

single segment of length ρ forming an angle $\phi \neq 0, \pi$ with the real axis, or the slit may lie on the real axis with the pieces of absolute value ≤ 1 identified with each other, in which case the slit appears as a fork with end points α, β . Accordingly, the boundary of V_3 consists of one surface corresponding to the parameters ρ, ϕ , and another surface with the parameters α, β .

The last chapter, written by Arthur Grad, gives an explicit determination of the region of values taken by $f'(z)$ at a fixed point z . It is an excellent illustration of the method in a case different from but of the same degree of difficulty as the determination of V_3 . The reader who is anxious to learn the technique from the point of view of actual application will find this chapter most rewarding.

The authors can be congratulated on the accurate work they have accomplished. Great professional skill and painstaking detailed analysis are dominating features throughout the book. Books of this sort are never easy to read, and they offer little to the impatient skimmer. This book is definitely written by conscientious authors for conscientious readers.

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BRIEF MENTION

Funzioni quasi-periodiche. By S. Cinquini. (Scuola Normale Superiore, Pisa, Quaderni Matematici, no. 4.) Pisa, Tacchi, 1950. 132+7 pp.

The class of Bohr almost periodic functions includes in particular the subclass of exponential polynomials $\sum a_n e^{i\lambda_n x}$, and Bohr's uniqueness theorem states that the whole class arises from the subclass on closing the latter by the norm

$$(1) \quad \|f\| = \sup_{-\infty < x < \infty} |f(x)|.$$

In the periodic case this corresponds to the C -functions, and the so-called generalizations of almost periodic functions are various modes for introducing closure norms that would give the analogues to all L_p -functions as well. The narrowest such generalization known is due to Stepanoff, and it uses the norm

$$(2) \quad \|f\| = \sup_{-\infty < x < \infty} \left(\int_0^l |f(x + \xi)|^p d\xi \right)^{1/p}$$

for some (and hence any) finite length l ; the purpose of the present tract is to give an account of the theory that would feature the Stepanoff functions in the main.

The tract presupposes but little and it is written in a pleasant comfortable style, so that not too much material need be expected from it; and, in fact, the author more or less carries it only to the point where he presents the following theorem which he had published previously:

If $\psi(w)$ is non-negative convex in $0 \leq w < \infty$, and $\psi(0) = 0$, if the Stepanoff function $f(x)$ is such that for $\psi(2|f(x)|)$ a certain integrability condition is fulfilled, and if a sequence $\{\sigma_n(x)\}$ of Bochner-Fejér polynomials of $f(x)$ is formally convergent towards it, then

$$\int_0^1 \psi |f(x + \xi) - \sigma_n(x + \xi)| d\xi \rightarrow 0$$

uniformly in $-\infty < x < \infty$.

The latter theorem generalizes a known theorem for pure periodic functions, and it confirms a general statement made by the reviewer repeatedly with illustrations [see for instance, *Properties of Fourier series of almost periodic functions*, Proc. London Math. Soc. (2) vol. 26 (1926) pp. 433–452], that any property of pure periodic functions whatsoever, no matter what its scope or origin, may be adapted to Stepanoff functions as well, either to the entire class as in the present case, or to some suitable sub-class containing all pure periodic elements occurring, as in many other cases.

We also recall that Stepanoff functions may also be viewed as particular cases of abstractly-valued functions [see for instance, Bochner, *Abstrakte fastperiodische funktionen*, Acta. Math. vol. 61 (1933) pp. 149–184, last section] and we venture to say that no comprehensive account of the theory of Stepanoff functions would be complete without introducing the viewpoints thus arising—but these remarks of ours are not intended to be a criticism of the work the author has accomplished according to his own plans.

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The problem of moments. By J. A. Shohat and J. D. Tamarkin. (Mathematical Surveys, no. 1.) American Mathematical Society, New York, 1943. Reprinted 1950. 14+144 pp. \$3.35.

In this printing the theorem on p. xiii on extending a non-negative linear functional has been corrected and a supplementary bibliography of papers up to 1949 has been added. Otherwise, except for minor corrections, this printing is identical with the printing of 1943.