tion between the images of $h(t)$ and $h(g(t))$ where $g(t)$ is a standard function such as t^{-1}, t^2, e^t, sinh t, and so on, interesting applications to hypergeometric series, the Parseval relation, again with interesting applications to Bessel function identities. Chapter XII introduces step- and discontinuous functions. This is the chapter in which the functions of number theory play such a conspicuous and welcome part. While the work is an excellent practice in operational technique, the results are also relevant for the theory of numbers. The discussion of difference equations in chapter XIII is another opportunity for deriving many relations between special functions.

Chapter XIV explains the technique of solving integral equations by means of the operational calculus. Chapter XV, on partial differential equations, is somewhat disappointing. It makes stimulating reading, the technique is explained carefully, and is illustrated by excellent examples; yet the work is more or less formal. The peculiar difficulties involved in verifying the solution are not even mentioned.

In chapter XVI the operational calculus is extended to several variables, and it is shown how this simultaneous operational calculus can be utilized for the solution of partial differential equations, and other problems.

Chapter XVII contains ten pages of general formulas and rules of the operational calculus, and is aptly labelled *Grammar.* Chapter XVIII, *Dictionary*, contains 27 pages of well-classified operational transform pairs.

The book developed from lectures given by van der Pol in 1938 and 1940, the original Dutch manuscript was prepared during the late war, and the English translation was edited by Dr. C. J. Bouwkamp. The translation deserves special praise for preserving the freshness and flavor of the original. It goes without saying that it is always clear what the authors mean, even where they do not express themselves idiomatically. Passages which may lead to misunderstandings, as for instance on p. 135 where the authors say "the exponents of x may increase" when they presumably mean "the exponents of x are assumed to increase" are very rare. A very special praise is also due to the printing of this book, one of the most beautifully printed of the recent mathematical books.

A. ERDÉLYI

Existence theorems in partial differential equations. By Dorothy L. Bernstein. (Annals of Mathematics Studies, no. 23.) Princeton University Press, 1950. 10+228 pp. $2.50.

An enormous number of papers have been written that deal with various aspects of the theory of partial differential equations. On the other hand, there are only very few books that give an adequate and up-to-date account of the main facts that have been established in this field, as anybody can testify who has ever had to teach a course on the subject. Dr. Bernstein's book, prepared under a contract with the Office of Naval Research, has as its purpose to collect existence theorems that may be of use in formulating problems for solution by computing machines. There is no doubt that existence theorems constitute an important guide for the arrangement of the computation. If the proof of the theorem is constructive, it may be possible to translate it immediately into a scheme for numerical computation. The theorem may also shed light on the nature of the dependence of the solution on the data, which often is of importance in judging the soundness of an approximation scheme. Thus a scheme that does not make use of all the data theoretically needed for determining the solution can be discarded as useless from the beginning. Unfortunately, existence theorems have been established only for relatively few of the types of problems encountered in applications for which numerical solutions are desired.

The following subjects are discussed by the author in greater detail and with outlines of proofs: First order equations (solution of Cauchy problem by the method of characteristics); Systems of first order equations (solution of Cauchy problem for analytic systems); Hyperbolic second order equations (characteristic Goursat problem and Cauchy problem for two independent variables; Cauchy problem for linear equations in more independent variables); Parabolic equations of second order (essentially the linear and quasi-linear cases); Second order elliptic equations (mostly the linear case); Equations of higher order (isolated results for special equations in addition to the Cauchy-Kowalesky theory).

This book is mostly concerned with problems in the small (except in the case of first order equations) and concentrates on existence theorems obtained by general methods rather than those based on explicit solutions. The author took great pains to state the precise assumptions needed in the proofs of the theorems given.

This reviewer feels that an even greater emphasis on methods and ideas in preference to isolated results would have served better the purposes for which this book is intended. Systems of first order equations might have been grouped with equations of higher order rather than with those of first order. There are a fairly large number of misprints and minor inaccuracies in the text that might have been eliminated by more careful proofreading. Many of the papers referred to in the text cannot be found in the bibliography.

Aside from these minor criticisms, this book should be of great value to any pure or applied mathematician working with partial differential equations, and not only to those interested in numerical solutions. A person engaged in research in the field of partial differential equations will find it necessary to go beyond the theorems stated to the original literature. However, the material given in the book should be ample enough to prevent his overlooking major contributions to a problem he is interested in, and to facilitate the task of informing himself on progress made by other workers in the field.

FRITZ JOHN

The Farey series of order 1025, *displaying solutions of the diophantine equation* $bx-ay=1$. Designed and compiled by E. H. Neville. (Royal Society Mathematical Tables, vol. 1.) Cambridge University Press, 1950. 29+405 pp.+1 plate. $18.50.

The Farey series of order n is the table obtained if one arranges in order of magnitude the rational numbers having denominators not greater than n and lying between 0 and 1 inclusive; each rational number is understood to be written in its "lowest terms," the numbers 0 and 1 being given in the forms 0/1 and 1/1 respectively. The table under review is by far the largest of its kind ever published in full and probably will remain so for some time to come, since the Farey series of order 1025 consists of 319,765 fractions.

Even though the fractions a/b and $(b-a)/b$ are given together, the Farey series of order 1025 itself occupies 400 pages. Except for the last page, each page contains 400 pairs of fractions, arranged in lines of 20 each. Besides the main table there is an appendix containing the following smaller tables: (1) the Farey series of order 50 with decimal equivalents, (2) the Farey series of order 64, (3) the denominators of the Farey series of order 100. In addition there is a brief introduction in which the author discusses the background, construction, and use of the tables. (See also the author's article *The structure of Farey series*, Proc. London Math. Soc. (2) vol. 51 (1949) pp. 132–144 and the third chapter of Hardy and Wright's *Introduction to the theory of numbers*.)

The principal interest of such a table, aside from its obvious usefulness in finding rational approximations to irrational numbers, lies in the fact that if a/b and x/y are consecutive fractions in the Farey series of order n, then $bx-ay=1$. Thus the present table can be thought of as a table of solutions of the diophantine equation