CORRECTION: DERIVATIVES OF INFINITE ORDER

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It has been pointed out to us by S. Mandelbrojt that our statement that $M_{n+1}/M_n$ is nondecreasing is incorrect except on the interval $(-\infty, \infty)$ (where $M^c$ must be replaced by $M^e$) and that for a finite interval there are in fact quasianalytic classes $C\{M_n\}$ which do not contain the class $C\{1\}$. However, Mandelbrojt has shown that our Theorem 2 is nevertheless correct; with his permission, we give his proof here. Theorem 2 states that, if $f(x)$ belongs to a quasianalytic class $C\{M_n\}$ in $a<x<b$ and if $f^{(n)}(x_0) \to L$ for one $x_0$ in $(a, b)$, then $f(x)$ is analytic in $(a, b)$ and consequently $f^{(n)}(x) \to Le^{-x}$ as $x \to x_0$ in $a<x<b$. There are two cases: either $\lim\inf M_n > 0$ or $\lim\inf M_n = 0$. In the first case $C\{1\} \subset C\{M_n\}$ trivially and our original proof applies. In the second case, let $\{n_j\}$ be a sequence such that $M_n^{1/n_j} \to 0$. Since $|f^{(n)}(x_0)| < k^{n_j}M_{n_j} \to 0$ and $f^{(n)}(x_0) \to L$, we must have $L = 0$. Given $\epsilon > 0$, there exist $p$ and $i$ such that $|f^{(n)}(x_0)| < \epsilon$ for $n > p$ and $k^{n_i}M_{n_j} < \epsilon$ for $j > i$. For $n > p$ let $j > i$ and $n_j > p$; then for $x$ in $(a, b)$ and $|x-x_0| < \epsilon$,

$$f^{(n)}(x) = f^{(n)}(x_0) + (x-x_0)f^{(n+1)}(x_0) + \cdots + f^{(n_j)}(x')(x-x_0)^{n_j-n}/(n_j-n)!,$$

where $x'$ is between $x_0$ and $x$. Then $|f^{(n)}(x)| \leq \epsilon \sum_{n=0}^{\infty} |x-x_0|^k/k! + \epsilon(\epsilon^{1/\epsilon} - 1)$, which shows that $f^{(n)}(x) \to 0$ uniformly between $x_0$ and $x$ (and so, by a repetition of the argument, if necessary, in $(a, b)$), and also that $f(x)$ is analytic.

In line 9 of page 523, replace $ae^x$ by $ke^x$.

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