THE FEBRUARY MEETING IN NEW YORK

The four hundred forty-third meeting of the American Mathematical Society was held at Columbia University on Saturday, February 26, 1948. The attendance was about two hundred seventy, including the following two hundred fifty-nine members of the Society:

Milton Abramowitz, C. R. Adams, E. J. Akutowicz, C. B. Allendoerfer, D. B. Ames, R. L. Anderson, T. W. Anderson, R. G. Archibald, L. A. Aroian, Helmut Aulbach, F. E. Baker, S. F. Barber, Joshua Barlaz, F. D. Bateman, P. T. Bateman, F. P. Beer, E. G. Begle, A. H. Berger, Stefan Bergman, Lipman Bers, Garrett Birkhoff, J. H. Blau, E. K. Blum, R. P. Boas, G. L. Bolton, Samuel Borofsky, D. G. Bourgin, S. G. Bourne, Paul Brock, A. B. Brown, J. H. Bushey, W. E. Byrne, W. R. Callahan, E. W. Cannon, P. G. Carlson, Jeremiah Certaine, K. Chandrasekharan, J. O. Chellevold, K. T. Chen, Y. W. Chen, S. S. Chern, P. L. Chessin, Randolph Church, J. A. Clarkson, G. R. Clements, E. A. Coddington, Carl Cohen, I. S. Cohen, R. M. Cohn, T. F. Cope, L. M. Court, W. H. H. Cowles, V. F. Cowling, J. E. Crawford, J. S. Cronin, C. W. Curtis, M. D. Darkow, D. A. Darling, Martin Davis, Philip Davis, A. S. Day, C. H. Dowker, T. C. Doyle, Jacques Dutka, Aryeh Dvoretzky, Samuel Eilenberg, Bernard Epstein, M. P. Epstein, R. M. Exner, W. H. Fagerstrom, A. B. Farnell, O. J. Farrell, Herbert Federer, J. M. Feld, F. G. Fender, R. S. Finn, Edward Fleisher, W. W. Flexner, R. H. Fox, Gerald Freilich, K. O. Friedrichs, David Gale, G. N. Garrison, L. L. Gavurin, Hilda Geiringer, Abe Gelbart, J. C. Gibson, B. P. Gill, Harriet Griffin, E. J. Gumbel, C. F. Hall, Philip Hartman, K. E. Hazard, C. M. Hebbert, Alex Heller, G. C. Helme, L. A. Henkin, A. P. Hillman, I. I. Hirschman, A. J. Hoffman, Banesh Hoffmann, T. R. Hollcroft, E. M. Hull, T. R. Humphreys, L. C. Hutchinson, Nathan Jacobson, A. R. Jacoby, R. A. Johnson, R. E. Johnson, Bjarni Jónsson, M. L. Juncosa, Robert Kahal, Shizuo Kakutani, Aida Kalish, Irving Kaplansky, Edward Kasner, M. E. Kellar, J. F. Kiefer, P. J. Kiernan, H. S. Kieval, S. A. Kiss, J. R. Kline, Morris Kline, E. G. Kogbetliantz, E. R. Kolchin, N. H. Kuiper, Wouter van der Kulk, Jack Laderman, A. W. Landers, R. E. Langer, J. R. Lee, Joseph Lehner, R. A. Leibler, R. B. Leipnik, Benjamin Lepson, W. J. LeVeque, Howard Levi, J. H. Lewis, Julius Lieblein, M. A. Lipschutz, Marie Litzinger, Charles Loewner, A. J. Lohwater, L. H. Loomis, E. R. Lorch, Lee Lorch, R. C. Lyndon, L. A. MacColl, H. M. MacNeille, Kurt Mahler, M. M. Maloney, Dis Maly, Irwin Mann, A. J. Maria, M. H. Maria, W. T. Martin, D. G. Mead, R. C. Meacham, A. E. Meder, K. S. Miller, W. H. Mills, G. C. Miloslavsky, Don Mittleman, Deane Montgomery, A. H. Moore, G. D. Mostow, Andrzej Mostowski, T. S. Motzkin, F. J. Murray, D. S. Nathan, C. A. Nelson, D. J. Newman, Morris Newman, P. B. Norman, I. L. Novak, C. O. Oakley, J. C. Oxtoby, F. W. Perkins, R. M. Peters, C. G. Plithides, E. L. Post, M. H. Protter, A. L. Putnam, Hans Rademacher, H. V. Rådström, L. R. Raines, G. N. Raney, John Rausen, G. E. Raynor, C. J. Rees, Helene Reschovsky, Moses Richardson, C. E. Rickart, R. F. Rinehart, J. F. Ritt, M. S. Robertson, Robin Robinson, S. L. Robinson, I. H. Rose, N. J. Rose, P. C. Rosenbloom, H. D. Ruderman, H. J. Ryser, C. W. Saalfrank, H. E. Salzer, Arthur Sard, Henry Scheffé, M. M. Schiffer, Pincus Schub, Abraham Schwartz, Abraham Seidenberg, Seymour Sherman, Edward Silverman, James Singer, M. H. Slud, P. A. Smith, J. J. Sopka, A. H. Sprague, George Springer, E. P. Starke, J. R. K. Stauffer, Fritz Steinhardt, F. M. Stewart, R. C. Stewart, R. C. Strodt, Walter Strodt, M. M. Sullivan, Fred Supnick,

R. T. Tear, Feodor Theilheimer, D. L. Thomsen, C. B. Tompkins, A. W. Tucker, J. W. Tukey, Annita Tuller, S. M. Ulam, D. H. Wagner, Sylvan Wallach, J. L. Walsh, M. A. Weber, J. H. Weiner, Alexander Weinstein, Louis Weisner, M. J. Weiss, J. G. Wendel, John Wermer, M. E. White, G. T. Whyburn, Albert Wilansky, A. B. Willcox, J. N. Williams, W. L. G. Williams, Jacob Wolfowitz, Y. K. Wong, M. Y. Woodbridge, Daniel Zelinsky, P. W. Zettler-Seidel, H. J. Zimmerberg, Leo Zippin.

On Saturday there were two general sessions for contributed papers, one at 10:00 A.M. in which Professor J. H. Bushey presided, and one at 3:30 P.M. in which Professor C. A. Nelson presided.

At 2:00 P.M. Dr. Stefan Bergman of Harvard University gave an address on *Some recent developments in the theory of analytic functions*. Professor W. T. Martin presided.

Abstracts of the papers presented follow below. Abstracts whose numbers are followed by the letter "*t*" were presented by title. Paper number 230 was read by Professor Tucker and paper number 254 was read by Mr. Kaplan. Dr. Rauch, Mr. Kaplan, and Mr. O'Brien were introduced by Professor T. R. Hollcroft.

ALGEBRA AND THEORY OF NUMBERS

221*t*. Richard Bellman: *On a generalization of an identity due to Wilton.*

Wilton, Proc. London Math. Soc. vol. 31 (1930) pp. 11–12, gave an identity relating the product $\zeta(s)\zeta(s')$, $\zeta(s+s'-1)$, and an infinite series of Dirichlet type, which, apart from its general interest, is of value in the estimation of the mean value of the square of the zeta-function. In the present paper a corresponding formula involving the product $\zeta^2(s)\zeta^2(s')$, $\zeta^2(s+s'-1)$, and a similar infinite series of Dirichlet type is obtained. Wilton's identity is derived using the Poisson summation formula. The present identity requires the Voronoi summation formula (which can be used similarly to obtain the classical approximate functional equation for $\zeta^2(s)$ due to Hardy and Littlewood) and some difficult results due to Atkinson concerning trigonometric sums with divisor functions as coefficients. It may also be applied to obtain mean value results, for $\sigma>1/2$. (Received February 11, 1949.)

222*t*. Richard Bellman: *On form-preserving linear transformations.* Preliminary report.

It is shown that only transformations of the form $x'=Ax$, where x has the components x_k, $1\leqq k\leqq N$, and A is an $N\times N$ matrix, which preserve the sums of kth powers, $k\geqq 3$, are the trivial ones. From this it follows easily that the only identities of the form (sum of kth powers)$\times$(sum of kth powers) equals (sum of kth powers), where the variables on the right are bilinear forms in the variables on the left, are again the trivial ones. (Received December 21, 1948.)

223*t*. Richard Bellman: *On some divisor sums associated with Diophantine equations.*

Asymptotic results for the sums $S_3=\sum_{n\leqq N} d(n)d_3(n+l)$, $S_4=\sum_{n\leqq N}d(n)d_4(n+l)$

are obtained. The number-theoretic functions $d_3(n)$, $d_4(n)$ are the coefficients in the Dirichlet series for $\zeta(s)^3$ and $\zeta(s)^4$, respectively, $d(n)$ is the divisor function. These sums arise when considering the number of positive, integral rational solutions of the diophantine equations $x_1x_2x_3-y_1y_2=l$, $x_1x_2x_3x_4-y_1y_2=l$, $y_1y_2\leqq N$, respectively. The problem reduces to that of finding estimates for the sums $\sum_{n\leqq N}d_3(kn+l)$, $\sum_{n\leqq N}d_4(kn+l)$ with sufficiently accurate error terms, which now depend upon k as well as N. These error terms are obtained, to the desired order of accuracy, follow-in a method of Hardy and Littlewood, using the mean value of the fourth power of the corresponding L-series mod k. (Received February 11, 1949.)

224*t*. Richard Bellman: *The average value of arithmetical functions.*

Consider the number-theoretical function, $\sigma_s(n)$, equal to the sum of the sth powers of the divisors of n. It is shown that $\sigma_s(f(n))$ has a mean value for any $s<0$, where $f(x)$ is any polynomial in x which is integer-valued for integer values of x. The mean value is a constant if n runs through the integers and equals $c/\log n$ if n runs through the primes. (Received December 21, 1948.)

225*t*. Sarvadaman Chowla and H. J. Ryser: *Combinatorial problems.* I.

Let v elements be arranged in v sets, and list the elements in a row and the sets in a column. Form an incidence matrix by inserting a one in row i and column j if the jth element belongs to the ith set, and a zero otherwise. Problem I: Arrange v elements into v sets such that (I_1) every set contains exactly k distinct elements, (I_2) every pair of sets has exactly $\lambda=k(k-1)/(v-1)$ elements in common $(0<\lambda<k<v)$. Problem II: Arrange v elements into v sets such that (II_1) each element occurs in exactly k distinct sets, (II_2) every pair of elements occurs in the v sets exactly $\lambda=k(k-1)/(v-1)$ times $(0<\lambda<k<v)$. Problem III: Arrange v elements into v sets fulfilling (I_1) and (I_2), (II_1), and (II_2). Problem (IV): Arrange v elements into v sets fulfilling (I_1) and (I_2) so that the incidence matrix is cyclic. A solution of any one of the problems I, II, and III is necessarily a solution of the remaining two. However, values for v and k exist for which Problem III has a solution and Problem IV has no solution. The theory is applied to Hadamard matrices, projective planes, block designs, and difference sets. (Received January 3, 1949.)

226*t*. Sarvadaman Chowla and H. J. Ryser: *Combinatorial problems.* II.

The theorems which follow concern the impossibility of Problem I described in the preceding abstract. Clearly the impossibility of Problem I for a given v and k implies the impossibility of Problems II, III, and IV. (1) if v is even and if $k-\lambda$ is not a square, then Problem I has no solution. (2) If $v\equiv 1 \pmod 4$ and if there exists an odd prime p such that p divides the squarefree part of $k-\lambda$ and if $(\lambda|p)=-1$, where $(\lambda|p)$ is the Legendre symbol, then Problem I has no solution. (3) If $v\equiv 3$ mod 4 and if there exists an odd prime p such that p divides the squarefree part of $k-\lambda$ and if $(-\lambda|p)=-1$, then Problem I has no solution. Theorems 2 and 3 are a rather straightforward generalization of a theorem of Bruck and Ryser on the nonexistence of certain finite projective planes, and for projective planes they give no new information (Bull. Amer. Math. Soc. Abstract 54-7-291). However, the proofs are now independent of the Minkowski-Hasse theory on the invariants of a quadratic form under rational transformations. (Received January 3, 1949.)

227t. Carl Cohen: *The Laplace transform of the generalized dilogarithm.*

Let $dl^{(k)}t$ be the integral from 0 to t of $x^{-1}dl^{(k-1)}x$, $dl^{(0)}t = -\log(1-t)$, then the Laplace transforms of this family of functions satisfy the law $(d/ds)[s \cdot L\{dl^{(k)}t\}] = -L\{dl^{(k-1)}t\}$, while $L\{dl^{(0)}t\} = s^{-1} \exp(-s)\, EI(s)$ is convergent if $s>0$. Take $\exp(t)$ instead of t as variable, then $(d/dt)[dl^{(k)}(\exp t)] = dl^{(k-1)}(\exp t)$ and $s \cdot L\{dl^{(k)}(\exp t)\} = \zeta(k+1) + L\{dl^{(k-1)}(\exp t)\} = \sum_{\mu=0}^{k-1} s^{-(\mu+1)}\zeta(k-\mu+1) + s^{-k}L\{dl^{(0)} \cdot (\exp t)\}$. $L\{dl^{(0)}(\exp t)\} = s^{-1}[\psi(1) \pm in\pi - \psi(s)]$, $s>0$, where $\psi(s) = (d/ds)[\log \Gamma(s)]$. The Laplace transform has been used to study the identities of the generalized dilogarithm. (Received November 26, 1948.)

228. R. H. Fox: *Free differential calculus and free groups.*

Iteration of a previously given formula (Bull. Amer. Math. Soc. Abstract 55-3-115) gives a "Taylor series expansion" $u = u^0 + \sum_j (\partial u/\partial x_j)^0 (x_j - 1) + \sum_{jk} (\partial^2 u/\partial x_j \partial x_k)^0 \cdot (x_j - 1)(x_k - 1) + \cdots$ about the point $(1, 1, \cdots)$ of any element u of the integral group ring P of a free group F. This expansion differs only by a translation $x_j = 1 + s_j$ from an expansion considered by Magnus (Math. Ann. vol. 111 (1935) pp. 259–280) for the case $u \in F$. The element u of P is determined uniquely by the coefficients of this series. Several known theorems follow from this: (1) The ring P has no divisors of zero (Higman, Proc. London Math. Soc. vol. 46 (1939) pp. 231–248); (2) the intersection of the subgroups of the lower central series of F consists of the identity element alone (Magnus, loc. cit.). (Received January 13, 1949.)

229. David Gale: *Convex cones and solutions of two-person games.*

A subset C of a finite-dimensional vector space V is called a *convex cone* if it is closed under the operations of addition and multiplication by non-negative scalars. The set of (convex) cones in V is closed under the operations of intersection ($\cap$) and algebraic sum ($+$). In addition there is an operation which attaches to each cone C its polar cone C^*, defined to be the set of all vectors y in V such that $y \cdot x \geqq 0$ for all x in C. For cones which are topologically closed the important property $(C^*)^* = C$ holds, and the set of cones becomes an orthocomplemented lattice under $+$, $\cap$ and $*$. If the unit vectors of V are taken to be the pure strategies of a zero-value, two-person game, the set of good mixed strategies will be a closed cone in the above sense. By making strong use of the lattice properties a complete characterization of the possible sets of good strategies is derived for any two-person game, a result obtained independently by S. Sherman and by H. F. Bohnenblust, S. Karlin and L. S. Shapley. (Received January 10, 1949.)

230. David Gale, H. W. Kuhn, and A. W. Tucker: *A game problem equivalent to a maximum problem.*

Let B be an m by n matrix, c a p-vector (p by 1 matrix), and D a p by n matrix (all real). Let X be the convex set of m-vectors x got by the linear mapping $x = By$ from the convex set of n-vectors y determined by the $n+p$ linear inequalities $y \geqq 0$ and $c \geqq Dy$ (where such vector inequalities hold for each component separately). A vector of a of X is called maximal if $x \geqq a$ for x in X implies $x = a$. The problem of finding a maximal a when B, c, D are given is closely related to econometric problems considered by G. B. Dantzig and T. C. Koopmans. Let G be the 0-sum 2-person game (see von Neumann and Morgenstern, *The theory of games and economic behavior*) associated with the $m+p$ by $n+1$ composite matrix formed from a, $-B$, $-c$, and D. A submatrix of a game matrix is called essential if both players have good mixed strategies in which the pure strategies corresponding to the rows and columns of the submatrix are used with positive probabilities. Theorem: A vector a in X is maximal if and only if the game G has value zero and the submatrix a is essential to G. (Received March 3, 1949.)

231*t*. Florence D. Jacobson and Nathan Jacobson: *Classification and representation of semi-simple Jordan algebras.*

The authors give a determination of the semi-simple special Jordan algebras over any field of characteristic 0. An *imbedding* of a Jordan algebra K is defined to be a linear mapping $a \to a^R$ of K into an associative algebra such that $(a \cdot b)^R = 2^{-1}(a^R b^R + b^R a^R)$. An imbedding in an algebra of linear transformations is called a *representation*. They show that the imbedding problem can be reduced to the problem of homomorphism of a certain universal associative algebra, and determine the structure of these universal associative algebras for semi-simple Jordan algebras of characteristic 0. These universal algebras are semi-simple. It follows that any representation of a semi-simple Jordan algebra is completely reducible. Moreover, the irreducible representations can be given by using these results. (Received January 28, 1949.)

232. Irving Kaplansky: *Primary ideals in group algebras.*

Let A be the L_1-algebra of any locally compact abelian group. It is shown that any closed primary ideal in A is maximal. The method is an extension of that of Segal and Ditkin, who proved the theorem for the group of integers or of real numbers. Some applications are noted. (Received January 13, 1949.)

233*t*. Jakob Levitzki: *A theorem on polynomial identities.*

Let S be a ring satisfying a polynomial identity $f(x_1, x_2, \cdots, x_n) = 0$ of degree $d \geqq 1$. If d is of minimal value we say in short: S is a *PI*-ring of degree $d = d(S)$. Denote by $N(S)$ the radical of S (that is, the sum of all nilpotent ideals). The following theorem is proved: If S is a *PI*-ring of degree d, then the nilpotent elements of S satisfy the relation $x^{[d/2]} \in N(S)$. This implies, for example, that if S is the ring of all k by k matrices over a field F, then $k \leqq [d/2]$. Scalar extension of the underlying field of an algebra yield further results. Other applications concern *PI*-nil-rings. Starting with $N_1(S) = N(S)$, R. Baer (*Radical ideals*, Amer. J. Math. vol. 65 (1943)) defined a transfinite ascending chain of ideals $N_\alpha(S)$ terminating in the lower radical $U(S)$. The author proves that each *PI*-nil-ring is an L-ring, that is, such that $S = U(S)$. Since each L-ring is semi-nilpotent (examples show that the converse is false) this includes a recent result of I. Kaplansky (*Rings with a polynomial identity*, Bull. Amer. Math. Soc. vol. 54 (1948)). If $\lambda = \lambda(S)$ denotes the length of an L-ring (that is, the smallest ordinal such that $N_\lambda(S) = N_{\lambda+}(S) = S$) we prove that if S is a *PI*-nil-ring—and hence an L-ring—then $\lambda(S)$ is finite. Moreover: $\lambda(S) \leqq \log d(S)/\log 2$ ($S \neq 0$; if $S = 0$ then $\lambda = d = 1$). In particular: $\lambda(S) < d(S)$ for $S \neq 0$. (Received January 3, 1949.)

234. R. C. Lyndon: *The representation of algebras of relations.*

A complete set of axioms is given which characterizes the class of all abstract relational algebras (cf. Jónsson and Tarski, Bull. Amer. Math. Soc. Abstract 54-1-89) that are isomorphic to algebras of concrete binary relations. This set is infinite, and the axioms of the set contain bound variables, in addition to the operations union, intersection, complement, converse, and relational product. It is shown that this class of algebras is not characterizable by any set of axioms containing only free variables. This provides a negative solution to the representation problem for relational algebras posed by Jónsson and Tarski (loc. cit. problem one). (Received January 10, 1949.)

235. Seymour Sherman: *Size, shape and situation of solutions in strategy simplices.*

Let $M_i =df.$ closed convex polyhedral subset of Sm_i; the unit simplex of real n_i-dimensional euclidean space U_i; $E_i=df.$ minimal face of Sm_i containing M_i; $\epsilon_i=df.$ number of vertices in E_i; $\sigma_i=df.\ n_i-\epsilon_i$; $\mu_i=df.$ the dimension of M_i; $m_i=df.$ number of maximal faces of M_i; $e_i=df.$ number of maximal faces of M_i contained in proper faces of E_i. Necessary and sufficient conditions that Sm_i and M_i, $i=1, 2$, represent the strategy simplices and solution polyhedra of some zero-sum, two-person, $n_1\times n_2$ game are (1) $\epsilon_1-\mu_1=\epsilon_2-\mu_2$, (2) $\sigma_2\geqq m_1-e_1$, and (3) $\sigma_1\geqq m_2-e_2$. The necessity of (1) has been shown by Bohnenblust, Karlen, and Shapley, *Solutions of discrete, two-person games* (unpublished), and Gale, Kuhn, and Tucker, *Multiplicity of solutions of two-person zero-sum games* (unpublished). Applications of the theorems are given. (Received December 20, 1948.)

236*t*. Robert Steinberg: *A geometric approach to the representations of the full linear group over a Galois field.*

Through a consideration of the permutation of the geometric entities of $PG(n-1, q)$ effected by the group G, which is essentially the collineation group of $PG(n-1, q)$ of all n-ary linear homogeneous substitutions of nonzero determinant with marks in $GF(q)$, a set $p(n)$ of irreducible representations of G is determined. This set immediately leads to a large number of irreducible representations of G, and a correspondence is seen to exist between sets of the representations so determined and sets of classes of conjugate elements of G. Through a favorable comparison of G with the symmetric group of degree n, much use is made throughout the paper of the methods of Frobenius in his determination of the irreducible representations of Sn (Berliner Berichte (1900) pp. 516–534). (Received December 20, 1948.)

Analysis

237. D. B. Ames: *Certain inversion formulas for the Laplace transform.*

The case of a periodic determining function $F(t)$, real or complex, is first discussed. Inversion formulas are derived which have the forms of the Fourier series of the functions $\int_0^t F(q)e^{-xq}dq$ and $F(t)e^{-xt}$. The coefficients involve the transform and its real and imaginary parts. Similar formulas hold when $F(t)$ is non-periodic and zero for $t>t_0>0$. From these results and extensions of them, several theorems are derived about the form of $F(t)$ when the transform vanishes at points on a vertical line, yielding a contrast with Lerch's theorem. Necessary and sufficient conditions are obtained for $F(t)$ to be null. By a natural generalization of results in the periodic case an inversion formula, of similar form, is derived for the general case where $F(t)$ is non-periodic and has a finite abscissa of absolute convergence. This holds when $F(t)$ is real and integrable but not necessarily bounded. It is shown, with no implication that the transform is analytic, that $F(t)$ is null if its transform vanishes at all points of any vertical line in the half-plane of absolute convergence. (Received January 6, 1949.)

238*t*. Stefan Bergman and M. M. Schiffer: *Kernel functions and conformal mapping*. I.

Let B be a domain in the complex plane bounded by analytic curves; let $g(z, \zeta)$ be Green's function of B. Define the functions $K(z, \bar{\zeta}) = -(2/\pi)g_{z\bar{\zeta}}$ and $l(z, \zeta) = (1/\pi)(z-\zeta)^{-2} - (2/\pi)g_{z\zeta}$ which are analytic in their arguments. For each analytic $f(z)$ with $\iint |f|^2 d\tau_B < \infty$ we have $\iint K(z, \bar{\zeta})f(\zeta)d\tau_B = f(z)$, $\iint \overline{l(z, \zeta)}f(\zeta)d\tau_B = (1/\pi) \cdot \iint \overline{(z-\zeta)^{-2}} \cdot f(\zeta)d\tau_B$. $K(z, \bar{\zeta})$ becomes infinite if z and ζ converge to the same boundary point of B: $l(z, \zeta)$ is regular in the closed region $B+C$. The identity (a) $\iint l(z,\zeta)\overline{l(\zeta, w)}d\tau_B = K(z,\bar{w}) - \Gamma(z, \bar{w})$, with $\Gamma(z, \bar{w}) = \iint (\zeta - z)^{-2}\overline{(\zeta - w)}^{-2} d\tau_{\tilde{B}}$, $\tilde{B}$ = complement of B, shows that $K-\Gamma$ is regular in the closed region $B+C$. One has the inequality (b) $\left| \sum l(z_i, z_k)x_i x_k \right| \leqq \sum K(z_i, \bar{z}_k)x_i\bar{x}_k$ for arbitrary constants x_i and points $z_i \in B$ $(i=1, \cdots, N)$. The transform $Tf(z) = (1/\pi)\iint (z-\zeta)^{-2}\overline{f(\zeta)}d\tau_B$ is defined in $B+\tilde{B}$; l is TK in B. The inversion formula $f(z) = (1/\pi)\iint (z-\zeta)^{-2}\overline{Tf(\zeta)}d\tau_{B+\tilde{B}}$ is proved. The integral equation $\phi_\nu = \lambda_\nu \cdot T\phi_\nu$ is studied by means of (a). (Received February 24, 1949.)

239*t*. Stefan Bergman and M. M. Schiffer: *Kernel functions and conformal mapping*. II.

If $w = \phi(z)$ maps the domain B upon a domain B_1 with corresponding kernels $K_1(w, \bar{\omega})$ and $l_1(w, \omega)$, one has the transformation formulas: $K_1(w, \bar{\omega})\phi'(z)\overline{\phi'(\zeta)} = K(z, \bar{\zeta})$, $l_1(w, \omega)\phi'(z)\phi'(\zeta) = l(z, \zeta) + U(z, \zeta)$ with $U(z, \zeta) = (1/\pi)[\log (\phi(z) - \overline{\phi'(\xi)}) - \log (z-\zeta)]_{z\zeta}$. Since the inequality (b) has to hold also in B_1, $\left| \sum [l(z_i, z_k) + U(z_i, z_k)]x_i x_k \right| \leqq \sum K(z_i, \bar{z}_k)x_i\bar{x}_k$ which gives, for K and l known and fixed, an inequality for all functions $\phi(z)$ univalent in B. One obtains easily by specialization Grunsky's necessary and sufficient conditions of univalence. The following lemma is used: Let $V(z, \zeta) = \sum d_{mn}z^m\zeta^n$ be symmetric in z and ζ and analytic around the origin; let $K(z, \bar{\zeta}) = \sum k_{mn}\, z^m \cdot \bar{\zeta}^n$ be the development of the K-kernel of B around the origin. If $\left| \sum d_{mn}\, x_m x_n \right| \leqq \sum k_{mn}x_m\bar{x}_n$ for every choice of x_n, then $V(z, \zeta)$ is regular analytic for both variables in B. (Received February 24, 1949.)

240*t*. Stefan Bergman and M. M. Schiffer: *Kernel functions and pseudo-conformal mapping*. I.

Let D^4 be a domain in the z_1, z_2-space bounded by two analytic hypersurfaces $\mathfrak{h}_1^3 = [z_2 = e^{i\lambda_1}]$ and $\mathfrak{h}_2^3 = [z_1 = h(z_2, \lambda), 0 \leqq \lambda \leqq 2\pi]$. $S^2 = \mathfrak{h}_1^3 \cap \mathfrak{h}_2^3$ is the distinguished boundary surface of D^4. The authors consider functions of the "extended class" $E(D^4)$, that is, real functions $H(x_1, y_1, x_2, y_2)$ which are harmonic in x_1, y_1 in every intersection $D^4 \cap [z_2 = z_2^0] = D^2(z_2^0)$ and such that $H(\mathrm{Re}[h(z_2, \lambda)], \mathrm{Im}\, [h(z_2, \lambda)], x_2, y_2)$ is harmonic in $|z_2| < 1$ for every λ. Every function which is the real part of an analytic function in z_1, z_2 belongs to $E(D^4)$. The generalized Green's formula $\{\phi, \psi\} = \iint \phi\psi_{n_1n_2}ds_1ds_2 = \iint \phi_{n_1}\psi_{n_2}ds_1ds_2 = \iint \phi_{n_2}\psi_{n_1}ds_1ds_2 = \iint \phi_{n_1n_2}\psi ds_1ds_2$ is proved for ϕ, $\psi \in E(D^4)$ where the integration is extended over S^2 and $\{\phi, \psi\} = \iiiint_{D^4}[\phi_{x_1x_2}\psi_{x_1x_2} + \phi_{y_1x_2}\psi_{y_1x_2} + \phi_{x_1y_2}\psi_{x_1y_2} + \phi_{y_1y_2}\psi_{y_1y_2}]dx_1dy_1dx_2dy_2$. Orthonormal functions of the class $E(D^4)$ in the $\{\phi, \psi\}$-metric are defined, the convergence of their kernel K is proved. (Received February 24, 1949.)

241*t*. Stefan Bergman and M. M. Schiffer: *Kernel functions and pseudo-conformal mapping*. II.

Fundamental solutions in the class $E(D^4)$ of the form $\log |z_1 - \zeta_1| \log |z_2 - \zeta_2| + \delta_1(z_2, \zeta_2) \log |z_1 - \zeta_1| + \delta_2(z_1, \zeta_1; \zeta_2) \log |z_2 - \zeta_2| + s(z_1, z_2; \zeta_1, \zeta_2)$, $s \in E(D^4)$, are introduced. δ_1, δ_2 are harmonic in $|z_2|<1$ and $z_1 \in D^2(\zeta_2)$, respectively. Four Green's functions $g^{(i)}(z_1, z_2; \zeta_1, \zeta_2)$ are defined as follows: $g^{(i)}$ is a fundamental solution with properly chosen δ_1 and δ_2 and satisfies on S^2 the conditions: $g^{(1)}=0$, $g^{(2)}_{n_1 n_2}=c_1(z_1)+d_1(z_2)$, $g^{(3)}_{n_1}=c_2(z_2)$, $g^{(4)}_{n_2}=c_3(z_1)$. They lead to the solution of boundary values problems for biharmonic functions. $L=g^{(1)}+g^{(2)}-g^{(3)}-g^{(4)}$ is an element of $E(D^4)$. Let K be the kernel of this class, then $K-L$ is orthogonal to all biharmonic functions in the $\{\phi, \psi\}$-metric. In the case of special domains, for example, product domains, one has $K=L$. This result represents a generalization of a previous theorem for harmonic functions of two real variables (see Bergman-Schiffer, Duke Math. J. vol. 14 (1947) p. 623). (Received February 24, 1949.)

242. Lipman Bers: *Isolated singularities of minimal surfaces. II.*

The results announced previously (Bull. Amer. Soc. Abstract 54-7-239) are generalized to the case of a solution of the minimal surface equation (1) $[\phi_x(1+\phi_x^2+\phi_y^2)^{-1/2}]_x + [\phi_y(1+\phi_x^2+\phi_y^2)^{-1/2}]_y=0$ with finitely many-valued gradients. If $\phi(x, y)$ has an isolated singularity at a finite point, say at $z=x+iy=0$, then $w=\phi_x-i\phi_y$ has a limit w_0 at 0. If $\phi(x, y)$ has an isolated singularity at ∞, then w has a *finite* limit at ∞. If the gradient at the singularity is finite, then $\phi(x, y)=\phi_0(x, y)+R(x, y)$ where $\phi_0(x, y)$ considered on the Riemann surface of $z^{1/m}$ (m a positive integer) is harmonic and $R(x, y)$ is small compared to $\phi_0(x, y)$. If the gradient w is infinite at the singular point, a new kind of singularity arises which is in general topologically different from any singularity of harmonic functions. Explicit parametric and asymptotic formulas for all possible singularities are given. (Received February 9, 1949.)

243. T. M. Cherry: *Uniform asymptotic expansions of Bessel, hypergeometric, and other functions with transition points.*

The functions concerned involve a large parameter ν, and satisfy a differentia equation of the form (1) $d^2y/dz^2+y\{\nu^2 f(z)+g(z, \nu^{-2})\}=0$, where $f(z)$ is regular at $z=0$ and vanishes to the first order at this point, and g is regular at $z=0$, $\nu^{-2}=0$. Thus in the real case the solutions are monotonic on one side of $z=0$, oscillating on the other. A method is given for approximating to the solutions of any such equation in terms of those of another such equation, for example, the Airy equation (2) $d^2y/dz^2-\nu^2 zy=0$. *The approximations are uniform to any desired order $2n$ in ν^{-1}, for z in a domain having the point $z=0$ in its interior*, and are found by transforming the equations (1) and (2) to forms which differ only by terms of order $\nu^{-2n}y$. They are made by means of asymptotic series proceeding in integral powers of ν^{-2}. In the case of the Bessel functions $J_\nu(\nu z)$, $Y_\nu(\nu z)$, the approximations by means of Airy functions are uniform for all values of z, and the $O(\nu^{-4})$ approximation gives accuracy to 7 significant figures for $\nu \geqq 5$, or to 5 figures for $\nu \geqq 2$. (Received December 10, 1948.)

244*t*. I. I. Hirschman and J. A. Jenkins: *Note on a result of Levine and Lifschitz.*

The connection between the integral orders of the zeros of a function represented by a Fourier series and the degree of lacunarity of the series has been studied by S. Mandelbrojt and after him by B. Levine and M. Lifschitz, Rec. Math. (Mat. Sbornik) N.S. vol. 9 (1941) pp. 693–711. This note contains certain refinements of the results previously obtained. (Received December 20, 1948.)

245*t*. Meyer Karlin: *Systems of extremals for the simplest isoperimetric problem.*

The system of extremals of the simplest isoperimetric problem $\int F(x, y, y')dx = \min$ while $\int G(x, y, y')dx = \text{constant}$ is shown to be a triply infinite system of curves of the form $y''' = Ay''^3 + By''^2 + Cy'' + D$ where A, B, C, D are functions of x, y, y'. For a given lineal element there corresponds ∞^1 curves. The locus of centers of curvatures of the evolutes of these curves is a cubic. Some special and degenerate cases are discussed. (Received December 9, 1948.)

246*t*. Norman Levinson: *Criteria for the limit-point case for second order linear differential operators.*

If $f(x) \leqq Kx^2$ for large x then it is shown that at most one solution of $u'' + f(x)u = 0$ is integrable squared over (x_0, ∞). This is a special case of the following: The equation $(pu')' + qu = 0$ is in the limit-point case if there exists a positive monotone increasing function $M(x)$ such that for large x, (a) $q(x) \leqq M(x)$, (b) $\int^\infty dx/(p(x)M(x))^{1/2} = \infty$, and (c) $\lim\sup p^{1/2}(x)M'(x)/M^{3/2}(x) < \infty$. (Received January 3, 1949.)

247*t*. Norman Levinson: *On the uniqueness of the potential in a Schrödinger equation for a given asymptotic phase.*

The solution $y(x, \lambda)$ of $y'' + (\lambda^2 - P(x))y = 0$ with initial values $y(0, \lambda) = 0$, $y'(0, \lambda) = 1$ will, in case $P(x)$ is small for large x, satisfy $y(x, \lambda) \to B(\lambda) \sin(\lambda x - \phi(\lambda))$ as $x \to \infty$ where $B(\lambda)$ and $\phi(\lambda)$ are continuous for $0 < \lambda < \infty$. In physics the problem of the determination of $P(x)$ for a given phase function $\phi(\lambda)$ arises. It is shown that if $\int_0^1 x|P(x)|dx + \int_1^\infty x^2|P(x)|dx < \infty$ and if $\phi(\infty) - \phi(+0) < \pi$ then $\phi(\lambda)$ determines $P(x)$ uniquely. It is also shown that under more general conditions $\phi(\lambda)$ determines $B(\lambda)$ and conversely. In case $P(x) \geqq 0$ then the condition at ∞ becomes $\int_1^\infty xP(x)dx < \infty$. The condition $\phi(\infty) - \phi(+0) < \pi$ is equivalent to the condition that the boundary value problem should have no discrete spectrum. (Received January 3, 1949.)

248. L. H. Loomis: *Note on a theorem of Mackey.*

The author gives a simpler proof, by new methods, of the following slight generalization of a theorem of Mackey. Let $\sigma \to U_\sigma$ be a weakly continuous representation of a locally compact group G by unitary operators on a Hilbert space H, and let $A \to \overline{A}$ be a nontrivial Boolean homomorphism from the open subsets of G with compact closures onto projection operators on H, such that $U_\sigma \overline{A} = \overline{(\sigma A)} U_\sigma$. Then H is the direct sum of subspaces H_i each of which is unitary-equivalent to $L^2(G)$ by a mapping T_i such that U_σ becomes left translation through $\sigma(T_i U_\sigma T_i^{-1}(f(x)) = f(\sigma^{-1}x))$ and $\overline{A}$ becomes multiplication by the characteristic function $A(x)$ of $A(T_i \overline{A} T_i^{-1}(f(x)) = A(x)f(x))$. (Received February 26, 1949.)

249*t*. S. H. Min: *On the order of $\zeta(1/2+it)$.*

The problem of finding an upper bound for θ such that $\zeta(1/2+it) = O(t^\theta)$ has been attacked by van der Corput and Koksma, Walfisz, Titchmarsh, Phillips and Titchmarsh. Their results obtained are, neglecting a factor involving $\log t$, $\theta \leqq 1/6$, 163/988, 27/164, 229/1392 and 19/116 respectively. The object in the present paper is to prove that $\zeta(1/2+it) = O(t^{15/92+\epsilon})$ for $\epsilon > 0$. The proof depends essentially on the estimation of a double exponential sum of the form $\sum\sum \exp(2\pi i\phi(x, y))$ where $\phi(x, y)$ is a function whose Hessian $H(x, y)$ vanishes along certain lines. The method used in this paper can be applied to problems in the analytic theory of numbers, whose solutions depend upon the estimation of similar exponential sums. (Received November 17, 1948.)

250. H. E. Rauch: *A "Poisson" formula and the Hardy-Littlewood theorem for matrix spaces.* Preliminary report.

An important type of "matrix space" of Siegel and Hua (cf. Hua, Amer. J. Math. vol. 66 (1944) pp. 470–488) is the space H consisting of the $n \times n$ complex matrices, Z, for which $I - ZZ^* > 0$ (positive definite). This space, viewed as a subset of the space of n^2 complex variables, presents a remarkable analogy to the unit circle, $1 - z\bar{z} > 0$ (of which it is a direct generalization), the characteristic feature being the existence of a transitive group of analytic homeomorphisms which can be written as $W = (AZ + B)(CZ + D)^{-1}$ where A, B, C, D are $n \times n$ complex matrices satisfying the matrix analogues of the usual conditions. Clearly, the boundary of H contains $U(n)$, the unitary group, as the analogue of $z\bar{z} = 1$. For the above and other matrix spaces the author has proved the following exact generalization of a well known theorem of Hardy and Littlewood on analytic functions in the unit circle: Let $f(Z)$ be an analytic function of n^2 complex variables, $Z \in H$, such that $\int_{U \in U(n)} |f(rU)|^\lambda dv \leqq M^\lambda$ where dv is the volume element on $U(n)$, $0 \leqq r < 1$, $\lambda > 0$. Then $\int_{U \in U(n)} \{\sup_{0 < r < 1} |f(rU)|\}^\lambda dv \leqq \alpha M^\lambda$ where α is independent of f. The proof combines previous methods of the author (C. R. Acad. Sci. Paris vol. 227 (1948) pp. 887–889) and a new formula directly generalizing the classical Poisson integral for real parts of analytic functions: $f(Z) = \int_{U \in U(n)} f(U)(\det|I - \bar{Z}Z'|)^n(\det\ |I - [U^*Z' + \bar{Z}U] + \bar{Z}Z'|)^{-n}dv$. (Received January 13, 1949.)

251*t*. H. M. Schwartz: *Contributions to existence theory of ordinary differential equations in the real domain.* I.

Carathéodory (*Vorlesungen über reele Funktionen*, 2d ed., pp. 665–688) has given far-reaching existence theorems for a system of ordinary differential equations in the real domain of the form (1) $y_i'(x) = f_i(x, y_1, \cdots, y_n)$ $(i = 1, 2, \cdots, n)$. His assumptions on f_i are as general as can be desired as far as their dependence on x is concerned, but they are unduly restrictive in what concerns their dependence upon the $y|f_i|$ <summable function of x (Theorem 2, p. 672). The author considers here some of the possibilities for relaxing the latter restriction. No attempt is made here at attaining at the same time full generality in the x-dependence of the f_i, but rather to single out some results which can be derived by simple means. A typical result is as follows. If the f_i are continuous for x in an interval I and all finite y, except possibly for a subset of I whose kth derived set vanishes where k is any positive integer, then the system (1) has a solution continuous in I and satisfying there any given initial values, provided that the f_i satisfy in I and for all finite y the following inequality: $|f_i| \leqq g(x) + h(x)\phi(\sum|y_i|)$, $\phi(t)$ monotonically increasing for $t > 0$, $\lim\sup_{t \to \infty}\phi(t)/t < \infty$, and g, h summable in I. (Received January 17, 1949.)

252. Edward Silverman: *A definition of Lebesgue area for surfaces in arbitrary metric spaces.* Preliminary report.

An area is defined for surfaces in arbitrary metric spaces which agrees with Lebesgue area for surfaces in Euclidean space. The procedure is to define the area of a triangle in a Banach space B in such a way as to permit the use of the original definition of Lebesgue area. Let this area be denoted by L_B. Surfaces are then studied in m, the space of bounded sequences. In particular, it is shown that isometric surfaces in m have the same L_m area. Since any surface can be mapped isometrically into m, its Lebesgue area is defined to be the area of an isometric image in m. Let this area be denoted by L. If B is finite-dimensional, then it is shown that $L=L_B$. (Received February 24, 1949.)

Applied Mathematics

253. T. C. Doyle: *Invariant theory of the general, ordinary, linear, homogeneous, second order, differential boundary problem.*

There is presented an invariantive formulation of the boundary problem treated in R. E. Langer's first Herbert Ellsworth Slaught memorial paper, *Fourier's series, the genesis and evolution of a theory*, Amer. Math. Monthly vol. 54 (1947). The solution $y(x, \lambda)$ of the differential equation, $y''+p(x)y'+[q(x)\lambda+r(x)]y=0$, is subjected to the boundary conditions $\sum_{\rho=1}^{2} [(\beta_{j\rho}^{-1}\lambda+\gamma_{j\rho}^{-1})y\cdot(a^\rho, \lambda)+(\beta_{j\rho}^{0}\lambda+\gamma_{j\rho}^{0})y(a^\rho, \lambda)]=0$, where the β's and γ's are constants and $y\cdot(x, \lambda)$ is the covariant derivative of $y(x, \lambda)$ under the most general group of transformations, $y=e^{\sigma(x)}\bar{y}$, $\bar{x}=\xi(x)$, preserving the differential equation. The Green's function is obtained in invariant form and its residues are computed by reduction of the system to canonical form. (Received December 7, 1948).

254. Sidney Kaplan and George O'Brien: *A note on Richardson's solution of the heat equation by finite difference techniques.*

Richardson's finite difference formula for the solution of the heat equation: $\partial^2\phi/\partial x^2=\partial\phi/\partial t$, namely: $(\phi_{k+1,j}-\phi_{k-1,j})/2\Delta t=(\phi_{k,j+1}-2\phi_{k,j}+\phi_{k,j-1})/(\Delta X)^2$ (L. F. Richardson, *The approximate arithmetical solution by finite differences of physical problems involving differential equations*, Philos. Trans. Roy. Soc. London, Ser. A vol. 210 (1910) p. 313) is proved to be unstable notwithstanding the fact that his formula possesses a remarkably small truncation error. The authors prove both theoretically and by examples that at a very early stage the solutions begin to oscillate, and at $X=0.4$, $t=0.017$, the values diverge beyond the boundary. The authors then exhibit a stable finite difference formula: $(\phi_{k+1,j}-\phi_{k,j})/\Delta t=(\phi_{k,j+1}-2\phi_{k,j}+\phi_{k,j-1})/(\Delta X)^2$ which can be used to solve this classical problem. (Received January 10, 1949.)

255*t*. Eric Reissner: *On bending of curved thin-walled tubes.*

The problem under consideration may be thought of as a problem of determining an axi-symmetrical stress distribution in a shell of revolution. Previous work on shells of revolution by H. Reissner, E. Meissner and others presupposed rotationally symmetric univalued displacements. It is shown that by admitting suitable multivalued expressions for displacements the known results may be generalized in such a way that the results for the problems of tube bending, previously studied by von Kármán, Lorenz and others by energy methods, are included. (Received January 13, 1949.)

256*t*. I. F. Ritter: *Noise abatement in numerical inversion of a matrix.*

"Noise" being defined as the accumulated influence of the rounding off errors, its occurrence in inverting a matrix by the elimination method is described and estimates for its magnitude are derived in a rigorous manner by J. von Neumann and H. H. Goldstine (Bull. Amer. Math. Soc. vol. 53 (1947)). In a manner quite trivial, compared with this elaborate solution of the involved estimation problem, an iteration process can be carried out by which, from a noisy approximation to the inverse, an unlimited number of correct digits of the true inverse can be computed. The effort required to remove the noise from the first solution does not exceed the work involved in obtaining the latter. The process is a matrix iteration, hence lends itself to automatic operation. If carried out on a desk calculator it is not impeded by limited width of the keyboard. It is particularly efficient in finding the numerical solution of a set of many linear equations to any desired precision. (Received November 19, 1948.)

257. Charles Tompkins: *Projection methods in calculation of some linear problems.*

The author deals with methods of calculation. In problem I he seeks a solution of equations $\sum A_{ik}x_k = b_i$; in problem II, to minimize the linear function $J = \sum c_k x_k$ subject to the restriction $\sum A_{ik}x_k \geqq b_i$, where $A_{ik} \geqq 0$. Problem II is of interest in connection with various linear supply problems concerning cost; in these the restrictive inequalities frequently include $x_i \geqq 0$. Successive projection from a point to the various hyperplanes represented by the equations of problem I (in terms of a multidimensional coordinate geometry with rectangular cartesian coordinates) is studied. Each projection is computationally short, and each produces a point nearer a solution (if one exists) by a factor definitely less than one. These two statements imply stability in the calculation and convergence. A possible helpful utilization of the fact of slow convergence in particular problems is noted. A similar attack is made on problem II; this attack involves motion from a point within the allowed region along a line whose direction numbers are the gradient of the linear function; when the boundary of the region is reached, reflection into the region is taken along another direction—one perpendicular to a bounding hyperplane reached. Slow convergence is again interpreted. (Received January 14, 1949.)

258. Alexander Weinstein: *On generalized potential theory and the equation of Darboux-Tricomi.*

It is shown that the two fundamental integrals of the equation $(^*)\phi_{xx}+\phi_{yy}+2py^{-1}\phi_y=0$ investigated in a previous paper (A. Weinstein, *Discontinuous integrals and generalized potential theory*, Trans. Amer. Math. Soc. vol. 63 (1948) pp. 342–354) are given by the formulas $\phi_1=\pi^{-1}b^{-1/2}y^{-p}Q_{p-1}(1+2\epsilon^2)$ and $\phi_2=\pi^{-1}b^{-1/2}y^{-p}Q_{-p}(1+2\epsilon^2)$ where $0<2p<2$, $y\geqq 0$, $b>0$, $\epsilon^2=[x^2+(y-b)^2](4by)^{-1}$, and P and Q are Legendre functions of the first and second kind. It is shown that $\phi_1=-\pi^{-1}b^{-1/2}y^{-p}P_{p-1}(1+2\epsilon^2)\log\epsilon+R_1(x, y-b)$ and $\phi_2=-\pi^{-1}b^{-1/2}y^{-p}P_{-p}(1+2\epsilon^2)\log\epsilon+R_2(x, y-b)$ where R_1 and R_2 denote power series. The coefficient of $\log\epsilon$ in both cases is $u(x, y)=\pi^{-1}b^{-1/2}y^{-p}F(p, 1-p, 1, -\epsilon^2)$ where F denotes the hypergeometric series. The function $u(x, y)$ is a solution of (*) and coincides, for $p=1/6$, with the solution given by F. Frankl (Bull. Acad. Sci. URSS. vol. 10 (1946) pp. 135–166) in his investigations on Darboux-Tricomi's equation and its applications to transonic flows. (Received January 7, 1949.)

Statistics and Probability

259*t*. B. O. Koopman: *The law of small numbers in Markoff processes.*

The author considers the probability of just s successes in a sequence of n trials forming a simple Markoff chain. The chain may be stationary (constant probabilities and correlations), semi-stationary (varying probabilities, constant correlations), or non-stationary (varying probabilities and correlations). In the limiting case of increasing n, probabilities of order $1/n$, and fixed correlations, a generating function is obtained which is of exponential form and reduces to the Poisson distribution generating function when the correlations are all zero. With its aid, the probabilities of s successes are given as simple expressions in terms of Laguerre polynomials, both in the stationary and semi-stationary cases. In the non-stationary case, the generating function is a simple exponential expression involving the generating function of the auto correlation. Various extension are considered to trials having many different outcomes, and to many-step processes. (Received December 13, 1948.)

260t. R. H. Fox: *On the asphericity of regions in a 3-sphere.*

If $X_1, \cdots, X_n$ are the 1-cells and $U_1, \cdots, U_m$ the 2-cells of a cell complex K normalized to contain only one vertex, then the fundamental group G of K is generated by $x_1, \cdots, x_n$ with defining relations $u_1=1, \cdots, u_m=1$, where x_j and u_i are carried by X_j and U_i respectively. In $\tilde{K}$, the covering space of K determined by a normal subgroup G_1 of G, the boundary of a 2-cell rU_i, $r\in R(G/G_1)$, the integral ring of G/G_1, is $r\cdot\sum_{j=1}^{n}(\partial u/\partial x_j)^{\psi\phi}X_j$ (cf. Bull. Amer. Math. Soc. Abstract 55-3-115), where ϕ is the natural homomorphism of the ring of the free group $[x_1, \cdots, x_n]$ onto the ring $R(G)$ of G and ψ the natural homomorphism of $R(G)$ onto $R(G/G_1)$. Using this, Whitehead's condition (Fund. Math. vol. 32 (1939), pp. 149–166) for the asphericity of the complement of a knot may be replaced by the condition: $\sum_i\xi_i(\partial u_i/\partial x_j)^{\phi}=0$, $\xi_i\in R(G)$ implies $\xi_i=0$ in the ring of the knot group in one of its common presentations. (Received January 13, 1949.)

Topology

261t. G. W. Whitehead: *On the characteristic cohomology class of a fibre bundle.*

Let X be a fibre bundle over a topological space B, with π: $X\rightarrow B$ as projection, and suppose that the fibre F is arcwise connected and that $\pi_i(F)=0$ for $i<n$. For each $b\in B$, let $F_b=\pi^{-1}(b)$. The characteristic cohomology class c^{n+1} of the bundle may be regarded as an element of the $(n+1)$st cohomology group of B modulo a fixed point b_0 with local coefficients in the system of local groups $\{\pi_n(F_b)\}$. Let d^n be the obstacle to contracting F_{b_0} over itself to a point; d^n belongs to the nth cohomology group of F_b with coefficients in $\pi_n(F_{b_0})$. The cohomology classes c^{n+1} and d^n are connected by the formula $\pi^*c^{n+1}=\delta d^n$, where π^* is the homomorphism induced by the mapping π: $(X, F_{b_0})\rightarrow(B, b_0)$, and δ is the coboundary operator. (Received December 7, 1948.)

262t. Leo Zippin: *Two-ended topological groups.*

Freudenthal has proved (see his *Neuaufbau der Endentheorie*, Ann. of Math. vol. 43 (1942) pp. 261–279) that a locally compact, connected, topological group G has at most two "ends"; these ends are ideal points which serve to make compact the group manifold, and groups with no ends are compact. In this note it is shown that the two-ended group manifolds are the topological product of a real axis by a compact connected set. The compact set is homeomorphic to the coset-space of G by a one-parameter subgroup. (Received February 26, 1949.)

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