THE OCTOBER MEETING IN NEW YORK

The four hundred fiftieth meeting of the American Mathematical Society was held at Columbia University on Saturday, October 29, 1949. The attendance was about 270, including the following 230 members of the Society.

Milton Abramowitz, M. I. Aissen, E. J. Akutowicz, C. B. Allendoerfer, R. D. Anderson, T. W. Anderson, R. G. Archibald, L. A. Aroian, Frederick Bagemihl, Joshua Barlaz, P. T. Bateman, F. P. Beer, E. G. Begle, Stefan Bergman, Lipman Bers, Garrett Birkhoff, D. W. Blackett, A. L. Blakers, A. A. Blank, S. G. Bourne, C. B. Boyer, Paul Brock, A. B. Brown, F. H. Brownell, L. J. Burton, Jewell H. Bushey, J. H. Bushey, W. R. Callahan, H. E. Campbell, P. G. Carlson, Jeremiah Certaine, K. T. Chen, Y. W. Chen, W. L. Chow, L. W. Cohen, A. J. Coleman, T. F. Cope, Natalie Coplan, L. M. Court, M. D. Darkow, Martin Davis, B. V. Dean, H. F. DeBaggis, J. L. Doob, Arnold Dresden, Aryeh Dvoretzky, Samuel Eilenberg, M. P. Epstein, R. M. Exner, J. M. Feld, F. A. Ficken, N. J. Fine, R. S. Finn, A. D. Fleshler, R. M. Foster, R. H. Fox, Gerald Freilich, Orrin Frink, G. N. Garrison, Hilda Geiringer, H. A. Giddings, J. B. Giever, B. P. Gill, G. H. Gleissner, H. E. Goheen, W. H. Gottschalk, Bernard Greenspan, Harriet Griffin, Emil Grosswald, E. J. Gumbel, V. H. Haefeli, F. C. Hall, F. S. Hawthorne, G. A. Hedlund, M. H. Heins, Alex Heller, Erik Hemmingsen, A. H. Henry, L. H. Herbach, A. A. Herschfeld, Einar Hille, Abraham Hillman, Joseph Hilsenrath, Banesh Hoffmann, T. R. Hollcroft, L. A. Hostinsky, E. M. Hull, Witold Hurewicz, L. C. Hutchinson, M. A. Hyman, Eugene Isaacson, Nathan Jacobson, Wenceslas Jardetzky, Børge Jessen, S. A. Joffe, Fritz John, R. A. Johnson, F. E. Johnston, Shizuo Kakutani, Edward Kasner, W. H. Keen, M. E. Kellar, L. S. Kennison, J. F. Kiefer, H. S. Kieval, S. C. Kleene, J. R. Kline, E. G. Kogbetliantz, E. R. Kolchin, Horace Komm, B. O. Koopman, Jack Laderman, A. W. Landers, J. P. LaSalle, V. V. Latshaw, Solomon Lefschetz, Marguerite Lehr, Benjamin Lepson, Howard Levi, D. C. Lewis, M. A. Lipschutz, S. R. Lipsey, L. H. Loomis, E. R. Lorch, Lee Lorch, Janet McDonald, Brockway McMillan, L. A. MacColl, H. M. MacNeille, Irwin Mann, A. J. Maria, M. H. Maria, W. T. Martin, W. S. Massey, A. E. Meder, Paul Meier, A. N. Milgram, K. S. Miller, W. H. Mills, Don Mittleman, E. E. Moise, Deane Montgomery, Marston Morse, H. H. Mostafa, G. D. Mostow, T. S. Motzkin, F. J. Murray, David Nelson, C. V. Newsom, A. V. Newton, P. B. Norman, I. L. Novak, C. O. Oakley, J. C. Oxtoby, T. E. Peacock, A. J. Penico, I. D. Peters, B. J. Pettis, Everett Pitcher, E. L. Post, Walter Prenowitz, M. H. Protter, Hans Rademacher, Hans Rådström, G. N. Raney, H. E. Rauch, G. E. Raynor, M. S. Rees, Helene Reschovsky, Moses Richardson, J. F. Ritt, I. F. Ritter, J. E. Robinson, P. C. Rosenbloom, H. D. Ruderman, Raphael Salem, H. E. Salzer, Arthur Sard, Samuel Schecter, Henry Scheffé, Eugene Schenkman, M. M. Schiffer, Pincus Schub, Abraham Schwartz, G. E. Schweigert, Wladimir Seidel, Atle Selberg, D. B. Shaffer, Daniel Shanks, C. E. Shannon, H. N. Shapiro, I. M. Sheffer, Seymour Sherman, James Singer, M. H. Slud, L. L. Smail, Ernst Snapper, Andrew Sobczyk, D. C. Spencer, George Springer, Fritz Steinhardt, Wolfgang Sternberg, J. J. Stoker, R. R. Stoll, M. M. Sullivan, Olga Taussky, D. L. Thomsen, John Todd, P. M. Treuenfels, H. C. Tsang, Oswald Veblen, S. I. Vrooman, H. V. Waldinger, J. L. Walsh, Alan Wayne, Alexander Weinstein, Louis Weisner, M. E. White, P. M. Whitman, D. V. Widder, Albert Wilansky, W. G. Wolfgang, Jacob Wolfowitz, Y. K. Wong,

M. A. Woodbury, D. M. Young, J. W. Young, J. A. Zilber, H. J. Zimmerberg, Leo Zippin.

At 1:30 p.m. there was a business meeting of the Society. At this time the Secretary presented certain changes in the By-Laws which had been recommended by the Council at the Summer Meeting. These changes were concerned with the time of the annual meeting of the Board of Trustees, the method of arranging for temporary replacements on editorial committees, the requirements for implementation of the publication of the Proceedings of the Society and the Memoirs of the Society. The changes were unanimously approved. President J. L. Walsh presided at the business meeting and the address which followed.

At 1:45 p.m. Professor R. H. Fox of Princeton University gave an address on *Covering spaces*.

There were two sessions for contributed papers at 3:00 p.m., one for papers in analysis, applied mathematics, and logic in which Dr. Mina S. Rees presided and one for papers in algebra, geometry, and topology in which Professor G. E. Schweigert presided.

Abstracts whose numbers are followed by the letter "*t*" were presented by title. Paper number 15 was read by Professor Seidel, and paper number 36 was read by Professor Cohen. Professor Milkman was introduced by Professor L. T. Wilson.

Algebra and Theory of Numbers

1. H. E. Campbell: *An extension of the "principal theorem" of Wedderburn.*

An ideal I of a linear associative algebra A over any field is called a ***first zero-trace ideal*** if the first trace of every element of I is zero. Every such I is contained in a unique maximal first zero-trace ideal T_1 called the ***first liberal.*** The difference algebra A/T_1 is separable and A is separable if and only if $T_1=0$. If $A\neq T_1$ there exists $S_1\cong A/T_1$ such that $A=S_1+T_1$. Similar results follow for second traces. Let N be the radical and T' the first (and second) liberal of A/N. Then the unique ideal T of A such that $T/N=T'$ is the *liberal.* Both traces of every element of T are zero and A/T is separable. If $A\neq T$ there exists $S\cong A/T$ such that $A=S+T$. The decomposition based on T_1 is the same as the well known theory based on N for fields of characteristic $p=0$ or $p>n$ (n order of A). If $p\leqq n$ there may be decomposition on T_1 but none on N or vice versa. Since $T=N$ when A/N is separable, the decompositions on T and N are the same when there is one on N. Also there may be a decomposition on T when there is none on N. Moreover, if $\overline{T}$ is an ideal of smallest order such that $A/\overline{T}$ is separable and $\overline{S}\cong A/\overline{T}$ such that $A=\overline{S}+\overline{T}$, then $\overline{T}$ is the liberal. (Received June 15, 1949.)

2*t*. A. H. Clifford: *Extensions of semigroups.*

If S is an ideal of a semigroup Σ, that is, $S\Sigma\subseteq S$ and $\Sigma S\subseteq S$, D. Rees (Proc. Cambridge Philos. Soc. vol. 36 (1940) pp. 387–400) defines the difference semigroup $\Sigma - S$ to be essentially that obtained by collapsing S into a single zero element, while the remaining elements of Σ retain their identity. Starting with given semigroups S and T, the latter having a zero element, the corresponding "extension problem" is to find every possible semigroup Σ containing S as an ideal such that $\Sigma - S \cong T$. A solution is given in terms of homomorphisms of T into the semigroups of left and right translations of S. (Received August 16, 1949.)

3. Benjamin Lepson: *Certain best possible results in the theory of Schnirelmann density.*

The $\alpha+\beta$ hypothesis, conjectured by Khintchine and proved by Mann, is discussed. This states that $\gamma \geqq \min(1, \alpha+\beta)$, where α, β, and γ are the densities of the sets A, B, and $A+B$ respectively, whenever both A and B contain zero. It is shown that, for any pair (α, β), there are sets A and B, each containing zero, for which the equality holds. If the above inequality holds for all sets A and B, where A and B contain fixed finite sets E and F respectively, then E and F both contain zero, in which case the same inequality is still best possible for every pair (α, β). (Received September 13, 1949.)

4*t*. F. I. Mautner: *Irreducible infinite-dimensional representations of certain groups.*

There are groups every one of whose elements has only a finite number of distinct conjugates, which have infinite-dimensional irreducible unitary representations. (Received July 12, 1949.)

5*t*. Eugene Schenkman: *A theory of subinvariant Lie algebras.* Preliminary report.

A theory of subinvariant Lie algebras over a field of characteristic 0 is developed analogous to Wielandt's theory (Math. Zeit. vol. 45 (1939) pp. 209–244) of subinvariant subgroups (that is, subgroups that occur in a composition series of a given group). The concept of subinvariance is used to prove the following main theorem: Let A be a Lie algebra over an arbitrary field with center zero; let $B = \bigcap_k A^k$ where $A^i = [A^{i-1}, A]$, and let d denote the dimension of the derivation algebra $D(B)$ of B, c the dimension of the center of B. Consider the chain of algebras $D_0 = A$, $D_1 = D(D_0), \cdots, D_n = D(D_{n-1}), \cdots$. Then for all n, dim $D_n \leqq d+c$. Since D_n has no center for all n, the dimension of D_n is nondecreasing as a function of n and it is immediate from the theorem that for some n the only derivations of D_n are inner. This latter result was announced by Chevalley for characteristic 0 (Proc. Nat. Acad. Sci. U.S.A. vol. 30 (1944) p. 274). It is also evident that the main theorem is applicable to the tower of automorphism groups of a Lie group. (Received September 12, 1949.)

6. Ernst Snapper: *Completely primary rings.* I and II.

Ring means commutative ring with unit element. A completely primary ring R is a ring whose radical $N(R)$ is a maximal ideal. Algebraic and trasncendental extensions $R \subset R'$ have been defined and investigated. The definitions are generalizations of notions studied by A. Fränkel and W. Krull. The investigation is based on the ideal theory of $R[x]$. First, all rings A with the property that $A/N(A)$ is a principal ideal ring are studied. ($R[x]$ has this property.) It is shown that the not nilpotent elements of A permit a unique factorization. Then, polynomial rings $B[x]$, where B is any ring, are investigated. To every nondivisor of zero f of $B[x]$ an integer $O(f)$, called the *order* of f, is associated. $O(f)$ is at least as important as the degree of f; a side result is that the radical and Jacobson radical of $B[x]$ always coincide. The theories of the rings A and $B[x]$ are then applied to $R[x]$. The ensuing theory of algebraic and transcendental extensions will be used in further papers to derive structure theorems for completely primary rings. (Received October 29, 1949.)

Analysis

7t. P. R. Garabedian (National Research Fellow): *A remark on the moduli of Riemann surfaces of genus 2.*

Let S be a closed Riemann surface of genus 2, and let $w_1(z)$ and $w_2(z)$ be a pair of normal integrals of the first kind on S. These integrals have about one set of canonical cuts on S the periods 0 or πi, and about the remaining canonical cuts they have the periods a_{11}, a_{12}, a_{21}, a_{22}, with $a_{12}=a_{21}$. It is shown that the three complex quantities a_{11}, a_{12}, a_{22} are a set of conformal moduli of the Riemann surface S. The proof is based on Riemann's relations $V_1=w_1(z)$, $V_2=w_2(z)$, $z\in S$, for the zeros V_1, V_2 of the theta-function $\theta(u_1, u_2)$ associated with S. Special symmetric cases are treated which are related to a paper of Ahlfors and Beurling (Comptes Rendus du Dixième Congrès des Mathématiciens Scandinaves, Copenhagen, 1947, pp. 341–351). (Received July 25, 1949.)

8t. E. R. Lorch: *The fundamental theorem for self-adjoint transformations.*

A brief proof is given of the fundamental theorem for self-adjoint transformations H (most general case) in Hilbert space $\mathfrak{H}$. The structure theorem is established in the following form: Let $\{\lambda_n\}$ be a monotone increasing set of real numbers, $n=0, \pm 1, \pm 2, \cdots$, without limit point; hence the set "covers" the real axis. Then there exist in $\mathfrak{H}$ closed linear manifolds $\mathfrak{M}_n$ which are orthogonal in pairs, which span $\mathfrak{H}$, and such that on $\mathfrak{M}_n$, H is bounded and satisfies $\lambda_n I\leqq H\leqq\lambda_{n+1}I$. The key to the proof is the improper Cauchy integral $K_{\lambda\mu}(m, n)=(2\pi i)^{-1}\int_C(\zeta-\lambda)^m(\mu-\zeta)^n(\zeta I-H)^{-1}d\zeta$. Here λ and μ are real numbers, $\lambda<\mu$; m and n are positive integers; and C is a simple closed contour containing λ and μ. The properties—principally of a multiplicative character—of the transformation $K_{\lambda\mu}(m, n)$ are derived with the help of the functional equation of the resolvent of H and the elements of the classic Cauchy theory. (Received July 26, 1949.)

9. Joseph Milkman: *Note on the functional equations*: $f(xy)=f(x)+f(y)$, $f(x^n)=nf(x)$.

Augustin-Louis Cauchy (*Cours d'analyse de l'école Royale Polytechnique*, part I, 1821, p. 109) gives a beautiful proof that the only continuous solution of the functional equation $f(x)+f(y)=f(xy)$, where $f(x)$ is defined for all real numbers x, is the function $f(x)=a\ln x$. Paul Erdös (Ann. of Math. (1946)) proved by analytic number theoretic methods that if $f(m)$ is additive and $f(m+1)\geqq f(m)$, $f(m)=c\ln m$. This implies the theorem, "If $f(m+1)>f(m)$ and $f(mn)=f(m)+f(n)$ hold for all positive integers m and n, then $f(m)=c\ln m$," which the author proves by showing that any solution $f(m)$ can be imbedded in a continuous solution $f(x)$, thus rendering the theorem a corollary of Cauchy's solution of $f(xy)=f(x)+f(y)$. The author also proves that if $f(r)$ is a monotonically increasing function on a set S which is everywhere dense among the positive real numbers and contains all positive integral powers of all members of the set S, for example, the rational numbers, and $f(r^p) = pf(r)$ for any prime p, then $f(r) = k \ln r$. (Received August 17, 1949.)

10. K. S. Miller: *On iterative methods in linear differential equations.*

The problem treated in this paper is that of estimating rates of convergence in solving homogeneous linear differential equations by an interative process. Let L be an ordinary linear differential operator of order n. It is shown that two operators M and N exist such that $L = M + N$ and (under certain restrictions on the boundary conditions) $T \equiv -MN^-$ is an integral operator with a symmetric kernel. [N^- is a "partial inverse" of N defined in terms of a Green's function.] In Theorem 2, it is proved that if $\epsilon_m(x) = Lu_m(x)$ where $u_m(x)$ is the mth iterant, then $\epsilon_m(x) = T\epsilon_{m-1}(x)$. Theorem 3 states: as m approaches infinity, $u_m(x)$ approaches a solution of the equation $Lu = 0$ satisfying the specified boundary conditions. Considered as a linear transformation in Hilbert space, T is self-adjoint with finite norm. Hence if $\{\psi_m(x)\}$ is a complete orthonormal set associated with T with characteristic values $\lambda_1, \lambda_2, \cdots$ (multiplicities included) and if $\epsilon_0(x) = \sum_{\alpha=1}^{\infty} b_\alpha \psi_\alpha(x)$ then $\|\epsilon_m(x)\|^2 = \sum_{\alpha=1}^{\infty} b_\alpha^2 \lambda_\alpha^{2m}$. Estimates of the rapidity of convergence can be drawn from the values of the λ_α. Applications to machine methods are discussed. (Received June 15, 1949.)

11. M. H. Protter. *On a boundary value problem for an equation of mixed type.* Preliminary report.

Consider the equation $K(y)u_{xx} - u_{yy} = 0$ (*) with $K(y)$ a nondecreasing function defined for $y \leqq 0$, continuous at $y = 0$ and with $K(0) = 0$. Let D be the domain bounded by the interval $[a, b]$ along the x-axis and the characteristic curves, $x = g_1(y)$, $x = g_2(y)$, through the end points of $[a, b]$. Suppose $F_0(x)$, $a \leqq x \leqq b$, and $G_0(y)$ are given sufficiently well-behaved functions. The existence and uniqueness of the solution $u(x, y)$ of (*) for (x, y) in D, satisfying the conditions $u(x, 0) = F_0(x)$, $u(g_1(y), y) = G_0(y)$, are shown. The method consists of approximating $K(y)$ by a step-function and then solving the corresponding purely hyperbolic equations. Bounds are obtained for the solution in terms of the given data; proceeding to the limit yields the result. (Received July 20, 1949.)

12t. Herman Rubin and M. H. Stone: *Postulates for generalizations of Hilbert space.*

The aim of this paper is to reduce the postulates of Jordan and von Neumann for Hilbert space. This is done by eliminating the triangle inequality from the assumed properties of the norm-function. (Received August 29, 1949.)

13t. I. E. Segal: *The class of functions which are absolutely convergent Fourier transforms.*

The set of functions on a locally compact abelian group G which are absolutely convergent Fourier transforms is dense (in the uniform topology) in the space of continuous functions on G which vanish at infinity (a function does this if the set where its absolute value exceeds any given positive number has compact closure), and is either all of that space, or of first category in it, according as G is finite or not. A corollary which is classical, but which is obtained here in a purely existential and noncomputational fashion, is that not every continuous function on the reals which tends to zero at infinity is an absolutely convergent Fourier transform, and similarly for Fourier series. (Received September 14, 1949.)

14t. I. E. Segal: *The two-sided regular representation of a unimodular locally compact group.*

If G is a unimodular locally compact group, then every bounded linear operator on the space H of functions on G square-integrable relative to Haar measure which commutes with all left translations $f(x) \to f(ax)$ $(a \in G; f \in \mathcal{H})$ on $\mathcal{H}$ is in the weak closure of the algebra generated by the right translations $f(x) \to f(xa)$ on $\mathcal{H}$. The special case of this theorem in which G is discrete was established by Murray and von Neumann, whose proof made essential use of discreteness. It follows that the lattice of all closed linear subspaces of $\mathcal{H}$ which are invariant under both left and right translations is a Boolean algebra. A second corollary is that if G is separable, then there exists an element of $\mathcal{H}$ whose (left and right) translations span $\mathcal{H}$. (Received September 14, 1949.)

15. Wladimir Seidel and Otto Szasz: *On positive harmonic functions and ultraspherical polynomials.*

In a forthcoming paper E. F. Beckenbach, W. Seidel, and Otto Szasz have studied recurrent determinants of ultraspherical polynomials and have shown, among other things, that the quadratic forms $\sum_{\mu,\nu=0}^{n} [P_{\mu+\nu}^{(\lambda)}(x)/P_{\mu+\nu}^{(\lambda)}(1)]u_\mu u_\nu$, where $P_n^{(\lambda)}(x)$ is the ultraspherical polynomial of degree n and order λ, are positive definite for $\lambda>0$, $n=0, 1, 2, \cdots$, and $x>1$. In the present paper it is shown that the hermitian forms $\sum_{\mu,\nu=0}^{n} [P_{|\mu-\nu|}^{(\lambda)}(x)/P_{|\mu-\nu|}^{(\lambda)}(1)]u_\mu \bar{u}_\nu$ are positive definite for $\lambda>0$, $n=0, 1, 2, \cdots$, and $-1<x<1$, a result conjectured by G. Szegö. This result follows from the fact that the harmonic function, $1/2+\Re\{\sum_{n=1}^{\infty} [P_n^{(\lambda)}(x)/P_n^{(\lambda)}(1)]z^n\}$ is positive in the unit circle $|z|<1$ for $-1\leqq x\leqq 1$ and $\lambda>0$. For the special case $\lambda=1$ this result was proved earlier by G. Szegö. (Received September 16, 1949.)

16. I. M. Sheffer: *On the solution of sum-equations.*

Consider the system of sum-equations (1) $\sum_{j=0}^{\infty} a_{nj}x_{n+j}=c_n$ $(n=0, 1, \cdots)$. Let (2) $A_n(t)=\sum_0^{\infty} a_{nj}t^j$ $(n=0, 1, \cdots)$ be analytic in $|t|<R$. There is a sequence of polynomials $\{H_n(1/t)\}$ in $1/t$, biorthogonal to the set $\{t^nA_n(t)\}$. Let (3) $\Phi(u, 1/t) = \sum_0^{\infty} H_n(1/t)u^n$, (4) $\Psi(1/u, t)=\sum_0^{\infty} (t/u)^n A_n(t)$, (5) $\Lambda(u, 1/t)=\sum_0^{\infty} H_n(1/t)u^nA_n(u)$. It is shown how convergence properties of series (3), (4), (5) and of series $\sum_0^{\infty} c_nH_n(1/t)$ determine solutions of system (1). In particular, if the analytic continuation of (3) (in the variable u) assumes the form (6) $\Phi(u, 1/t)=\sum_{j=1}^{k} R^j(1/t)\Theta_j(u)+\sum_0^{\infty} H_n^*(1/t)u^n$, with appropriate conditions (too lengthy to cite here) on the functions that appear on the right-hand side, then the homogeneous system (1) (that is, with $c_n\equiv 0$) has solutions determined by the functions $R_j(1/t)$. Examples are given in the way of illustration. (Received September 9, 1949.)

17t. G. M. Wing: *The mean convergence of orthogonal series.*

The mean convergence of Fourier-Bessel series is studied. The functions $\phi_n(x) = 2^{1/2}J_\nu(\mu_n x)/J_{\nu+1}(\mu_n)$, where $\nu\geqq 1/2$ and $\{\mu_n\}$ is the sequence of successive positive roots of $J_\nu(x)$, form an orthonormal set on (0, 1) with weight function x. Let $\int_0^1|f(x)|^p x dx<\infty$ for some p, $4/3<p<4$, and define $a_n=\int_0^1\phi_n(x)f(x)xdx$. It is proved that $\lim_{N\to\infty} \int_0^1|f(x)-\sum_{n=1}^{N} a_n\phi_n(x)|^p xdx=0$. The methods used are similar to those of M. Riesz (*Sur les fonctions conjugées*, Math. Zeit. vol. 27 (1927) pp. 218–244) and H. Pollard (*The mean convergence of orthogonal series*, Trans. Amer. Math. Soc. vol. 62 (1947) pp. 387–403; vol. 63 (1948) pp. 355–367). An example is given to show that the theorem fails for $1 \leq p < 4/3$ (or $p > 4$). It is also proved that if $\int_0^1 |f(x)|^p dx < \infty$ for some $p > 1$ and $b_n = \int_0^1 f(x)\phi_n(x)x^{1/2}dx$ then $\lim_{N\to\infty} \int_0^1 |f(x) - \sum_{n=1}^N b_n\phi_n(x)x^{1/2}|^p dx = 0$. These results are used to study the more general problem in which the μ_n are the zeros of $xJ_\nu'(x) + HJ_\nu(x) = 0$ (H a real number). Similar theorems are established in this case. Polynomials orthogonal on the interval $(-1, 1)$ are also investigated and some results of Pollard on mean convergence of Jacobi series are generalized. (Received August 17, 1949.)

Applied Mathematics

18t. Hilda Geiringer: *Linear differential equations of the plane stress problem for a perfect plastic body.*

For the perfect plastic body the state of stress is restricted to a manifold $F(\sigma_1, \sigma_2, \sigma_3) = 0$ (σ_i principal stresses). The nonlinear plane stress problem, $\sigma_3 = 0$, determined by two equilibrium conditions and $F(\sigma_1, \sigma_2, 0) \equiv F(\sigma_1, \sigma_2) = 0$ has been linearized by v. Mises. With ξ, η denoting principal stress directions he introduced $\theta = (x, \xi)$ and some parameter s for which $F(\sigma_1(s), \sigma_2(s)) \equiv 0$ as coordinates in a "stress graph" and derived (if $j = \partial(s, \theta)/\delta(\xi, \eta) \neq 0$) linear differential equations for x and y dependent on s and θ. Much simpler equations hold for $X = x \cos \theta + y \sin \theta$, $Y = y \cos \theta - x \sin \theta$. With $\sigma_i' = d\sigma_i/ds$, $f(s) = (\sigma_2 - \sigma_1)/\sigma_1'$, $g(s) = (\sigma_2 - \sigma_1)/\sigma_2'$, and subscripts denoting partial differentiation: $X_\theta - Y = gY_s$, $Y_\theta + X = fX_s$, $X_{\theta\theta} - fgX_{ss} = -X + X_s \cdot (f'g + f - g)$, and $Y_{\theta\theta} - fgY_{ss} = -Y + Y_s \cdot (g'f + f - g)$. These equations which are hyperbolic, parabolic, elliptic according as fg is greater than, equal to, or less than 0, form the basis of the linearized theory. After choice of a specific yield condition all methods for linear equations, for example, Bergman's operator method, or expansion in series of functions are available. For the slope of the characteristics in the "physical plane" (the images of the fixed characteristics in the stress graph for which $ds/d\theta = (fg)^{1/2}$) there holds $dy/dx = \tan(\theta \pm \phi)$ with $\tan^2 \phi = f/g$. By means of this mapping the initial value problem is solved approximately by a simple graphic procedure. (Received September 8, 1949.)

19t. Hilda Geiringer: *Parabola-yield-condition for the perfect plastic body.*

For the "parabola-limit" (v. Mises, 1949) $F(\sigma_1, \sigma_2) = (\sigma_1 + \sigma_2)^2 - 4aK(\sigma_1 - \sigma_2) - 4a^2K^2 = 0$, $a = 1 + 2^{1/2}$, the *plane stress* problem becomes hyperbolic throughout, exhibiting remarkable mathematical simplifications. With notations of the preceding abstract, and $g(s)h'(s) + h(s) = 0$, $X/gh = \psi_s$, $Y/h = \psi_\theta$, $j = \partial(s, \theta)/\partial(\xi, \eta) \neq 0$, a *linear* equation of the problem is $L = \psi_{ss} - fg\psi_{ss} - \psi_s \cdot (fg' - f - g) = 0$, satisfied, for the parabola-limit, by products of trigonometric and hypergeometric functions. The fixed characteristics in the "stress graph" are circles (congruent sin-lines) for s and θ polar (rectangular) coordinates; for the characteristics in the "physical plane" $dy/dx = \tan(\theta + \phi)$, where $\tan^2 \phi = (a - s)/(a + s)$. With $s = a \sin t$, L reduces to $\psi_{\theta\theta} - \psi_{tt} + \psi_t \cdot (\cos t)^{-1} = 0$, $|t| < \pi/2$, to which, for example, Bergman's operator method can be applied. With a view to more general use a space generalization is derived: With $J_1 = \sigma_1 + \sigma_2 + \sigma_3$, $J_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$, $F(\sigma_1, \sigma_2, \sigma_3) = (4a^2K^2 - J_1)^2 - 16a^2K^2(J_1^2 - 4J_2)$ for $|J_1| \leq 2aK$, while $F = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - 4a^2K^2$ for $|J_1| \geq 2aK$.

For $\sigma_3=0$ this reduces to $F(\sigma_1, \sigma_2)=0$, it intersects each "ray" in centric symmetric points and approximates well the v. Mises (1913) limit. (Received September 8, 1949.)

20t. Hilda Geiringer: *Simple wave solutions for the plane stress problem of the perfect plastic body.*

Solutions of the plane plasticity problem (two equilibrium conditions, one yield condition, $F(\sigma_1, \sigma_2)=0$) are studied such that each straight line of a continuous one-dimensional set in the x-y-plane carries a constant value of the stress tensor Σ. It follows that such solutions exist only in the hyperbolic domain, that these straight lines form a family of characteristics, $\{C_1\}$, that each C_1 is mapped onto a point (s, θ) of the "stress graph," all points lying on a characteristic curve Γ, and that all "cross characteristics," $\{C_2\}$—and consequently the entire "simple wave"—are likewise mapped onto Γ. It can be shown that Σ is the same function of the slope $m=\tan\delta$ for all sets of straight lines. For the "elliptic limit," $F\equiv\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2-4K^2$, the wave extends over 180°. The solution is (upper sign for σ_1, lower for σ_2): $\sigma_{1,2}=K(3m\mp(4+m^2)^{1/2})/(3+3m^2)^{1/2}$, $\theta=\pi/4+\arctan m-2^{-1}\arctan m/2$ $(-\pi/2\leqq\delta\leqq\pi/2)$. The cross characteristics, $\{C_2\}$, in the case of the "centered wave," have the polar equation $r^2\cos\delta=r_0^2$. For the "parabola-limit" $F\equiv(\sigma_1+\sigma_2)^2-4aK(\sigma_1-\sigma_2)-4a^2K^2$ the wave extends over 270°: $\sigma_{1,2}=2^{-1}Ka$ $(2\cos 2\delta/3\mp\sin^2 2\delta/3)$, $\theta=2\delta/3$ $(0\leqq\delta\leqq 3\pi/2)$. For the centered wave the $\{C_2\}$ have the equation $r=r_0$ $(\sin 2\delta/3)^{-3/2}$, while for the principal stress lines $r=r_0(\sin\delta/3)^{-3}$ and $r=r_0$ $(\cos\delta/3)^{-3}$ respectively. (Received September 8, 1949.)

21. Fritz John: *On the motion of floating bodies. II.*

Uniqueness and existence proofs are given for the simple harmonic motion of a heavy liquid of constant depth h in the presence of a partly immersed obstacle. Let R be the region in xyz-space bounded by the average free surface S_F, the average immersed obstacle surface S_I, and the bottom surface S_B. It is assumed that no point of S_F lies vertically above a point of S_I. The motion is described by the complex velocity potential W. W satisfies Laplace's equation in R, $W_n=0$ on S_B, $W_n=\lambda W$ on S_F, and $W=W'+W''$, where W' is the "primary" wave which is regular everywhere, and W'' satisfies Sommerfeld's radiation condition. It is proved that W is uniquely determined by W' and by the normal derivative W_n of W on S_I. Conversely, there exists a motion for given W_n on S_I and given W', if it is assumed that S_I is convex and that S_I and S_F intersect at right angles. (Received September 8, 1949.)

22t. R. R. Reynolds: A note on orthogonalization.

Let f and v be column matrices of functions f_m, which are complete and orthonormal with respect to a metric [ ] in a space S, and v_m, which are only complete in S. A triangular matrix Γ exists such that $f=\Gamma v$; from the orthonormality relation $[ff^*]=I$ (f^* is the transposed conjugate of f) it follows that $I=\Gamma[vv^*]\Gamma^*$, or (a) $[vv^*]=D=\Delta\Delta^*$, (b) $\Gamma=\Delta^{-1}$, (c) $D^{-1}=\Gamma^*\Gamma$. Thus the orthonormalization coefficients are determined from an auxiliary matrix Γ appearing in the so-called square root method of inverting D; this is accomplished in the steps (a), (b), (c), and was noted by Schur (J. Reine Angew. Math. (1917)) and applied by Banachiewicz (Bulletin International de l'Académie Polonaise des Sciences et des Lettres. Ser. A (1938)). (Received September 16, 1949.)

23. D. M. Young: *Iterative methods for solving the finite difference analogue of the Dirichlet problem.*

The finite difference analogue of the Dirichlet problem with N interior mesh points can be solved systematically either by: (i) replacing the trial value at each point by the average of the values at four adjacent points and iterating ($u^{i+1}(u_1, u_2, \cdots, u_N) = R(u^i)$); or (ii) traversing the points in a prescribed order σ and averaging as in (i) except that new values are used as they are obtained ($u^{i+1} = L_\sigma(u^i)$). It is proved for the symmetric transformation R that if $R(v) = \mu v$ there exists v^* such that $R(v^*) = (-\mu)v^*$. Although the matrix of the Liebmann operator L_σ is non-symmetric, under the usual choice of σ the N-dimensional function space S consists of: an invariant subspace S_0, associated with the eigenvalue 0, of dimension $N-s/2$ (where s is the number of nonzero eigenvalues of R); and a complementary subspace S_1 such that the normal form of the corresponding submatrix is diagonal. To each pair v, v^* corresponds $v_1 = \mu^{p+q}v = (-\mu)^{p+q}v^*$ such that $L_\sigma(v_1) = \mu^2 v_1$, where $x = ph, y = qh, h =$ mesh size, and the ordering σ is taken with increasing $(x+y)$. The rate of convergence of L_σ is twice that of R. Moreover the eigenvalues of L_σ are all real. A counterexample is given to show that there exists an ordering σ for which the results are not valid. (Received September 14, 1949.)

GEOMETRY

24t. M. C. Foster: *Transformations on rectilinear congruences.*

Each line of a congruence is associated with that point on the unit sphere at which the normal is parallel to the line. Accordingly, a congruence is defined by the coordinates (a, b) of the point in which an arbitrary line pierces the tangent plane at the associated point. A second congruence (A, B) is associated with the original by various transformations on (a, b), such as the affine transformation and inversion. The conditions are developed whereby a congruence of one type is transformed into another. (Received September 16, 1949.)

25. T. S. Motzkin: *The dual curve for $p \neq 0$.* Preliminary report.

While much of the classical theory of algebraic plane curves subsists for an algebraically closed coordinate field of nonzero characteristic, three main new features appear. 1. Duality is no longer a contact transformation. Every (irreducible) curve is the dual C' of infinitely many curves C. $C'' = C$ if (I): the duality $C \rightarrow C'$ is birational. (I) holds if and only if $p = 2$ and C has not an inflexion everywhere. 2. If (I) is not true then the order of contact of the tangent P' at a general point P is some $p^e - 1$, and P' may have more than one point of contact. Plücker's formulae change accordingly. 3. The tangent $P'_0 = \lim P'$ of a branch with origin P_0 may differ from its quasitangent $P_0^* = \lim P_0 P$, and P_0 from the quasiorigin $P_0''^* = \lim P_0' P'$. A branch has four independent characteristic numbers and, for curves fulfilling (I), together with the dual branch six or five (for $p = 0$ two, its cusp number s and the dual s'). For $p = 3$, $s \equiv s'$; the dual of a cusp is a cusp, of an inflexion a triple cusp. For $p = 2$, $s \equiv 0$; every cusp is to be counted twice. (Received September 6, 1949.)

26t. H. P. Mulholland: *On Geöcze's problem for nonparametric surfaces.*

While previous solutions of this problem, by A. Mambriani (Annali della Scuola

Normale Superiore di Pisa vol. 13 (1944) pp. 1–17) and by the author (Proc. London Math. Soc., forthcoming) are independent of earlier restricted solutions, in the present paper the author settles the problem comparatively shortly by following up an earlier approach due to T. Radó. Let $f(x, y)$ be everywhere continuous, R be an oriented rectangle, $A(R)$ the Lebesgue area and $A_Y(R)$ the W. H. Young area of the surface $S(R)$: $z=f(x, y)[(x, y)\in R]$, $A^*(R)$ the lower limit of the areas of polyhedra $z=p(x, y)[(x, y)\in R]$ *inscribed* to $S(R)$ as the maximum face-diameter tends to 0, $2^{-1}hk\Phi(x, y; h, k)$ the area of the triangle with vertices $(x, y, f(x, y))$, $(x+h, y, f(x+h, y))$, $(x, y+k, f(x, y+k))$, and $I^*(R, q)$ be $\limsup \iint_R \Phi(x, y; h, k)dxdy$ as $h, k \to 0$, $h/k=q$ (a constant). The desired proof that $A^*(Q)=A(Q)$, where Q is $[0, 1; 0, 1]$, rests on lemmas: (i) $A^*(R) \leqq I^*(R, q)$ (cf. T. Radó, *Length and area*, New York, 1948, V. 3.53, pp. 548–549); (ii) $I^*(R, q_R) - A(R) \leqq \{8A(R)[A(R)-Y(R)]\}^{1/2}$, where $Y(R)$ is the magnitude of the vector-area of the curve bounding $S(R)$ and q_R depends on its direction; (iii) $A^*(R)$ almost always increases by subdivision; (iv) $A(R)=A_Y(R)$ (a known result). (Received September 6, 1949.)

27. Walter Prenowitz: *Spherical geometries and multigroups.*

Spherical geometries are defined by postulates similar to those of J. R. Kline (Ann. of Math. (2) vol. 18 (1916)) in terms of *point* and *between*, but involving no continuity or dimensional restriction. Let an abelian multigroup be called *regular* if (1) it has an identity 0 satisfying $a+0=a$, (2) each element a has an inverse $-a$ satisfying $a+(-a)\supset 0$, (3) subtraction is related to addition in the usual way, $a-b=a+(-b)$. The theory of regular multigroups which bears very strong analogies to that of abelian groups is outlined. A spherical geometry G is converted into a regular multigroup M by adjoining an "ideal point" o and properly defining $+$, the join of two points. The geometry of G is reduced to the algebra of M, for example spherical subspaces correspond to submultigroups, halfspaces to cosets. Spherical geometries are characterized by the fact that their associated multigroups satisfy (4) the idempotent law $a+a=a$ and (5) each element other than 0 has order 3. (Received September 15, 1949.)

28. W. G. Wolfgang: *On the construction of the equation of the n-dimensional analogue of the conic sections.*

The author generalizes to n dimensions a method given by Brink in his book, *Essentials of analytic geometry*, of constructing the equation of the conic passing through five given points. Given $[n(n+3)]/2$ points in n-space such that no $n+1$ of them lie in an $(n-1)$-space, construct the equation of the n-dimensional analogue of the conic (the locus of an equation of the second degree in n variables) passing through them as follows. (i) Choose any $2n$ of the given points and group them in $n(n-1)+2$ groups of n points each, then write the equations, $\Pi_\nu=0$, of the n-dimensional linear forms passing through each of these groups. (ii) Pair these equations so that neither member of a pair intersects the other member in any of the given points. Relabel the equations so that these pairs are denoted by: $\Pi_{2\nu}\cdot\Pi_{2\nu+1}=0$. (iii) Form the equation: $\sum_{\nu=0}^{n(n-1)/2} \lambda_\nu(\Pi_{2\nu}\Pi_{2\nu+1})=0$, $\lambda_0=1$. (iv) Substitute the remaining $n(n-1)/2$ points in this equation and obtain $n(n-1)/2$ equations in $n(n-1)/2$ unknowns. (v) Solve these equations simultaneously and denote the values by α_ν. (vi) Then $\sum_{\nu=0}^{n(n-1)/2} \alpha_\nu(\Pi_{2\nu}\cdot\Pi_{2\nu+1})=0$, $\alpha_0=1$, is the equation of the nondegenerate analogue of the conic passing through the given points. (Received October 5, 1949.)

Logic and Foundations

29. S. C. Kleene: *A symmetric form of Gödel's theorem.*

It has been remarked that recursively enumerable sets behave surprisingly similarly to analytic sets and general recursive sets to Borel sets. There is a theorem which says that two disjoint analytic sets can always be separated by a Borel set. In this note, two disjoint recursively enumerable sets C_0 and C_1 are constructed which cannot be separated by a general recursive set. Hence there is no exact parallelism between the two theories. Given any two disjoint recursively enumerable sets D_0 and D_1 such that $C_0 \subset D_0$ and $C_1 \subset D_1$, a number f can be found such that $f \notin D_0 + D_1$. The sets C_0 and C_1 are obtained by rearranging Rosser's proof of Gödel's theorem, in which the existence of an undecidable proposition is inferred from simple consistency only instead of ω-consistency. The rearrangement makes the treatment of the proposition and of its negation symmetrical. (Received September 18, 1949.)

30*t*. A. R. Schweitzer. *An analysis of Sir James Jeans' Physics and philosophy.* I.

Jeans' treatise, *Physics and philosophy* (Cambridge and New York, 1943; reprinted, 1946) has to do with nature, events of nature and their pattern; pictorial representation and mathematical description of this pattern. Two such descriptions which, in Jeans' view, are believed to be "complete and perfect" are provided by Einstein's generalized theory of relativity and the new quantum theory due to Heiseinberg; de Broglie and Schrödinger, and Dirac (ibid. pp. 12, 68, 118; pp. 158, 174). Relativity is approached by discussing space, time and space-time historically and critically. As a basis for criticism Jeans introduces (ibid. pp. 42, 43) a doctrine of three worlds of modern science: the world of the electron or the minute-scaled world of atomic physics; the "man-sized" world; and the world of the nebulae or the vast-scaled world of astronomy. Jeans assumes (ibid. pp. 43, 70) that the same laws of nature prevail throughout the range of phenomena of the three worlds. (Received September 13, 1949.)

31*t*. A. R. Schweitzer: *An analysis of Sir James Jeans' Physics and philosophy.* II.

Jeans' view of space associated with the concepts "right" and "left" (ibid. p. 65) is discussed by the author in detail by referring to the author's foundations of geometry. Transition to philosophy is effected by Jeans by recognizing that in modern science certainty must be replaced by probability. Much use is made by Jeans of analogy in the course of his exposition and prominence is given to simplicity as a working hypothesis in the evolution of physics. Philosophically, Jeans is committed to positivism. The author refers to Sir A. S. Eddington's combination of a modification of relativity ("intermediate" relativity) and quantum theory: *Fundamental theory* (posthumous volume, Cambridge, 1946). (Received September 13, 1949.)

32*t*. A. R. Schweitzer: *An examination of Whitehead's Process and reality.* I.

Whitehead's *Process and reality* (New York, 1929) representing the "philosophy of organism" is classed among treatises having a heraclitean motive including Schleiermacher's philosophy of dependence and Bergson's philosophy of movement. More specifically, Whitehead's philosophy is interpreted as a theory suggested by mathematics as a science of extension (space, time, space-time) and by mathematical physics (including atomic physics). The latter subject is considered mainly in Part II, Chapter III: The order of nature; the former is treated in Part II, Chapter II: The extensive continuum and in Part IV: The theory of extension. Both aspects are recognized by Whitehead in his theory of prehension (Part III: pp. 337, 365; compare Part II, p. 227 and Part IV, p. 433). This generalization leads to Whitehead's cosmology (ibid. p. 365). (Received September 13, 1949.)

33*t*. A. R. Schweitzer: *An examination of Whitehead's Process and reality*. II.

Geometry is described by Whitehead philosophically as the "investigation of the morphology of nexus"; subsequently geometry is discussed in a sense familiar to mathematicians. Arithmetical theorems seem to Whitehead the "most obviously metaphysical" among propositions. Historically, Whitehead regards his philosophy as a continuation of Plato and Locke and the inversion of Kant's philosophy. A concluding part of the author's paper is concerned with Whitehead's theory of orientation and his doctrine of relations. Whitehead's references to orientation include opposites; discrimination between positive and negative applied to prehension; polarity applied to actual entity (actual occasion), becoming, concrescence and creative urge; order applied to society and nexus. Whitehead describes relation as a genus of contrasts or modes of synthesis of entities in one prehension. The author discusses Whitehead's essay, *An enquiry concerning the principles of natural knowledge* (Cambridge, 1919) as a preliminary to *Process and reality*. (Received September 13, 1949.)

STATISTICS AND PROBABILITY

34. E. J. Gumbel: *Orthogonal least squares applied to probability papers*.

A probability paper for a continuous unlimited statistical variate x which permits a linear reduction $x=u+y/\alpha$ is a rectangular grid for the observed variate x, and the reduced variate y, where the probability $F(y)$ is written instead of y. The simplest plotting procedure is the use of the mean frequency $m/(n+1)$ for the mth among n observations. The tiresome calculation of the cross product $(\overline{xy})_n$ may be avoided by minimizing the orthogonal distances which leads to the equations $u=\bar{x}-\bar{y}_n/\alpha$ and $\alpha=\sigma_n/s_x$, where $\bar{x}$ and s_x are the observed mean and standard deviation, and $\bar{y}_n$, σ_n are the corresponding theoretical values. These two moments may be calculated as functions of n from a table of $y(F)$. For symmetrical distributions, $\bar{y}_n$ vanishes, and u is estimated as the observed mean. For the standardized normal distribution, the expression $z_{2,n}$ defined by $F(z_{2,n})=\sigma_n$ approaches quickly a linear function of $\hat{y}_n$, where $\hat{y}_n$ is defined as the solution of $F(\hat{y}_n)=n/(n+1)$. For the asymptotic distribution of extreme values of the exponential type $z_{1,n}$ defined by $F(z_{1,n})=\bar{y}_n/\gamma$ and $z_{2,n}$ defined by $F(z_{2,n})=\sigma_n 6^{1/2}/\pi$ are also linear functions of $\hat{y}_n$. These relations simplify the calculations of $\bar{y}_n$ and σ_n as functions of n. (Received September 19, 1949.)

35. B. O. Koopman: *Necessary and sufficient conditions for Poisson's distribution*.

An infinite sequence of sets of n independent trials ($n=1, 2, 3, \cdots$) is considered; $p_{n,k}$ is the probability that, in the nth set of trials, the kth trial shall succeed. It is proved in this paper that a necessary and sufficient condition for obtaining the Poisson distribution (of given total numbers of successes in the nth set of trials) as a limit $(n\to\infty)$ is that: (a) $p_{n,1}+\cdots+p_{n,n}$ approach a limit, and (b) the maximum $p_{n,k}$ $(1\leqq k\leqq n)$ approach zero (as $n\to\infty$). The special case in which the probabilities are in fixed ratios $(p_{n,1}:p_{n,2}:p_{n,3}:\cdots)$ independent of n is considered and connected with certain divergent series of non-negative terms. Applications are given, in some of which the ratio of the maximum to minimum $p_{n,k}$ in the set $(1\leqq k\leqq n)$ increases as any given positive power of n as $n\to\infty$. (Received October 3, 1949.)

Topology

36. L. W. Cohen and Casper Goffman: *The metrization of uniform space.*

It is shown that a necessary and sufficient condition for a uniform space S to be metrizable with distances in an ordered abelian group is that the neighborhood system $U_\xi(x)$, $\xi<\xi^*$, $x\in S$, satisfy the axioms 1, 2, 3, 4′ in the authors' paper *A theory of transfinite convergence*, Trans. Amer. Math. Soc. vol. 66 (1949) pp. 65–74. (Received August 15, 1949.)

37. G. D. Mostow: *A theorem on locally euclidean groups.*

The *derived subgroup* of a topological group is defined as the closure of the commutator subgroup. If G is a locally compact, connected topological group, then its derived series, that is, series of successive derived subgroups, becomes stationary at some finite stage. The *core* of such a group is defined as the subgroup in the derived series which coincides with its derived subgroup. *Theorem.* Let G be a locally euclidean group and let C denote its core. Then G/C is a Lie group. In the special case that the core consists of the identity element alone, this theorem asserts that a solvable locally euclidean group is a Lie group. (Received September 16, 1949.)

38. Everett Pitcher: *A model for the homotopy theory of spheres.* Preliminary report.

A model for the homotopy theory of the n-sphere is constructed by use of deformations inherent to the theory of critical points of the length integral. Methods based on exact homomorphism sequences are used to prove theorems which have the following corollaries: (1) it is false that $\pi_{2n}(S^n)=0$ for two consecutive values of $n\geqq 2$; (2) it is false that $\pi_{2n+2}(S^n)=0$ for two consecutive values of $n\geqq 3$; (3) it is false that $\pi_{2n+6}(S^n)=0$ for two consecutive values of $n\geqq 5$. Of special interest is a class of homomorphisms of $\pi_r(S^n)$ into $\pi_{r-2}(S^{p(n-1)-1})$, defined for all r, with p determined so that $p(n-1)-1\leqq r\leqq(p+1)(n-1)-2$ (and believed to be equivalent to the generalized Hopf homomorphism of G. W. Whitehead when $p=2$). (Received September 16, 1949.)

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