$$\cdot\mathfrak{h}^1 = \sum_{i=1}^{n} \mathfrak{h}_i^1 + (p - n - m + 1)\mathfrak{g}. \tag{9}$$

Proof. In constructing 1-simplexes $('0''0), \cdots, ('0^{(m)}0)$ (not belonging to $\cdot\mathfrak{M}^2$), we get $\cdot\mathfrak{M}^{*2}$, $\cdot\mathfrak{M}_1^{*2}$, $\cdots$ as in the lemma. By (7) and (8), we have

$$\cdot\mathfrak{h}_i^{*1} = \mathfrak{h}_i^1 + (p_i - 1)\mathfrak{g}, \tag{10}$$

where $\cdot\mathfrak{h}_i^{*1}$ is the homology group of $\cdot\mathfrak{M}_i^{*2}$. The newly constructed simplexes form a connected 1-complex whose 1-dimensional homology group contains the identity only. Therefore from a famous theorem (cf. Seifert-Threfall, p. 179), by (5) we get

$$\cdot\mathfrak{h}^{*1} = \textstyle\sum_{i=1}^{n} \mathfrak{h}_i^1 + (p - n)\mathfrak{g}, \tag{11}$$

where $\cdot\mathfrak{h}^{*1}$ is the homology group of $\cdot\mathfrak{M}^{*2}$. Therefore (9) is finally established in virtue of (11) and (8)′.

Theorem (3.5) may be extended analogously.

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A NOTE ON EQUICONTINUITY

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During a recent seminar discussion of his paper *Transitivity and equicontinuity* [1],[1] W. H. Gottschalk proposed the following question:

"Is the center of every algebraically transitive group of homeomorphisms on a compact metric space equicontinuous?"

An affirmative answer to the above question is given in this note.

1. **Definitions.** We let X and Y be compact metric spaces and let d be the metric for Y.

A set F of functions on X into X is *algebraically transitive* if corresponding to each pair p and q of points in X there exists $f \in F$ such that $f(p) = q$.

A sequence $[g_n]$ of functions on X into Y converges to a function

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[1] Numbers in brackets refer to the bibliography at the end of the paper.

g uniformly at a point $p \in X$ if $\epsilon > 0$ implies that there exists $N > 0$ and a neighborhood V of p such that $d(g_n(x), g(x)) < \epsilon$ whenever $x \in V$ and $n > N$.

We shall need to make use of the fact that if $[g_n]$ is a sequence of continuous functions on X into Y which converges pointwise to a function g on X, then the sequence converges uniformly at each point of a set residual in X. This fact has been proved by Kuratowski in [**2**]. Although the notation implies that Kuratowski is restricting himself to more special spaces than those with which we are dealing, the proof given in [**2**] is actually valid for any compact metric spaces X and Y.[2]

2. **A more general theorem.** We shall now prove a theorem which yields as a corollary the answer to Gottschalk's question.

THEOREM. *Let F be a set of continuous functions on X into X and G a set of continuous functions on X into Y, such that to each $f \in F$ there corresponds a continuous function f^* on Y into Y such that $g = f^* g f$ for all $g \in G$. If F is algebraically transitive then G is equicontinuous.*

PROOF. It is well known that in order to prove G equicontinuous it is sufficient to prove that every sequence in G has a uniformly converging subsequence. This is the converse of Ascoli's theorem.

Let S be any sequence in G. Choose a subsequence $[g_n]$ of S which converges at some point $p \in X$. This is possible since Y is compact. We shall prove that $[g_n]$ converges uniformly on X.

We first prove that $[g_n]$ converges pointwise on X. Let x be any point of X. Since we are assuming that F is algebraically transitive, we may choose $f \in F$ such that $f(x) = p$. There exists, by hypothesis, a continuous function f^* on Y into Y such that $g = f^* g f$ for all $g \in G$. Since $f(x) = p$ and $[g_n]$ converges at p, we see that $[g_n f(x)]$ is a converging sequence in Y. Since f^* is continuous on Y, it follows that the sequence $[f^* g_n f(x)]$ converges. This sequence is the same, however, as $[g_n(x)]$.

Since we now know that $[g_n]$ converges pointwise on X, we may let g_0 be the limit of the sequence of functions. The sequence $[g_n]$ converges to g_0 uniformly at each point of a set residual in X, and since X is a compact metric space this residual set is non-empty. Let q be a point at which $[g_n]$ converges uniformly to g_0.

[2] The theorem is true for functions on any topological space X into a separable metric space Y. The author has a proof of this fact which will be included in a later paper on applications of semi-continuous set-valued functions.

We now prove that $[g_n]$ converges uniformly at each point of X. Let x be a point of X and choose $f \in F$ such that $f(x) = q$. There exists f^*, continuous on Y into Y, such that $g = f^*gf$ for all $g \in G$. The function g_0 may not belong to G, but since g_0 is the pointwise limit of a sequence of elements of G it is easy to verify that $g_0 = f^*g_0f$. Suppose $\epsilon > 0$. There exists $\delta > 0$ such that if u and v belong to Y and $d(u, v) < \delta$ then $d(f^*(u), f^*(v)) < \epsilon$. There exists $N > 0$ and a neighborhood U of q such that $d(g_n(y), g_0(y)) < \delta$ whenever $n > N$ and $y \in U$. There exists a neighborhood V of x such that $f(V) \subset U$. It is now easy to see that if $z \in V$ and $n > N$ then $d(f^*g_nf(z), f^*g_0f(z)) < \epsilon$. We thus obtain $d(g_n(z), g_0(z)) < \epsilon$ whenever $z \in V$ and $n > N$. This proves that the convergence is uniform at x.

If a sequence of functions converges uniformly at each point of a compact space, then the sequence converges uniformly on the entire space. Therefore $[g_n]$ converges uniformly to g_0 on X.

COROLLARY 1. *If F is an algebraically transitive group of homeomorphisms of X onto X and G is a group of homeomorphisms of X onto X such that $gf = fg$ whenever $f \in F$ and $g \in G$, then G is equicontinuous.*

COROLLARY 2. *If F is an algebraically transitive group of homeomorphisms of X onto X, then the center of F is equicontinuous.*

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