462t. Werner Leutert: On the convergence of approximate solutions of the heat equation to the exact solution.

It is shown that an approximate solution of the heat equation can be obtained from a three line difference equation by using only half of the particular solutions of the form $e^{ax}e^{at}$. The approximate solution will converge to the exact solution for all positive values of the mesh ratio $r = \Delta t/(\Delta x)^2$ and it will be stable in the sense that small changes in the initial condition vanish as the time $t$ is increased. von Neumann's test shows instability for all values of $r>0$. (Received July 31, 1950.)

463t. Bertram Yood: On fixed points for semi-groups of linear operators.

Let $G$ be a semi-group of bounded linear operators on a normed linear space $X$, and $G^*$ be the family of adjoints of elements of $G$. Sets of conditions are given on $G$ which imply the existence of a nonzero fixed element for $G^*$ (in $X^*$). In particular if $X$ is the space of bounded functions on a set $S$, the results show, as a special case, the existence of a finitely-additive measure defined for all subsets of $S$ invariant under a solvable group of 1-1 transformations of $S$ onto $S$. This fact is due to von Neumann (Fund. Math. vol. 13 (1929)). (Received September 14, 1950.)

APPLIED MATHEMATICS


The automata moves in an artificial environment having positions or states $q_i (i=1, \cdots, N_q)$. It has a repertory of moves that it can make, each called $m_{ij}$ ($j =1, \cdots, N_m$). From state $q_i$ by move $m_{ij}$ it goes to a new uniquely determined state $q_k$, that is, $(q_i, m_{ij}) = q_k$. Each state $q_i$ is characterized by an aspect $a_i$ having the value $\pm 1$. The $a_i$ is a "drive" in the psychological sense, and when $a_i$ is positive the automata is active. In state $q_i$ the automata initially randomly chooses an $m_{ij}$ where all the $m$'s have an equal probability. In the case $(q_i, m_{ij}) = q_{i+1}$ whose $a_{i+1}$ is negative (drive extinguished), then the probability is increased for choice $m_{ij}$ when in state $q_i$. In $(q_i, m_{ij})$ there is a transfer relation such that when some $m_{i+k}$ of $q_{i+k}$ has a probability greater than $2/N_{i+k}$, then the probability of taking $m_{ij}$ in $q_i$ is also increased. The automata as postulated can learn its way through a maze, learning from the goal backwards; it can remember the solution to two or more mazes; it forgets unused information; and its behavior is not predictable. (Received September 5, 1950.)


Starting with the definition of stability in the case of linear varying-parameter systems: a system is stable if and only if every bounded input produces a bounded output, it is shown that the necessary and sufficient condition for stability is that the impulsive response of the system $W(t, \tau)$ should belong to $L(0, \infty)$ for all $t$ ($W(t, \tau)$ is the response at $t$ to a unit impulse applied at $t-\tau$). The system function of a linear varying-parameter system is related to $W(t, \tau)$ through $H(s; t) = \int_0^\infty W(t, \tau)e^{-st}d\tau$. From this it follows that the system function of a stable system is analytic in the right half and on the imaginary axis of the $s$-plane for all $t$. This result can be applied with advantage to the investigation of stability of linear varying-parameter systems. In particular, it yields useful criteria of stability for differential equations having periodic coefficients. (Received September 14, 1950.)
ABSTRACTS OF PAPERS

466t. L. A. Zadeh: Initial conditions in linear varying-parameter systems.

Consider a linear varying-parameter system \( N \) whose behavior is described by an \( n \)-th order linear differential equation \( L(p; t)v(t) = u(t) \). Let \( u(t) \) be zero for \( t < 0 \) and let the initial values of \( v(t) \) and its derivatives be \( v^{(o)}(0) = \alpha_v \) (\( o = 0, 1, \ldots, n-1 \)). Let \( H(s; t) \) be the system function of \( N \). When the system is initially at rest (that is, all \( \alpha_v \) are zero), the response of \( N \) to \( u(t) \) may be written as \( v(t) = \mathcal{L}^{-1}\{H(s; t)U(s)\} \) (see abstract 56-6-465). When, on the other hand, some of the \( \alpha_v \) are not zero, the expression for the response to a given input \( u(t) \) becomes \( v(t) = \mathcal{L}^{-1}\{H(s; t)[U(s) + A(s)]\} \), where \( A(s) \) is a polynomial in \( s \) and \( p_0 \) given by \( A(s) = \left\{ \frac{[L(s; 0) - Lp_0(0)]}{(s-p_0)} \right\}v \) (\( p_0 \) represents a differential operator such that \( p_0 v^{(o)}(0) = \alpha_v \)). \( A(s) \) is essentially the Laplace transform of a linear combination of delta-functions of various order (up to \( n-1 \)) such that the initial values of the derivatives of the response of \( N \) to this combination are equal to \( \alpha_v \). (Received September 14, 1950.)

TOPOLOGY


J. H. C. Whitehead has defined (Ann. of Math. vol. 42 (1941) pp. 409-428) a product which associates with elements \( \alpha \in \pi_p(X) \) and \( \beta \in \pi_q(X) \), an element \( [\alpha, \beta] \in \pi_{p+q-1}(X) \). The authors show how to define three new products, as follows: (a) A product which associates with elements \( \alpha \in \pi_p(A) \) and \( \beta \in \pi_q(X, A) \), an element \( [\alpha, \beta] \in \pi_{p+q-1}(X, A) \). (b) A product which associates with elements \( \alpha \in \pi_p(A, B) \) and \( \beta \in \pi_q(A \cap B) \), an element \( [\alpha, \beta] \in \pi_{p+q-1}(A \cap B) \). Here the sets \( A \) and \( B \) are covering of the space \( X = A \cup B \), and \( \pi_p(A \cap B) \) is the \( p \)-dimensional homotopy group of this covering which has been introduced by the authors (Bull. Amer. Math. Soc. Abstract 56-3-208). (c) Let \( (X; A, B) \) be a triad (see A. L. Blakers and W. S. Massey, Proc. Nat. Acad. Sci. U.S.A. vol. 35 (1949) p. 323), then there is a product which associates with elements of \( \pi_p(A/B) \) and \( \pi_q(X(X; A, B)) \) an element of \( \pi_{p+q-1}(X; A, B) \). The bilinearity of these three new products is established under suitable restrictions, and relationships between the various products are proved. The behavior of the products under homomorphisms induced by a continuous map or a homotopy boundary operator is also studied. (Received August 30, 1950.)

468t. A. L. Blakers and W. S. Massey: The triad homotopy groups in the critical dimension.

Let \( X^* = X \cup \xi^* \cup \xi^* \cup \cdots \cup \xi^* \) be a space obtained by adjoining the \( n \)-dimensional \( (n > 2) \) cells \( \xi^* \) to the connected, simply connected topological space \( X \). Let \( \xi^* = \xi^* \cup \cdots \cup \xi^* \) and \( \xi^* = X \setminus \xi^* \). Assume that the space \( \xi^* \) is arcwise connected, and that the relative homotopy groups \( \pi_p(X, \xi^*) \) are trivial for \( 1 \leq p \leq m \), where \( m \geq 1 \). Then it is known that the triad homotopy groups \( \pi_q(X^*; \xi^*, X) \) are trivial for \( 2 \leq q \leq m+n-1 \). The authors now show that under the assumption of suitable "smoothness" conditions on the pair \( (X, \xi^*) \) (for example, both \( X \) and \( \xi^* \) are compact A.N.R.'s), there is a natural isomorphism of the tensor product \( \pi_n(\xi^*, \xi^*) \otimes \pi_{m+n}(X/\xi^*) \) onto the triad homotopy group \( \pi_{m+n}(X^*; \xi^*, X) \). This isomorphism is defined by means of a generalized Whitehead product. The Freudenthal "Ein­hängung" theorems in the critical dimensions can easily be derived from this theorem;