results are obtained. The group of automorphisms of the free monogenic loop is the free cyclic group. A monogenic loop defined by a finite, nonzero number of relations has only a finite number of endomorphisms. This means that the isomorphism problem can be solved for monogenic loops. Any countable loop can be embedded in a monogenic loop. A monogenic loop is constructed which is isomorphic to a proper factor loop of itself. (Received August 14, 1950.)

456t. D. C. Murdoch: *Intersections of primary ideals in a noncommutative ring.*

Let R be a noncommutative ring in which the ascending chain condition holds for two-sided ideals. An ideal $\mathfrak{q}$ is right primary if when a is not in $\mathfrak{q}$, $aRb \subseteq \mathfrak{q}$ implies $b \in \mathfrak{r}(\mathfrak{q})$, where $\mathfrak{r}(\mathfrak{q})$ is the radical of $\mathfrak{q}$ in the sense of McCoy (Amer. J. Math. vol. 71 (1949) pp. 823–833). The radical of a right primary ideal is prime. If $\mathfrak{a}$ is the intersection of a finite number of right primary ideals then $\mathfrak{a}$ is right primary with radical $\mathfrak{p}$ if and only if all its right primary components have radical $\mathfrak{p}$. Hence any ideal $\mathfrak{a}$ which is the intersection of right primary ideals has a short representation in which the right primary components all have different radicals. In any two short representations of $\mathfrak{a}$ the number of right primary components is the same and the radicals of the components of one representation are equal in some order to the radicals of the components of the other. (Received September 12, 1950.)

457t. H. J. Ryser: *A combinatorial theorem with an application to latin rectangles.*

Let A be a matrix of r rows and n columns composed entirely of zeros and ones, where $1 \leqq r < n$. Let there be exactly k ones in each row, and let $N(i)$ denote the number of ones in the ith column of A. If for each i, $k-(n-r) \leqq N(i) \leqq k$, then $n-r$ rows of zeros and ones may be adjoined to A to obtain a square matrix with exactly k ones in each row and column. Let T be an array of r rows and s columns, formed from the integers $1, \cdots, n$ in such a way that the integers in each row and column are distinct. Let $N(i)$ denote the number of times that i occurs in T. The preceding theorem implies that a necessary and sufficient condition in order that T may be extended to an n by n latin square is that for each $i=1, \cdots, n$, $N(i) \geqq r+s-n$. The latter result generalizes M. Hall's existence theorem for latin squares (Bull. Amer. Math. Soc. vol. 51 (1945) pp. 387–388). (Received September 18, 1950.)

Analysis

458t. P. R. Garabedian: *A partial differential equation arising in conformal mapping.*

Let $K(z, t)$ and $k(z, t)$ be the kernel functions in a plane region D which are associated, respectively, with the norms $\iint_D |f(z)|^2 \rho dxdy$ and $\iint_D u(x, y)^2 \rho dxdy$ of analytic functions $f(z)$ and real harmonic functions $u(x, y)$ in D, where ρ is a positive weight function. These kernels are given in terms of the Green's functions $G(z, t)$ and $g(z, t)$ of the partial differential equations $\partial/\partial\bar{z}(\rho^{-1}\partial/\partial z)G=0$ and $\Delta\rho^{-1}\Delta g=0$ by the formulas $K(z, t) = -2(\pi\rho(z)\rho(t))^{-1}(\partial^2 G(z, t)/\partial z\partial\bar{t})$ and $k(z, t) = -(8\pi\rho(z)\rho(t))^{-1}\Delta_z\Delta_t g(z, t)$. The Friedrichs eigenfunctions ψ_ν and eigenvalues μ_ν for $\mu = |\iint_D \psi^2 dxdy| / \iint_D |\psi|^2 dxdy$ $=$ maximum, $\partial\psi/\partial\bar{z}=0$, are given in terms of the eigenfunctions U_ν of the partial differential equation $\partial^2 U/\partial\bar{z}^2 + \mu\partial^2\overline{U}/\partial z\partial\bar{z}=0$ by the formula $(1-\mu_\nu^2)\psi_\nu$ $=\partial U_\nu/\partial\bar{z}+\mu_\nu\partial\overline{U}_\nu/\partial z$. Both problems have the same eigenvalues μ_ν. The eigenfunctions m_ν and eigenvalues λ_ν for the problem $\lambda = \iint_D |\partial m/\partial z|^2 dxdy / \iint_D |\partial m/\partial\bar{z}|^2 dxdy$

$=$minimum, $\partial^2 m/\partial\bar{z}^2=0$, are given by the formulas $\partial m_\nu/\partial\bar{z}=\psi_\nu$, $\partial m_\nu/\partial z=\partial U_\nu/\partial z$, $\lambda_\nu=1-\mu_\nu^2$. For $\rho\equiv 1$, the identity $\partial^2 g/\partial z\partial\bar{t}=G(z, t)-(\pi/2)\sum(1-\mu_\nu^2)^{-1}U_\nu(z)\overline{U_\nu(t)}$ is proved. Applications are made to the case where D is an ellipse. (Received September 20, 1950.)

459t. P. R. Garabedian: *Asymptotic identities among periods of integrals of the first kind.*

In a multiply-connected domain D of the z-plane bounded by curves $C_1, \cdots, C_{g+1}$, the harmonic measures $\omega_k(z)$ with boundary values 1 on C_k and 0 on the C_j with $j\neq k$, $k=1, \cdots, g$, have an interpretation as the real parts of the normal integrals of the first kind on a symmetric closed Riemann surface of genus g associated with D. Since the conformal type of D depends on $3g-3$ parameters, the periods $P_{ij}=\oint_{C_j}(\partial\omega_i/\partial n)ds$ of the integrals of the first kind satisfy $(g-2)(g-3)/2$ nontrivial identities. Asymptotic forms of these identities are obtained which correspond to degeneration of D when, first, all the C_j shrink to points a_j and, second, the C_j with $j\leqq g$ shrink to points a_j and C_{g+1} is the unit circle. In the first case, the asymptotic identities correspond to those which exist among the Euclidean distances between the points a_j; in the second case the identities originate from those which exist among the non-Euclidean distances amongst the a_j in the unit circle. In a further study, Poincaré's identity $(P_{12}P_{13}P_{24}P_{34})^{1/2}+(P_{13}P_{14}P_{23}P_{24})^{1/2}=(P_{12}P_{14}P_{23}P_{34})^{1/2}$ for $P_{ij}\to 0$, $i\neq j$, and P_{ii} bounded away from zero and infinity is derived. (Received September 20, 1950.)

460t. P. R. Garabedian: *The classes L_p and conformal mapping.*

Let D be a region of the z-plane bounded by disjoint smooth curves $C_1, \cdots, C_n$, and let t be a fixed point of D. We denote by $K(z)$ the analytic function in D with $K(t)=1$ and with $\alpha=\oint_C|K(z)|^p ds=$minimum, $p>1$; and we denote by $L(z)$ the function analytic in D except for a simple pole of residue $1/2\pi i$ at $z=t$ and with $\beta=\int_C|L(z)|^q ds=$minimum, $q>1$. For $p^{-1}+q^{-1}=1$ we obtain $L(z)K(z)dz=\alpha^{-1}|K(z)|^p ds=\beta^{-1}|L(z)|^q ds$ on C, and the minimum values α and β satisfy $\alpha^{1/p}\beta^{1/q}=1$. Proofs are based on Hölder's inequality and on a consideration of the class L_p of all complex-valued functions $f(z)$ defined on C with $\oint_C|f|^p ds<\infty$ and with $\oint_C f\phi dz=0$ for every function $\phi(z)$ analytic in $D+C$. Further normalizations are discussed, and it is shown in one case where t is the point at infinity that all extremal functions with any value of $p\geqq 1$ are related and are given in terms of the Szegö kernel function. Some of the results are extended to the case of values of p with $0<p<1$. (Received September 20, 1950.)

461t. J. G. Herriot: *An example of an analytic function with a singular line.*

The function $f(z)=\int_0^\infty(1-\tanh X\tanh zx)dx$, $0<z\leqq 1$, was first encountered in the study of the electrostatic capacity of a symmetric lens. It is easily seen that the above integral represents a function analytic in the half-plane $R(z)>0$. A series representation of $f(z)$ is obtained and used to continue $f(z)$ analytically throughout the rest of the complex plane with the exception of the negative real axis. The negative real axis is shown to be a line of singularities of $f(z)$. This is done by showing that $f(z)$ becomes infinite as z approaches any point of the form $-(2p+1)/(2q+1)$ where p and q are integers greater than one and $2p+1$ and $2q+1$ are relatively prime; z moves so that its real part is constant. (Received August 15, 1950.)

462t. Werner Leutert: *On the convergence of approximate solutions of the heat equation to the exact solution.*

It is shown that an approximate solution of the heat equation can be obtained from a three line difference equation by using only half of the particular solutions of the form $e^{i\beta x}e^{\alpha i}$. The approximate solution will converge to the exact solution for all positive values of the mesh ratio $r=\Delta t/(\Delta x)^2$ and it will be stable in the sense that small changes in the initial condition vanish as the time t is increased. von Neumann's test shows instability for all values of $r>0$. (Received July 31, 1950.)

463t. Bertram Yood: *On fixed points for semi-groups of linear operators.*

Let G be a semi-group of bounded linear operators on a normed linear space X, and G^* be the family of adjoints of elements of G. Sets of conditions are given on G which imply the existence of a nonzero fixed element for G^* (in X^*). In particular if X is the space of bounded functions on a set S, the results show, as a special case, the existence of a finitely-additive measure defined for all subsets of S invariant under a solvable group of 1-1 transformations of S onto S. This fact is due to von Neumann (Fund. Math. vol. 13 (1929)). (Received September 14, 1950.)

Applied Mathematics

464t. C. N. Mooers: *Automata with learning.* Preliminary report.

The automata moves in an artificial environment having positions or states $q_i(i=1, \cdots, N_q)$. It has a repertory of moves that it can make, each called m_{ij} $(j=1, \cdots, N_i)$. From state q_i by move m_{ij} it goes to a new uniquely determined state q_k, that is, $(q_i, m_{ij})=q_k$. Each state q_i is characterized by an aspect a_i having the value $+1$ or -1. The a_i is a "drive" in the psychological sense, and when a_i is positive the automata is active. In state q_i the automata initially randomly chooses an m_{ij} where all the m's have an equal probability. In the case $(q_i, m_{ij})=q_{i+1}$ whose a_{i+1} is negative (drive extinguished), then the probability is increased for choice m_{ij} when in state q_i. In (q_i, m_{ij}) there is a transfer relation such that when some $m_{i+1,k}$ of q_{i+1} has a probability greater than $2/N_{i+1}$, then the probability of taking m_{ij} in q_i is also increased. The automata as postulated can learn its way through a maze, learning from the goal backwards; it can remember the solution to two or more mazes; it forgets nonused information; and its behavior is not predictable. (Received September 5, 1950.)

465t. L. A. Zadeh: *On stability of linear varying-parameter systems.*

Starting with the definition of stability in the case of linear varying-parameter systems: a system is stable if and only if every bounded input produces a bounded output, it is shown that the necessary and sufficient condition for stability is that the impulsive response of the system $W(t, \tau)$ should belong to $L(0, \infty)$ for all t ($W(t, \tau)$ is the response at t to a unit impulse applied at $t-\tau$). The system function of a linear varying-parameter system is related to $W(t, \tau)$ through $H(s; t)=\int_0^\infty W(t, \tau)e^{-s\tau}d\tau$. From this it follows that the system function of a stable system is analytic in the right half and on the imaginary axis of the s-plane for all t. This result can be applied with advantage to the investigation of stability of linear varying-parameter systems. In particular, it yields useful criteria of stability for differential equations having periodic coefficients. (Received September 14, 1950.)