## GEORGE ABRAM MILLER
### 1863–1951

George Abram Miller was born on July 31, 1863, and died on February 10, 1951. He received the degrees of Bachelor of Arts (1887) and Master of Arts (1890) from Muhlenburg College, and of Doctor of Philosophy (1892) from Cumberland University. He was Professor of Mathematics (1888–1893) at Eureka College; Instructor at the University of Michigan (1893–1895) and Cornell University (1897–1901); Assistant and Associate Professor at Leland Stanford University (1901–1906); Associate Professor, Professor, and Professor Emeritus at the University of Illinois (1906–1951).

In October, 1891, Professor Miller was elected to membership in the New York Mathematical Society which had had a roster of twenty-three members at the beginning of the year. This Society became the American Mathematical Society in 1894. Later lists date Miller's membership from 1898; he became a life member in 1900. He was a member of the Council from 1901 to 1904. He was one of the founders of the San Francisco Section and its Secretary until 1906. He was Chairman of the Chicago Section from 1907 to 1909. He was Vice President of the Society in 1908.

He was a member of the London Mathematical Socety, the Deutschen Mathematiker Vereinigung, a corresponding member of the Sociedad Matematica Española, and an honorary life member of the Indian Mathematical Society. He was one of the organizers of the Mathematical Association of America at the end of 1915, a Vice President in 1916, and President in 1921. He was Secretary of Section A of the American Association for the Advancement of Science in 1899 and again from 1907 to 1912. He was a member of the National Academy of Sciences, and a fellow of the American Academy of Arts and Sciences. He was made an Honorary Doctor of Letters by Muhlenburg in 1936.

At Michigan in 1893 Miller lived in the home of F. N. Cole. Under Cole's direction and with his encouragement he started on the study of finite groups. His first publications were two short papers in volume 3 (1894) of the Bulletin of the New York Mathematical Society. In the next fifty-three years he contributed upward of 820 papers to the educational, scientific, and mathematical journals of eleven countries; about 450 of them made direct contributions to the theory of finite groups, the last in 1946. His course was set while he was at Michigan. He spent the years 1895 to 1897 at Leipzig and Paris where he at-

tended the lectures of Sophus Lie and Camille Jordan. By the end of the year of his return, he had published 27 papers in the mathematical journals of America, England, France, and Germany. By 1900 he was considered to be the foremost student of groups in this country, and by 1910 as unsurpassed by any living student of the subject.

His first two papers completed the determination of the 200 substitution groups of degree eight and the 258 of degree nine. In each case he found two groups that had been missed by his predecessors. His immediate predecessors were Cayley, Cole, and Jordan. In the same year he published his own list of 994 intransitive groups of degree ten. None of these lists has been questioned since. In 1900 he was awarded a prize by the Academy of Sciences of Cracow for his work on groups of degree ten; this was the first award by a foreign academy to an American for work in pure mathematics. The fact that he was thirty years old and a Doctor of Philosophy when he started, and the rapidity with which his work progressed beyond any patterns he might have followed, arouses a curiosity about the years that went before.

He was born on a farm near Lynville, Pennsylvania, in a German-speaking family. He began teaching in the public schools at seventeen to earn money to prepare for and attend college, and for the first year after graduation was Principal in the public schools of Greeley, Kansas. At Muhlenburg the offerings in mathematics were calculus in the junior year, and astronomy and meteorology in the senior year. The Professor of Mathematics had been appointed to teach mathematics, physics, astronomy, and geology; at one time he taught the advanced classes in Latin also. When Miller went to Eureka in 1888, a knowledge of calculus was required for the bachelor's degree. Beginning in 1890, Miller offered courses for post-graduates leading to the degree of Doctor of Philosophy at the end of two years. In the meantime, he had spent part of the summer of 1889 at Johns Hopkins and the summer of 1890 at Michigan; neither university was in session during those summers. Bolza gave a series of lectures on groups at Johns Hopkins in 1889, but it is not known that Miller had any contact with the mathematicians there during that summer. He probably met Cole in the summer of 1890, but the steady expansion of his offerings at Eureka and the fact that a course in groups was not among them makes it quite certain that his attention had not turned to groups at that time. In 1891–1892 he was enrolled as a graduate student at Cumberland. Graduate work could be done by correspondence and his must have been done so, for he was teaching six hours a day at Eureka. The requirements for the doctor's degree at

Cumberland were the possession of a bachelor's degree, the completion of a number of courses specified by the text-books, and a thesis for which examinations in the required courses could be substituted. The Professor of Mathematics at Cumberland was a Civil Engineer who was also the engineering faculty and who, when not occupied by his duties at the university, was mapping Tennessee for the U. S. Coast and Geodetic Survey. This busy man offered to teach the advanced courses in mathematics by correspondence for $8 per month. Presumably Miller did not require the instruction, for the following year Cumberland added a year of residence to the requirements for the doctor's degree in mathematics.

It should be noted that in 1890, when Miller first offered courses in advanced mathematics at Eureka, the College of New Jersey offered three courses beyond the calculus, all open to seniors and all taught by Fine; the state universities of Illinois, Indiana, and Ohio offered nothing beyond the calculus; Michigan with Cole and Ziwet offered several advanced courses and the doctor's degree. Johns Hopkins had been offering instruction in advanced mathematics since 1876. In 1890 Harvard offered eight courses taught by Byerly, White, J. M. Peirce, and Dr. Osgood and offered research in four areas under the direction of the same four men; there was one graduate course offered, taught by B. O. Peirce, and a seminar on various topics meeting once a week and directed by Byerly, J. M. Peirce, and B. O. Peirce. There were fifteen graduate students including Bôcher who was on a fellowship at Göttingen where he received his doctor's degree at the end of that year. A new era in instruction in advanced mathematics at Harvard was said to have been inaugurated by Cole's lectures on groups in 1886.

It seems clear that Miller came to mathematics and to Michigan without benefit of benign pressures from without. If his taking up the study of groups upon his close association with Cole be interpreted as environment working upon Miller, the rest of his life exhibits Miller working on his environment. Lie's lectures on *Groups and invariants* apparently contained too little groups and too much invariants, for Miller's notes stop after the second lecture. He had a strong liking for Lie and he credits Lie with starting him on a systematic consideration of commutators and commutator subgroups. His attitude toward Lie's continuous groups was essentially the same as Lie's attitude toward finite groups—tolerance and respect but no enthusiasm. Jordan's lectures he followed through the year. They were concerned mostly with questions of primitivity and imprimitivity. Jordan had published in 1872 a list of primitive groups through degree

seventeen; Miller had made one correction in 1894, he made the last in 1899. It seems likely that Miller would have made these corrections if he had never gone to Paris.

He published two books: *Determinants* (1892), written for the class in advanced algebra at Eureka; and *Historical introduction to mathematical literature* (1916). He wrote a part of each of two other books: *The algebraic equation*, pp. 211–260 of Monographs on Topics of Modern Mathematics (1911), edited by J. W. A. Young; and *Finite groups*, pp. 1–192 (1916), by Miller, Blichfeldt, and Dickson.

He wrote papers explaining groups to non-specialists, papers explaining mathematics to teachers of elementary mathematics and to non-mathematicians, a few papers on subjects of more general interest, and papers on the history of mathematics. His expository papers on groups began with about forty pages in fourteen installments in volumes 2 and 3 (1895–1896) of the American Mathematical Monthly; his last paper of this type was published in 1942. He urged the use of groups in elementary number theory, and in trigonometry. His most important paper of a more general nature was probably *Some thoughts on modern mathematical research* (1912). This was published in Science, in the Journal of the Indian Mathematical Society, and republished in Reports of the Smithsonian Institution as one of the most significant scientific publications of the year.

His interest in the history of mathematics dates from his days at Eureka. His first publication on a historical subject was in 1901, and his last in 1947. In his *Historical introduction* he says "The caliber of a mathematician can probably be judged just as accurately from the errors to which he pays attention as from the new results which he announces. In both cases, he can devote himself to trivialities or to big things." We might add that a man may concern himself so much with little things as to distract attention from bigger things he has done. In his papers on history he gave attention to too many things of questionable importance, like the anomaly of two first International Congresses of Mathematicians, or the authenticity of a dialogue between Euclid and a student used to dramatize the fact that the Greeks had an interest in mathematics that was not utilitarian. One could wish that he had not gone on to pillory the seventy times seventh errors in a popular history of mathematics. His own *Historical introduction*, as well as a history of elementary mathematics which he left in manuscript, is better than that.

His technical papers were all directed at the fundamental problem of finite groups, namely, the determination of what groups exist and methods of distinguishing one group from another. He carried

the determination of substitution groups through the transitive groups of degree seventeen, listing only the primitive groups of degrees fifteen and sixteen. Jordan's list of 1872 was far from complete, containing, for example, only twelve of the twenty primitive groups of degree sixteen.

His publications on abstract groups began in 1896 when he gave all the groups of orders smaller than 48. This paper was the first to determine the 15 groups of order 24 and the 51 groups of order 32. Cayley had proved that there are only two groups of each of the orders four and six in 1854 and later that there are five groups of each of the orders eight and twelve. The fourteen groups of order 16 had been determined previously by Kirkman and by J. W. A. Young. Early in 1896 Le Vavasseur had announced that he had found 75 groups of order 32 and had not completed his investigation; later he stated that the number was 50, but finally agreed that 51 was correct. In his last long technical paper, in 1930, Miller determined the 294 groups of order 64.

He was the first to answer many of the questions that every student of finite groups must ask. For example, he was the first to determine that the order of the product of two elements is in no way limited by the orders of the two elements; the first example of a commutator subgroup containing elements which are not commutators was given by Fite in a dissertation prepared under Miller's direction, and Miller later gave an infinite system of such groups; he was the first to prove the simplicity of the multiply transitive Mathieu groups of degrees 22, 23, and 24.

He was always looking for a key that would unlock the problem of the groups of order $n$. In 1902 he gave a method for the determination of all the groups of order $p^m$, and that is the method he used on the groups of order 64. He tried the attack of determining the orders $n$ for which a specified small number of groups exist. A large number of his papers determine the groups which have what look like arbitrary queer sets of properties; all are motivated by the desire to find out what sorts of questions one should ask of a group in order to be able to identify it.

One would have to be intrepid to attempt to extend Miller's work on groups by his methods; on the other hand, very little of it has been extended by any other methods. His determination of substitution groups of low degree could be carried on. Thirty-four years elapsed between his determination of the groups of order 32 and those of order 64 and in the meantime only Potron's incorrect determination of the latter was published. His proof in 1900 that there is no simple

group of odd composite order that can be written transitively on fewer than fifty-one letters is the last definitive word on such groups.

He wrote in 1935 that he was fortunate to have begun the study of groups at the right time. He was fortunate also in his marriage in 1901 to Miss Cassandra Boggs, who was a charming lady and a sympathetic helpmeet for forty years. He was fortunate in his vigorous constitution, which enabled him to write ten papers in 1944 when he was eighty-one and to go to his office regularly through the first five months of his eighty-eighth year. He was fortunate in his mental and moral endowments, which led him to success in a worthy career.

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