suggested by a heuristic argument used by physicists in connection with the quantum mechanics equation $\psi''+8\pi^2mh^{-2}(E-V(x)\psi=0$ (Brillouin, Wentzel, Kramer). In chap. 9 it is proved that if $q'>0$, $q''\geqq 0$, $q''\leqq(q')^\gamma(1<\gamma<4/3;\ x\geqq x_0)$ and $f$ is $L^2(0,\ \infty)$, then $f(x+0)+f(x-0)=2\sum c_n\psi_n(x)$, provided $f$ is of bounded variation near $x$. In chap. 10 the author proves the following summability theorem. If continuous $q\rightarrow\infty$ monotonically and $f$ is $L^2(0,\ \infty)$, then $f(x)=\lim_{v\rightarrow\infty}\sum_n(v+\lambda_n)^{-1}vc_n\psi_n(x)$ for every $x$ for which $\int_0^\eta|f(x+y)-f(x)|dy=o(\eta)$, as $\eta\rightarrow 0$ (that is, almost everywhere). With the aid of the above result the expansion theorem of chap. 9 is then proved anew, which presents an analogy with the situation in the ordinary Fourier theory.

Some developments analogous to those in the book under review have been carried out in the field of partial differential equations by T. Carleman [Arkiv för Mathematik, Astronomi och Fysik vol. 24 B (1934) pp. 1–7] and by the present reviewer [Ann. of Math. vol. 43 (1942) pp. 1–55; also, Rec. Math. (Mat. Sbornik) N.S. 20 (1947) pp. 365–430]. The field of partial differential equations being so much more difficult than that of ordinary differential equations, much more remains to be done. The book of Titchmarsh may serve as a useful guide in the line of investigation just mentioned.

W. J. Trjitzinsky

*The theory of functions of real variables.* By L. M. Graves. New York and London, McGraw-Hill, 1946. 10+300 pp. $4.00.

The theory of functions of real variables occupies a central position in present day analysis, and it is typical for graduate schools in the United States to offer students in mathematics a one year introduction to the subject. It has always been a problem to find a suitable text for such a course, needing, as it does, something shorter and crisper than one of the standard treatises. The volume under review is written with this end in view.

The first chapter is a short exposition of the ideas and methods of deductive logic. The notions of negation, conjunction and alternation of propositions are given symbolic notations, and regarded as undefined (but "generally understood") operations: a list of laws by which their use is to be governed (for example, double negation, excluded middle) is discussed. The calculus of classes is briefly described, and the student is introduced to a technique of translating English sentences involving logical and class relationships into briefer symbolic formulae. We are warned about possible paradoxes which may arise through unguarded use of these notions, and the writer describes "common sense" safeguards which he feels are generally adequate to avoid trouble.

Chap. II gives the traditional construction of the reals, starting with Peano's axioms and proceeding via equivalence classes and Dedekind cuts to the real numbers, which are shown to form a complete ordered field. In chap. III point sets in $n$ dimensions are treated, and a chain of theorems giving their fundamental topological properties culminates in the Heine-Borel theorem. The concepts of continuity and limit for functions of one or several real variables are defined in chap. IV; superior and inferior limits, semi-continuity and uniform continuity are discussed, and their relations and properties exhibited by a series of standard theorems, including max and min theorems for semi-continuous functions. The definition of the limit of $f(x)$ at $x=a$ is such that if it exists it must equal $f(a)$ (when the latter is defined), contrary to most texts. A point with which the reviewer believes some readers might take issue is the author's addition of ideal points to Euclidean spaces of all dimensions, leading, for example, to the statement that if a function is continuous on a closed set $S$, it is uniformly continuous on $S$.

Chap. V furnishes a detailed exposition of the fundamental definitions and theorems on differentiation of functions of one variable, including mean value theorems and a discussion of derivates. Total differentiability for functions of several variables, and theorems on partials of various orders, receive due attention. The Riemann integral for functions of one variable is treated rapidly and succinctly in chap. VI, and chap. VII is devoted to various aspects of uniform convergence, including a brief introduction to multiple series, the function space $C$, and non-differentiable functions.

The next two chapters, while well within the spirit of the book's title, have an element of novelty. Chap. VIII deals with functions defined implicitly by a relation $f(x, y)=0$. In connection with investigating the solvability for one variable in terms of the other, some fixed point theorems are proved, involving a short excursion into the theory of simplicial partitions. Chap. IX is on existence theorems for ordinary differential equations.

The last three chapters are on integration theory. Chaps. X and XI deal with Lebesgue integration, using the Riesz device of approach by step functions for the definition. The detailed discussion is carried out for functions of one variable, but a good deal of side comment is furnished concerning generalizations. We are led to an introductory exposition of the spaces $L_p$, and thence to a discussion of orthonormal sets for the case $p=2$. (It is rather surprising not to see Hilbert's name mentioned in this connection.) The last chapter is on several types of Stieltjes' integration.

Very few misprints or misstatements have come to the reviewer's attention. Especial mention should be made of the valuable and well chosen lists of references to the literature which conclude each chapter.

It seems to the reviewer that in scope and choice of subject matter this text is nicely calculated to suit the needs of introductory classes in real variable theory. On the basis of having used the text for such a class for one term, he would suggest only one respect in which it proved to be somewhat troublesome: namely, in the free use from the start of the logical symbols introduced in chap. I. It is suggested that students embarking on this subject have a good many new ideas, and an essentially novel ideal of precision, to struggle with, both of which are inherent in the subject itself. It seems open to question whether we really help them by replacing a possibly tedious, but clear, English sentence by such a formula as

$$m \neq p : \supset : \exists q \ni \cdot m + q = p \cdot V \cdot \exists n \ni \cdot m = p + n$$

for them to cope with in addition, almost at the outset of their voyage.

JAMES A. CLARKSON

*Topological methods in the theory of functions of a complex variable.* By Marston Morse. (Annals of Mathematics Studies, no. 15.) Princeton University Press, 1947. 2+145 pp. $2.50.

The theory of analytic functions is related to topology in two ways. On one hand it can serve as a powerful tool in the study of topological questions. On the other hand many of the basic theorems in the theory of functions are essentially topological in character and can be proved by such methods. It is the latter observation that forms the starting point for Morse's booklet. If the purely topological properties of analytic functions are to be isolated, it is natural to study the class of functions which, in the small, share the topological properties of analytic functions. One of the main problems will then be to find out to what extent this new class retains the topological properties in the large.

The author actually considers two classes of topologically defined functions, *interior transformations* and *pseudo-harmonic functions*. They arise, respectively, from analytic and harmonic functions by sense-preserving local homeomorphisms. Interior transformations