\( \mathfrak{h}_i^1 = \sum_{i=1}^{n} \mathfrak{h}_i^1 + (p - n - m + 1)\mathfrak{g} \).

**Proof.** In constructing 1-simplexes \((\mathfrak{f}''_0'), \cdots, (\mathfrak{f}''(m_0)')\) (not belonging to \(\mathfrak{M}^2\)), we get \(\mathfrak{M}_i^{\mathfrak{g}}, \mathfrak{M}_i^{\mathfrak{g}^2}, \cdots\) as in the lemma. By (7) and (8), we have

\[ \mathfrak{h}_i^* = \mathfrak{h}_i^1 + (p_i - 1)\mathfrak{g}, \]

where \(\mathfrak{h}_i^*\) is the homology group of \(\mathfrak{M}_i^{\mathfrak{g}^2}\). The newly constructed simplexes form a connected 1-complex whose 1-dimensional homology group contains the identity only. Therefore, from a famous theorem (cf. Seifert-Threfall, p. 179), by (5) we get

\[ \mathfrak{h}_i^* = \sum_{i=1}^{n} \mathfrak{h}_i^1 + (p - n)\mathfrak{g}, \]

where \(\mathfrak{h}_i^*\) is the homology group of \(\mathfrak{M}_i^{\mathfrak{g}^2}\). Therefore (9) is finally established in virtue of (11) and (8)'.

Theorem (3.5) may be extended analogously.

**A Note on Equicontinuity**

**M. K. Fort, Jr.**

During a recent seminar discussion of his paper *Transitivity and equicontinuity* [1], W. H. Gottschalk proposed the following question:

"Is the center of every algebraically transitive group of homeomorphisms on a compact metric space equicontinuous?"

An affirmative answer to the above question is given in this note.

**1. Definitions.** We let \(X\) and \(Y\) be compact metric spaces and let \(d\) be the metric for \(Y\).

A set \(F\) of functions on \(X\) into \(X\) is *algebraically transitive* if corresponding to each pair \(p\) and \(q\) of points in \(X\) there exists \(f \in F\) such that \(f(p) = q\).

A sequence \([g_n]\) of functions on \(X\) into \(Y\) converges to a function...
If \( \epsilon > 0 \) implies that there exists \( N > 0 \) and a neighborhood \( V \) of \( p \) such that \( d(g_n(x), g(x)) < \epsilon \) whenever \( x \in V \) and \( n > N \).

We shall need to make use of the fact that if \( [g_n] \) is a sequence of continuous functions on \( X \) into \( Y \) which converges pointwise to a function \( g \) on \( X \), then the sequence converges uniformly at each point of a set residual in \( X \). This fact has been proved by Kuratowski in [2]. Although the notation implies that Kuratowski is restricting himself to more special spaces than those with which we are dealing, the proof given in [2] is actually valid for any compact metric spaces \( X \) and \( Y \).

2. A more general theorem. We shall now prove a theorem which yields as a corollary the answer to Gottschalk's question.

**Theorem.** Let \( F \) be a set of continuous functions on \( X \) into \( X \) and \( G \) a set of continuous functions on \( X \) into \( F \), such that to each \( f \in F \) there corresponds a continuous function \( f^* \) on \( Y \) into \( Y \) such that \( g = f^*g \) for all \( g \in G \). If \( F \) is algebraically transitive then \( G \) is equicontinuous.

**Proof.** It is well known that in order to prove \( G \) equicontinuous it is sufficient to prove that every sequence in \( G \) has a uniformly converging subsequence. This is the converse of Ascoli's theorem.

Let \( S \) be any sequence in \( G \). Choose a subsequence \( [g_n] \) of \( S \) which converges at some point \( p \in X \). This is possible since \( Y \) is compact. We shall prove that \( [g_n] \) converges uniformly on \( X \).

We first prove that \( [g_n] \) converges pointwise on \( X \). Let \( x \) be any point of \( X \). Since we are assuming that \( F \) is algebraically transitive, we may choose \( f \in F \) such that \( f(x) = p \). There exists, by hypothesis, a continuous function \( f^* \) on \( Y \) into \( Y \) such that \( g = f^*g \) for all \( g \in G \). Since \( f(x) = p \) and \( [g_n] \) converges at \( p \), we see that \( [g_nf(x)] \) is a converging sequence in \( Y \). Since \( f^* \) is continuous on \( Y \), it follows that the sequence \( [f^*g_nf(x)] \) converges. This sequence is the same, however, as \( [g_n(x)] \).

Since we now know that \( [g_n] \) converges pointwise on \( X \), we may let \( g_0 \) be the limit of the sequence of functions. The sequence \( [g_n] \) converges to \( g_0 \) uniformly at each point of a set residual in \( X \), and since \( X \) is a compact metric space this residual set is non-empty. Let \( q \) be a point at which \( [g_n] \) converges uniformly to \( g_0 \).

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2 The theorem is true for functions on any topological space \( X \) into a separable metric space \( Y \). The author has a proof of this fact which will be included in a later paper on applications of semi-continuous set-valued functions.
We now prove that \([g_n]\) converges uniformly at each point of \(X\). Let \(x\) be a point of \(X\) and choose \(f \in F\) such that \(f(x) = q\). There exists \(f^*\), continuous on \(Y\) into \(Y\), such that \(g = f^*gf\) for all \(g \in G\). The function \(g_0\) may not belong to \(G\), but since \(g_0\) is the pointwise limit of a sequence of elements of \(G\) it is easy to verify that \(g_0 = f^*g_0f\). Suppose \(\epsilon > 0\). There exists \(\delta > 0\) such that if \(u\) and \(v\) belong to \(Y\) and \(d(u, v) < \delta\) then \(d(f^*(u), f^*(v)) < \epsilon\). There exists \(N > 0\) and a neighborhood \(U\) of \(q\) such that \(d(g_n(y), g_0(y)) < \delta\) whenever \(n > N\) and \(y \in U\). There exists a neighborhood \(V\) of \(x\) such that \(f(V) \subseteq U\). It is now easy to see that if \(z \in V\) and \(n > N\) then \(d(f^*g_nf(z), f^*g_0f(z)) < \epsilon\). We thus obtain \(d(g_n(z), g_0(z)) < \epsilon\) whenever \(z \in V\) and \(n > N\). This proves that the convergence is uniform at \(x\).

If a sequence of functions converges uniformly at each point of a compact space, then the sequence converges uniformly on the entire space. Therefore \([g_n]\) converges uniformly to \(g_0\) on \(X\).

**Corollary 1.** If \(F\) is an algebraically transitive group of homeomorphisms of \(X\) onto \(X\) and \(G\) is a group of homeomorphisms of \(X\) onto \(X\) such that \(gf = fg\) whenever \(f \in F\) and \(g \in G\), then \(G\) is equicontinuous.

**Corollary 2.** If \(F\) is an algebraically transitive group of homeomorphisms of \(X\) onto \(X\), then the center of \(F\) is equicontinuous.

**Bibliography**


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