

tity representation to  $G_0$ . The remaining space groups are then derived in detail and classified within the rhombohedral, hexagonal, monoclinic, rhombic, tetragonal, and cubic systems.

The two final sections are devoted to the study of special families of space groups in  $n$  dimensions, such as those arising from the cyclic, symmetric, and alternating groups on  $n$  symbols.

The book is clearly written and self-contained, except in the section beginning on p. 91 where the ternary arithmetic classes are listed. Here the reader without previous knowledge of the notations of crystallography may have some difficulty reading the rather condensed summary of the 73 ternary arithmetic classes. The groups of motions in the plane are illustrated by excellent figures, but no attempt is made to illustrate the 230 space groups by drawings such as are given by Wyckoff. The emphasis in the book is clearly on the mathematical derivation rather than the pictorial representation of the 230 space groups.

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*Methods of mathematical physics.* By Harold Jeffreys and Bertha S. Jeffreys. New York, Macmillan; Cambridge University Press, 1946. 9+679 pp. \$15.00.

The book starts with a substantial chapter on real variable—Dedekind sections, sequences, series, continuity, integration, mean value theorems. Chapters 2, 3, and 4 cover vectors, cartesian tensors, and matrices, and these are followed by chapters on multiple integrals and potential theory. Operational methods and their applications occupy two chapters, and a long chapter is devoted to numerical methods. A short chapter on calculus of variations brings us to what may be regarded as the mid-point of the book, attained almost entirely without the use of complex numbers.

The essential elements of the theory of functions of a complex variable are covered in two chapters. This opens up a wide field, and chapters follow on conformal representation, Fourier's theorem, factorial (gamma) functions, linear differential equations of the second order, asymptotic expansions, equations of wave motion and heat conduction (three chapters), Bessel functions and applications, confluent hypergeometric functions, Legendre functions, elliptic functions. The book ends with explanatory notes, an appendix on notation, and an index.

Each chapter has a set of examples, a stimulating collection culled from examinations of the Universities of Cambridge, London, and Manchester.

It is obviously impossible to cover in a single volume, even a large one, all those methods of mathematical physics which a wide variety of readers might wish to see included. It is natural that authors should allow their particular interests to influence their choice of material, and one is not surprised to find in the book much of the contents of two short books by Harold Jeffreys—*Operational methods in mathematical physics* (now out of print) and *Cartesian tensors*. The authors admit that the choice of subject-matter has been difficult. Their stated intention is to “provide an account of those parts of pure mathematics that are most frequently needed in physics,” and they set themselves the general rule to include a method if it has applications in at least two branches of physics. They have not tried to give a detailed account of any branch of physics, regarding that as a matter for special text books.

The authors' style is crisp and at times rather condensed, but it expands as the occasion requires in a humorous, historical, or caustic vein. This gives the book a human atmosphere rare in mathematical publications, but naturally enough leads at times to statements which may arouse the antagonism of the reader; for example, the statement on p. 76 (footnote) that the reduction of a general force system to a force and a couple is the sort of thing that occurs only in examination questions! In most books with a scope comparable to that of the book under review, intrinsic evidence is present to show that the authors are either mathematicians or physicists; there is a rawness or incompleteness in the domain less familiar to the authors. In the present instance the authors have preserved a good balance. Physics is the goal, but the mathematics is handled with feeling and care. To quote from the Preface: “We maintain therefore that careful analysis is more important in science than in pure mathematics, not less. We have also found repeatedly that the easiest way to make a statement reasonably plausible is to give a rigorous proof.”

Nevertheless the cloven hoof of the physicist peeps out from time to time, and is very much in evidence in the first few pages. Once the mathematical reader has found nests of intervals on p. 6 and a reassuring epsilon on p. 7, he knows that all is well. But to reach this haven of rest he has first to weather a stormy sea. On p. 1 it is written that the formulae of algebra “may still be correct when we replace the letters in them by something other than numbers, and it is to this fact that the possibility of mathematical physics is due.” The authors do not have in mind here replacement by matrices or operators, but rather replacement by the quantities of physics in which a unit of measurement is involved, for example, replacement of the

number 3 by 3 cm. "When a particular application to a measured system is made we naturally give the symbols their actual values in terms of the measures, which will include a statement of the units; but in the general theory the unit is irrelevant. The symbols will then be said to stand, not for numbers, but for *physical magnitudes*." Does this mean that when we write  $e^x = 1 + x + x^2/2 + \dots$ , we are to have in the backs of our minds the idea that  $x$  stands for 3 cm. or 5 gm. rather than a pure number? Surely this is not an honest picture of the psychology of the modern mathematical physicist. If it is, we have traced to its source a subtle divergence of points of view which keeps mathematicians and physicists from enjoying a full measure of mutual esteem. It is a divergence which takes place at the level of high school algebra, and must be felt as soon as a student wonders whether he should write for his answer " $x = 3$ " or " $x = 3$  miles." Venturing an opinion apparently in direct contradiction to that of the authors, it seems to me that the power of modern mathematical physics lies in the fact that it can and does formulate its problems in terms of pure numbers. Nevertheless, mathematicians should not dismiss the question of units as a storm in a teacup; a little experience with electromagnetic units will reduce the arrogant to a spirit of respectful humility. The best bridge between measured quantities and pure mathematics seems to lie in the formulation of a vector algebra of measured quantities, as was once pointed out to me by Professor L. Infeld. On the whole, the authors should not be criticized for introducing the matter, which is obviously of importance, but it is to be regretted that they did not deal with it more completely and satisfactorily. Confusion on an elementary level is the worst confusion of all.

A few small points may be noted. The juxtaposition of vectors, tensors, and matrices brings together a happy family group. As refresher material, it is excellent. But will the novice understand what a scalar is from the following: "Any physical measurement is the assignment of a single magnitude. Such magnitudes are called *scalars*" (p. 49), and "A scalar is a single quantity, the same for all axes" (p. 53)? What about the  $x$ -coordinate of a point? It is a scalar by first definition, but not by the second. The definition of a scalar function of position on p. 82 is not as clear as it might be; we have merely to say that the function has at each point a value independent of the coordinate system, but the authors somewhat obscure this simple idea

The statement following equation (21) on p. 109 is not correct; the adjugate of an antisymmetric matrix  $a$  is antisymmetric if the

order of  $a$  is even, symmetric if the order of  $a$  is odd.

The index appears to be adequate; one misprint in it was accidentally noticed: Group velocity, for 477 read 479.

The production conforms to the excellent standards of the Cambridge University Press. Those in whom the conservatism of publishers engenders a deep rage will note with satisfaction that the historic Cambridge Press starts sentences with mathematical symbols (lower case!) and adheres to the practice (anathema to some publishers) of cutting in figures on the right hand sides of both odd and even pages, so as to leave an unbroken margin. Very few misprints have been noticed, and they are not likely to confuse the reader. In places a dearth of commas forces a rereading, but this is no harm.

The authors show good sense with regard to terminology and notation, and are not afraid to make suggestions. In the case of a theorem bearing a hyphenated name (for example, Newton-Bessel) they make it clear that the second name is merely a label, and does not imply a division of priority. It is to be hoped that this practice may prevail; the creators of new theorems should welcome it: reference to the Smith-Robinson theorem would imply (a) that Smith has created at least two theorems, and (b) that Robinson vouches for the correctness of one of them. The word "limit" is overworked, and the authors suggest "termini" for "limits of integration." For Hankel functions they use  $Hs_n(x)$  and  $Hi_n(x)$ , the "s" and the "i" meaning respectively "superior" and "inferior" in reference to paths of integration in the Schlöfli integrals; they use also  $Kh_n(x)$  for Heaviside's function, equal to  $2/\pi$  times the usual  $K_n(x)$ .

The book assumes a knowledge of calculus, and might serve as a text book for students who proceed from that basis in the direction of applied mathematics or mathematical physics. One hesitates to affirm or deny its merits as a text; so much depends in such cases on the establishment of a triple bond of sympathy between student, teacher, and author. Under favorable conditions it should work well; as for unfavorable conditions, let me requote the quotation from Oliver Heaviside which heads the chapter on Operational methods: "Even Cambridge mathematicians deserve justice."

J. L. SYNGE

*Applied Bessel functions.* By F. E. Relton. London and Glasgow, Blackie, 1946. 7+191 pp. 17s. 6d.

This text furnishes an introduction to those properties of the Bessel and allied functions which are of use in solving boundary value problems. The role of these properties in applied mathematics and