BOOK REVIEWS

Foundations of economic analysis. By P. A. Samuelson. Cambridge, Harvard University Press, 1947. 11+447 pp. $7.50.

Professor Samuelson has used mathematics expertly to produce this unified treatment of economic theory. His *Foundations* provides a synthesis of statical economic theory, a survey of welfare economics and other important special topics, and an exploratory discussion of the problem of stability of equilibrium for dynamical economic systems.

The exposition is thoroughly mathematical. It is likely that most economists will find it very hard reading even though, as is noted in the Introduction: " . . . The pure mathematician will recognize all too readily the essentially elementary character of the tools used." Nevertheless, the book will undoubtedly lead to a greatly increased use of mathematics by economists in their future work. It sets a new high standard for the mathematical development of unified economic theory.

The pure mathematical aspects of the work are discussed separately in two appendices. The first appendix is concerned with the problem of maximum conditions for functions, and stresses maxima problems for quadratic forms treated from the matrix point-of-view. The second appendix is a sixty page self-contained treatment of the elements of the theory of systems of difference equations, and is included because of the essential use made of theorems in this field for the development and study of dynamical economic systems. Although the interest centers in systems of linear difference equations, there are some theorems concerning systems of differential equations and mention of mixed difference-differential equations.

The choice of material for the appendices was made on the basis of need for support of the main text. For this purpose, it was necessary to stress the topics that are not generally known to economists. As a result, the selection seems poorly balanced when gauged on the basis of general mathematical importance. I suspect also that most economists would like to have more in explanation of the elementary properties of matrices, and a brief discussion of the solutions of systems of linear difference equations in terms of ordinary trigonometric functions, even at the expense of additional pages.

Professor Samuelson set out to formulate a general theory of economics that would unify various particular fields with respect to analogous central features. He remarks: "Only after laborious work

in each of these fields did the realization dawn upon me that essentially the same inequalities and theorems appeared again and again, and that I was simply proving the same theorems a wasteful number of times." He insists also that the theorems he proves be "operationally meaningful" in order that they can be treated as hypotheses about empirical data that could conceivably be refuted.

The unified treatment of the theory of consumer's behaviour (Chap. V) is a convincing sample of success with synthesis in operationally meaningful terms. It is the general notion that a consumer has an ordinal preference field $U=F[\phi(X)]$ where ϕ is some one cardinal index of utility such that preferences between combinations of goods X and Y are in accordance with $\phi(X)>\phi(Y)$, $\phi(X)=\phi(Y)$, or $\phi(X)<\phi(Y)$, where $\phi(X)>\phi(Y)$ implies that X is preferred to Y. (It is assumed that ϕ is continuous and differentiable and that $F'(\phi)>0$.) Now if total income is I and if the price of good i is p_i, the main problem is to derive the demand function $X_i(p_1, p_2, \cdots, p_n, I)$ for the quantity x_i of good i that the consumer would purchase, subject to the restriction $I=\sum p_i x_i$, if he wished to maximize U. It is shown that all restrictions on the demand function can be derived from the single condition that the form $K_{ij}=\delta x_i/\delta p_j+x_j\delta x_i/\delta I$ be symmetric and negative semi-definite. A constructive proof is sketched also to show that when this condition is satisfied there exists a $\phi(X)$ that satisfies the properties of a preference field. Professor Samuelson concludes that: "Despite its lofty beginnings, the pure theory of consumer's behavior, when its empirical meaning is finally distilled from it, turns out to be one simple hypothesis on price and quantity behavior." Here, then, is a concise hypothesis that provides the basis for an imposing theoretical economic structure that should soon be put to trial by some clever experimental economist.

The pure mathematician who wishes to sample an important segment of economic theory, written competently in his favorite language, will find the *Foundations* pleasant reading. He may well also be stimulated, especially by the discussion of dynamic economic theory in Part II, to extend some of the results reported by Professor Samuelson and this would constitute a well-deserved widening of the sphere of influence of this splendid book.

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Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability. Ed. by J. Neyman. Berkeley, University of California Press, 1947. 12+447 pp. $7.50.

The thirty papers which are collected in the volume cover such a