the end of the paper.