order of a is even, symmetric if the order of a is odd.

The index appears to be adequate; one misprint in it was accidentally noticed: Group velocity, for 477 read 479.

The production conforms to the excellent standards of the Cambridge University Press. Those in whom the conservatism of publishers engenders a deep rage will note with satisfaction that the historic Cambridge Press starts sentences with mathematical symbols (lower case!) and adheres to the practice (anathema to some publishers) of cutting in figures on the right hand sides of both odd and even pages, so as to leave an unbroken margin. Very few misprints have been noticed, and they are not likely to confuse the reader. In places a dearth of commas forces a rereading, but this is no harm.

The authors show good sense with regard to terminology and notation, and are not afraid to make suggestions. In the case of a theorem bearing a hyphenated name (for example, Newton-Bessel) they make it clear that the second name is merely a label, and does not imply a division of priority. It is to be hoped that this practice may prevail; the creators of new theorems should welcome it: reference to the Smith-Robinson theorem would imply (a) that Smith has created at least two theorems, and (b) that Robinson vouches for the correctness of one of them. The word "limit" is overworked, and the authors suggest "termini" for "limits of integration." For Hankel functions they use $\mathrm{Hs}_n(x)$ and $\mathrm{Hi}_n(x)$, the "s" and the "i" meaning respectively "superior" and "inferior" in reference to paths of integration in the Schläfli integrals; they use also $\mathrm{Kh}_n(x)$ for Heaviside's function, equal to $2/\pi$ times the usual $K_n(x)$.

The book assumes a knowledge of calculus, and might serve as a text book for students who proceed from that basis in the direction of applied mathematics or mathematical physics. One hesitates to affirm or deny its merits as a text; so much depends in such cases on the establishment of a triple bond of sympathy between student, teacher, and author. Under favorable conditions it should work well; as for unfavorable conditions, let me requote the quotation from Oliver Heaviside which heads the chapter on Operational methods: "Even Cambridge mathematicians deserve justice."

J. L. SYNGE

Applied Bessel functions. By F. E. Relton. London and Glasgow, Blackie, 1946. 7+191 pp. 17s. 6d.

This text furnishes an introduction to those properties of the Bessel and allied functions which are of use in solving boundary value problems. The role of these properties in applied mathematics and engineering is fully illustrated by numerous examples both in the text and in the exercises. Prerequisites for the reading of the text are some elementary calculus and a course in elementary (or possibly intermediate) differential equations. However, the author states (and frequently proves) those properties of differential equations which are used in the text.

Essentially, the text treats four topics: (1) zeros and growth properties of Bessel functions of the first and second kinds; (2) growth properties of related functions; (3) integral expansions of Bessel functions; (4) applications to ordinary and partial differential equations.

After defining the error, gamma, and beta functions, the author discusses those properties of linear differential equations which are to be used in the treatment of the Bessel functions. Considerable emphasis is placed on normal forms and zeros of second order equations. The cylinder functions C_n (functions which satisfy a particular form of the Bessel differential equation) are defined by means of two recurrence relations. By means of these relations, the author obtains the structure of the zeros of the cylinder functions and then the nature of the graph of these functions. Further, various transformations of the dependent and independent variables which lead to cylinder functions are discussed. In Chapter IV, the power series expansion for the Bessel function of the first kind J_n ($J_n = x^{-n}C_n$) is obtained. Some numerical values for the first zero of different integral J_n are determined and Bessel identities involving Lommel integrals are discussed. Before attempting to obtain the second integral Y_n of the Bessel equation for integral n, the author merely states various propperties of the indicial equation. Proofs of these statements will be found in texts on intermediate or advanced differential equations (see *Intermediate differential equations* by E. D. Rainville, Wiley, 1943). Actually, the second integral of the zero order is obtained from the fact that $(dC_n/dn)_{n=0}$ is a solution of the zero order Bessel equation. The function Y_n for integral n is obtained by a similar device.

The properties of modified Bessel functions (imaginary argument) and the definitions of the "ber" and "bei" functions are studied in the latter part of the text. The first class of functions are related to the Bessel functions in the same manner that the hyperbolic functions are related to the circular functions. Hence, they can possess zeros only at the origin and only the growth of these functions is discussed. This is done by use of the recurrence formulas.

Various integral expansions for Bessel functions are furnished in Chapters IX and X. They include: (1) the well known relations between integrals of the circular functions and the Bessel functions of the first kind; (2) the Lipschitz and Sonine integrals for Bessel functions; (3) Weber's discontinuous integral; (4) some asymptotic formulas for the various Bessel functions.

Following each discussion of a type of Bessel function, the author furnishes an application to problems in applied mathematics. In particular, Chapter V is devoted to five boundary value problems whose solutions involve Bessel functions of the first kind. Of these, the following should be of interest to the engineer: (1) transverse vibrations of a non-uniform string; (2) stability of a vertical wire and deep cantilever beam. Again, in Chapter VIII, some applications to hydrodynamics (tidal motion), elasticity (buckling of a circular plate, vibrations of a disk), and heat conduction are given. In each case, the differential equation is solved by the separation of variables and then the appropriate Bessel functions are introduced in studying the solutions of the resulting ordinary differential equation.

The reviewer feels that this book should furnish an excellent introduction to Bessel functions and their applications in applied mathematics.

N. COBURN