orthonormal sets for the case $p=2$. (It is rather surprising not to see Hilbert's name mentioned in this connection.) The last chapter is on several types of Stieltjes' integration.

Very few misprints or misstatements have come to the reviewer's attention. Especial mention should be made of the valuable and well chosen lists of references to the literature which conclude each chapter.

It seems to the reviewer that in scope and choice of subject matter this text is nicely calculated to suit the needs of introductory classes in real variable theory. On the basis of having used the text for such a class for one term, he would suggest only one respect in which it proved to be somewhat troublesome: namely, in the free use from the start of the logical symbols introduced in chap. I. It is suggested that students embarking on this subject have a good many new ideas, and an essentially novel ideal of precision, to struggle with, both of which are inherent in the subject itself. It seems open to question whether we really help them by replacing a possibly tedious, but clear, English sentence by such a formula as

$$m \neq p : \supset : \exists q \ni \cdot m + q = p \cdot V \cdot \exists n \ni \cdot m = p + n$$

for them to cope with in addition, almost at the outset of their voyage.

JAMES A. CLARKSON

Topological methods in the theory of functions of a complex variable. By Marston Morse. (Annals of Mathematics Studies, no. 15.) Princeton University Press, 1947. 2+145 pp. $2.50.

The theory of analytic functions is related to topology in two ways. On one hand it can serve as a powerful tool in the study of topological questions. On the other hand many of the basic theorems in the theory of functions are essentially topological in character and can be proved by such methods. It is the latter observation that forms the starting point for Morse's booklet. If the purely topological properties of analytic functions are to be isolated, it is natural to study the class of functions which, in the small, share the topological properties of analytic functions. One of the main problems will then be to find out to what extent this new class retains the topological properties in the large.

The author actually considers two classes of topologically defined functions, *interior transformations* and *pseudo-harmonic functions*. They arise, respectively, from analytic and harmonic functions by sense-preserving local homeomorphisms. Interior transformations have been studied previously by Stoilow and Whyburn, from different points of view, while the concept of pseudo-harmonic functions is new except for pioneer work by Morse himself and in collaboration with M. H. Heins. It is clear from the definition that the local behavior of these functions at an interior point is perfectly known, but on the boundary of the region of definition a careful and detailed analysis is required, even in the case of relatively strong conditions of regularity.

The main theorems are enumerative in character, dealing with the number of singularities and similar characteristics. As a typical case, take the formula

$$M + m - S - s = 2 - v,$$

where M is the number of logarithmic poles of a pseudo-harmonic function U in a region G, m the number of relative minima of U on the boundary B, S and s the number of saddle points (critical points) on G and B, v the number of contours. For harmonic functions, which remain such on the boundary, formulae of this type are of course familiar as more or less immediate consequences of Cauchy's integral theorem, except for the fact that the number m of relative minima is usually not displayed. One recognizes, however, that m serves to measure the turning of the tangent to the image of B under the analytic transformation $U+iV$ whose real part is U. In proving this theorem under more general assumptions a delicate point is the counting of singularities on the boundary; instead of counting half-multiplicities, as one would ordinarily do in the theory of analytic functions, Morse shows good cause for adopting a different procedure. In fact, the main point in the proof, which of course proceeds along purely topological lines, consists in a careful scrutiny of the saddle-points on the boundary.

The first four chapters deal with this and similar questions. Various types of boundary conditions are discussed in great detail, and certain enumerative theorems which have received special attention in the theory of functions are singled out. Throughout, important details are investigated with all the skill expected from the author. If the reader has any doubts, they will not be concerned with the validity of the arguments, but rather with the degree of importance that can be attached to these questions. Since the appreciation of Morse's work must necessarily hinge on such considerations, it is a pity that the limited size of this tract has prevented its author from presenting his material against a broader background. Personally, the reviewer believes that a good case could be made for Morse's point of view.

In the fifth and last chapter deformations of locally simple curves are studied. The main theorem that two locally simple curves can be deformed into each other (by an admissible transformation) if and only if they have the same angular order is proved in its full generality. An admissible transformation is defined as one under which the intermediate curves are uniformly locally simple. The same question is treated for the case of O-deformations under which the curves are not allowed to pass through a point O. Here the characteristics are the angular order and the order with respect to O. These are of course highly important questions whose systematic treatment in the non-regular case has been long overdue.

There is finally a brief introduction to the deformation for interior transformations which has been more extensively treated in the author's joint paper with M. H. Heins (Acta Math. vol. 80 (1947)). This interesting theory aims at a complete characterization of the homotopy properties of meromorphic functions as opposed to those of more general interior transformations. As an introduction to this theory the book serves a very laudable purpose.

L. Ahlfors

Vector and tensor analysis. By Louis Brand. New York, Wiley, 1947. 9+439 pp. $5.50.

This volume bears a relation to the average elementary book on vector analysis similar to the relation of the author's well known *Vectorial mechanics* to the average elementary book on mechanics. In each case all of the expected topics are included and treated thoroughly, carefully, and with rigor and sophistication. But a number of related subjects are brought into the picture.

Thus in the volume under review one finds chapters on motor algebra, quaternions, dyadics, the application of vectors to perfect fluids and to elementary differential geometry. A long chapter on tensor analysis carries the subject beyond covariant differentiation and includes surface geometry in tensor notation. The author expects to include further applications of tensor analysis such as those made by Einstein and Kron in a later volume.

Many exercises are worked out in the text. These often include interesting results not found in conventional texts. These, together with some of the exercises to be worked out, should do a great deal for the reader interested in increasing the breadth of his mathematical education.

The book can be used as the basis of an elementary course in vector