CORRECTION: DERIVATIVES OF INFINITE ORDER

R. P. BOAS, JR., AND K. CHANDRASEKHARAN

It has been pointed out to us by S. Mandelbrojt that our statement[1] that M^o_{n+1}/M^o_n is nondecreasing is incorrect except on the interval $(-\infty, \infty)$ (where M^o must be replaced by M^c) and that for a finite interval there are in fact quasianalytic classes $C\{M_n\}$ which do not contain the class $C\{1\}$. However, Mandelbrojt has shown that our Theorem 2 is nevertheless correct; with his permission, we give his proof here. Theorem 2 states that, if $f(x)$ belongs to a quasianalytic class $C\{M_n\}$ in $a<x<b$ and if $f^{(n)}(x_0)\to L$ for one x_0 in (a, b), then $f(x)$ is analytic in (a, b) and consequently $f^{(n)}(x)\to Le^{x-x_0}$ in $a<x<b$. There are two cases: either $\liminf M_n^{1/n}>0$ or $\liminf M_n^{1/n}=0$. In the first case $C\{1\}\subset C\{M_n\}$ trivially and our original proof applies. In the second case, let $\{n_j\}$ be a sequence such that $M_j^{1/n_j}\to 0$. Since $|f^{(n_j)}(x_0)|<k^{n_j}M_{n_j}\to 0$ and $f^{(n)}(x_0)\to L$, we must have $L=0$. Given $\epsilon>0$, there exist p and i such that $|f^{(n)}(x_0)|<\epsilon$ for $n>p$ and $k^{n_j}M_{n_j}<\epsilon$ for $j>i$. For $n>p$ let $j>i$ and $n_j>p$; then for x in (a, b) and $|x-x_0|<1$,

$$f^{(n)}(x) = f^{(n)}(x_0) + (x - x_0)f^{(n+1)}(x_0) + \cdots + f^{(n_j)}(x')(x - x_0)^{n_j-n}/(n_j - n)!,$$

where x' is between x_0 and x. Then $|f^{(n)}(x)| \leqq \epsilon \sum_{k=0}^{\infty} |x-x_0|^k/k! + \epsilon=\epsilon(e^{|x-x_0|}+1)$, which shows that $f^{(n)}(x)\to 0$ uniformly between x_0 and x (and so, by a repetition of the argument, if necessary, in (a, b)), and also that $f(x)$ is analytic.

In line 9 of page 523, replace ae^x by ke^x.

Brown University and
Institute for Advanced Study

Received by the editors September 10, 1948.

[1] R. P. Boas, Jr. and K. Chandrasekharan, *Derivatives of infinite order*, Bull. Amer. Math. Soc. vol. 54 (1948) pp. 523–526; p. 524.