of weak duality, which alone can give a meaning to the notion of weak topology, is not touched at all. There are other topics which one misses in chapters 5 and 6, where they would have been in their proper setting, such as the discussion of finite-dimensional topological vector spaces, or of locally compact vector spaces. On the whole, in the reviewer's opinion, the book suffers from a lack of balance, due to the overemphasis laid on chapter 7, at the expense of more relevant matters. However, the author has done a very valuable service to mathematicians in bringing together in book form a large number of results which up to now were scattered in periodicals, and not always very explicitly. His style moreover deserves high praise for its remarkable clarity and thoroughness, so that the book genuinely vindicates its claim of being self-contained, although of course the motivation for the whole theory can only be understood with a considerable background of functional analysis.

J. DIEUDONNÉ

Rekursive Funktionen. By R. Péter. Budapest, Akademischer Verlag, 1951. 206 pp.

Although recursions have been used since Archimedes, and have played a part in foundational investigations by Dedekind (1888), Peano (1891), and Skolem (1923), the theory of recursive functions consists largely of two recent developments, which we call here the "special theory" and the "general theory."

The stimulus to the special theory came from Hilbert's lecture *Über das Unendliche* (published 1926) in which he proposed to attack the continuum problem of set theory by showing that there is no inconsistency in supposing that the number-theoretic functions are all definable by use of forms of recursion associated with the transfinite ordinals of Cantor's second number class. (This program has not yet been carried out, though Gödel in 1938 used an analogous idea to show the consistency of the continuum hypothesis within axiomatic set theory.) For Hilbert's proposal it was necessary to show that higher forms of recursion do give new functions; and the first demonstration of the existence of a function definable by a double recursion but not by use only of simple or "primitive" recursion was given by Ackermann in 1928 in a paper entitled *Zum Hilbertschen Aufbau der reellen Zahlen.* Beginning in 1932, Rósza Péter has published a series of papers, examining the relationship of various special forms of recursion, and showing the definability of new functions by successively higher types of recursion, which establish her as the leading contributor to the special theory of recursive functions.

The general theory of recursive functions dates from the formulation nearly simultaneously (in publications appearing in 1934–1937) of three notions, those of general recursive function (Herbrand-Gödel), λ-definable function (Church-Kleene), and computable function (Turing-Post), which were proved to be equivalent, and were proposed (first by Church in 1936) as exact mathematical equivalents of the intuitive notion of an effectively calculable function.

Both the special and the general theory have developed in close association with applications to logic and the foundations of mathematics.

In line with Mrs. Péter's research interests, the special theory occupies about two-thirds of the present book, namely the first fifteen chapters, with the following titles (translated by the reviewer), and also a part of the last chapter. §1. The usual definition of number-theoretic functions by passage from n to $n+1$. §2. Recursive functions and relations. §3. Course-of-values recursion. §4. Simultaneous recursion. §5. Recursion in which substitutions take place for the parameter. §6. Recursion on several variables. §7. Reductions. §8. Elementary functions. §9. Example of a number-theoretic function which is not primitive recursive. §10. Nested recursion. §11. The diagonal procedure and the multiple recursions. §12. Transfinite recursions. §13. Recursions of higher order. §14. The normal form of the multiple recursions. §15. The "Gödelizing" of recursion of higher order.

The remaining third of the book is devoted to the general theory and applications, with chapter headings: §16. General recursive functions. §17. The explicit form of the general recursive functions. §18. Possibilities for the further simplification of the explicit form. §19. Example of a function which is not general recursive. §20. Computable functions. §21. History and applications. §22. Undecidability effectively of the question, which systems of equations define general recursive functions. §23. The question of the general decidability of the arithmetical problems. §24. Extension of the concept of recursiveness. Applications to analysis.

The aim of the book is primarily to give an elementary exposition of the existing theory, rather than to push into new territory. But the part dealing with the special theory is especially complete, and the latter chapters of this part contain material not covered or covered only summarily in the literature. It is of great value to have in this book for the first time a connected account of the special theory.

In the latter part of the book, the general recursive functions themselves (in the absolute sense) and their explicit form are treated very fully, but the associated notions and applications are treated less fully or only cited. In writing this book Mrs. Péter has carried out a considerable undertaking; and to go further would have constituted a still greater one, and required either a much larger book or a more compact style.

Only a minimum of knowledge of elementary number theory, analysis, and set theory including transfinite ordinals is presupposed and none of mathematical logic. Mrs. Péter aims to make the subject intelligible to the beginner by working out the treatment of many topics (particularly in the special theory) on an example, whence the reader can surmise how the treatment would go in general (or consult the literature). This method has both advantages and disadvantages. No student can complain that he has lost contact with the reality for want of concrete examples; but an unwary reader may be oppressed by the immense amount of detail involved in working out the examples and proofs.

S. C. KLEENE

Infinite matrices and sequence spaces. By R. G. Cooke. London, Macmillan, 1950. 14+347 pp. 42s.

This book, which might be considered a continuation of Chapter XII of Dienes' *Taylor series*, is a useful and welcome adjunct to the recent book by Hardy, *Divergent series*, Oxford, 1949. The overlap between these is slight since the present book is largely concerned with the study of general properties of classes of regular summability transformations.

Chapter 1 introduces several special classes of infinite matrices and certain of the special problems that arise in connection with their algebraic properties such as the ever-present need for the consideration of the validity of interchange of limit operations which leads, for example, to the failure of the associative law. Chapters 2 and 3 deal with the existence of left- and right-hand inverses and annihilators in rings of matrices, the notion of a "bound" (norm) and of weak convergence of sequences of matrices, and the special problem of solving the equations $AX=XD$ and $AX-XA=I$ for specified A and diagonal D. In Chapter 4, the class of K-matrices which transform convergent sequences into convergent sequences is characterized, as well as the subclass of Toeplitz T-matrices and the analogous