BOOK REVIEWS

Regular polytopes. By H. S. M. Coxeter. London, Methuen, 1948; New York, Pitman 1949. 20+321 pp. $10.00.

The study of polytopes (that is, polygons and polyhedra of three or higher dimensions) appears to interest more different kinds of people than any other branch of mathematics with the possible exception of number theory. Its beauties inspired the rug merchant, P. S. Donchian, patiently to construct a remarkable set of models representing all varieties of polytopes. It enabled the housewife, Alicia Boole Stott, a daughter of George Boole, to capitalize on her unusual powers of geometrical visualization in spite of her meager mathematical education. It provided the struggling young lawyer, Thorold Gosset, with an amusing and constructive pastime during his long waits between clients. And equally well it has attracted the attention of many famous mathematicians such as Klein, Poincaré, Poinsot, Schläfli, Cayley, Euler, and Goursat, to mention only a few. Nor is it solely a "pure" discipline devoted to beauty but not utility, for it has been cultivated by a number of crystallographers such as Fedorov.

Coxeter has spent a major portion of his mathematical career digging out the obscure references in early works, in making personal contact with contemporary gifted amateurs, and in developing his own outstanding contributions to the field. In this book he has poured forth all his devotion and scholarship and has produced a work which will be the standard treatise in this field for many years. It is beautifully illustrated with photographs of Donchian's models and with numerous drawings. Its value as a reference book is greatly enhanced by historical material at the end of each chapter, by tables giving the essential combinatorial and metric properties of polytopes of many varieties, by an exhaustive bibliography, and by a carefully constructed index. It is a particular pleasure to record this last feature; for its omission in so many mathematical books published in England greatly detracts from their value.

In his preface Coxeter follows the lead of Birkhoff-MacLane and says: "Anyone familiar with elementary algebra, geometry, and trigonometry will be able to appreciate this book." In a literal sense this is true, but let no one be deceived—this is a serious book, full of advanced ideas, and worthy of careful study by professional mathematicians. In the elementary section Coxeter gives a brief résumé of the Platonic solids, and then discusses other solids related to these which, though not regular, have many regular features. Examples are the cuboctahedron, the rhombic dodecahedron, and the zonohedra.

The serious mathematics begins with the third chapter in which Coxeter introduces the symmetry groups of the Platonic solids. After a full discussion of this important topic, he turns to degenerate polyhedra such as tessellations and honeycombs and their groups. These lead to results of crystallographic importance. Under the heading "The Kaleidoscope" he then describes the discrete groups generated by reflections. The exposition is greatly illuminated by his own "graphical notation" which makes complicated relations self-evident. The treatment of three-dimensional solids closes with a chapter on regular solids which are not quite polyhedra in the strict sense. These are obtained from the Platonic solids by "stellating" (adding pointed solid pieces) or "faceting" (removing solid pieces). This process raises the number of regular polyhedra from five to nine.

The remaining two-thirds of the book is devoted to polytopes of higher dimensions. The general program is similar to that carried out for ordinary polyhedra. It is shown that in four-space there are six regular polytopes and that in n-space ($n>4$) there are only three regular polytopes. Explicit constructions are given for these and metrical properties are derived. There are even photographs of models of three-dimensional projections of some of these hypersolids. The methods used are increasingly analytic, but the underlying geometry is never lost among the formulae.

I have only one regret about this theory, and Coxeter should not be blamed for this. I refer to the formidable terminology, doubtless invented by mathematicians with a far better education in classical languages than myself. Dry-sounding words like "enantiomorphous" and "great stellated triacontahedron" tend to obscure the geometrical treasures of the subject. This is a place where a judicious use of American slang would greatly improve the situation.

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Mathematical theory of human relations. An approach to a mathematical biology of social phenomena. By N. Rashevsky. Bloomington, Principia, 1947. 14+202 pp. $4.00.

The name Rashevsky is virtually synonymous with the term "mathematical biophysics," a term which he coined to designate a field created and developed by himself and under his own direction. In this field he is the author of two books, one recently reissued in revised and greatly expanded form, and the editor of a quarterly journal, the Bulletin of Mathematical Biophysics.

Professor Rashevsky entered this field by way of a theoretical study of the behavior of liquid drops within which constituents drawn from