§8.7 (which is defined to be the Abel limit) extremely disturbing, especially in the light of the well known fact that Abel and Cesaro summability coincide for bounded sequences.

R. CREIGHTON BUCK

Randwertprobleme und andere Anwendungsgebiete der höheren Analysis für Physiker, Mathematiker und Ingenieure. By F. Schwank. Leipzig, Teubner, 1951. 6+406 pp. $5.47.

The book is a compendium of those portions of mathematical analysis beyond advanced calculus which are of great interest to engineers and physicists. It is an excellent book for physicists and engineers in that it provides a lucid introduction to a good selection of mathematical techniques and theories, with references for further reading, examples, practical applications, and an almost encyclopedic bibliography of applications. From the point of view of the mathematical reader the principal merit of Schwank's book is the wide range and amazingly large number of practical applications described or quoted in the various chapters.

In a preface, G. Hamel explains that Schwank's book is neither a textbook, nor a work of reference. Although the presentation is consequential and covers the ground thoroughly, the book is not as systematic as a textbook, and not as complete as a book of reference. It is written for the physicist or engineer with only a modest mathematical knowledge, and a desire to learn more about some of the more advanced mathematical techniques. The author aims at precision wherever it can be attained. In view of the readers for whom the book was written, it is quite clear that precision and rigour could not be maintained throughout the book, and examples and descriptions are called in when general formulations and proofs would seem out of place.

The material is organized in six chapters and a mathematical appendix.

Chapter I is an introduction to boundary value problems. The vibrations of a string are discussed in detail, and boundary conditions, normal modes of vibration, characteristic values, characteristic functions, orthogonality, Fourier expansion, and other relevant notions are introduced. D'Alembert's solution of the one-dimensional wave equation is also given. The last two sections of this chapter are devoted to the re-formulation of the problem of the vibrating string in terms of integral equations and calculus of variations respectively.

Chapter II is devoted to complex variables. It starts from the beginning with complex numbers, and proceeds to functions of a complex variable, Cauchy-Riemann equations, conformal mapping, complex integrals, power series expansions, singularities, residues. It concludes with an example of the evaluation of a real integral by means of contour integration.

Partial differential equations are discussed in Chapter III. This is the longest chapter in the book (141 p., about one third of the whole book); it was probably the most difficult chapter to plan and write, and quite possibly it is the least satisfactory one to read. Although the potential equation, wave equation, and diffusion equation all occur in this chapter, the classification of partial differential equations into elliptic, hyperbolic, and parabolic equations is not introduced, and characteristics are not mentioned at all. In view of the great difficulty of the subject this omission is understandable, but it is hardly justifiable. Quite apart from the vital importance, in certain branches of applied mathematics, of the distinction between elliptic and hyperbolic regions, and of the method of characteristics, it is a fact that the strikingly different nature of, say, Laplace's equation and the wave equation cannot be appreciated without knowing that they are representatives of two different types of partial differential equations. An extensive study of the potential equation, wave equation, and diffusion equation does not make sense as an introduction to partial differential equations unless one realizes that (roughly speaking) every (linear) partial differential equation behaves like one of the three types. Regarded as an introduction to potential, separation of variables, and Bessel and Legendre functions, the chapter seems adequate.

After a brief introduction, the study of potentials (both two- and three-dimensional) is taken up, the Laplace and Poisson equations are obtained, and some properties of harmonic functions are derived from Green's formula. Applications to fluid mechanics, electricity, and elasticity follow. Separation of variables in spherical polars leads to spherical harmonics and an introduction to Legendre functions (of the first kind).

Separation of variables in the two-dimensional wave equation in polar coordinates leads to an introduction to Bessel functions (all kinds). Separations in elliptic, or ellipsoidal, coordinates are shown to lead to Mathieu and Lamé functions, and transition to integral equations and variational problems are briefly mentioned.

Next follow several conduction of heat problems (mostly treated by separation of variables). In the (brief) section on the three-dimensional wave equation Kirchhoff's formula, Huygens' principle, retarded potentials, and similar matters are mentioned. The chapter concludes with a section on the partial differential equation $\Delta\Delta u = 0$ and its applications.

Integral equations are the subject of Chapter IV. After a brief introduction, Fredholm-type integral equations with degenerate ("polynomial") kernels are discussed. Neumann's expansion (in the general case), Fredholm's theory, and the theory of symmetric kernels, with the classical theorems, follow. The sections on estimates, approximations, and numerical methods will be particularly useful. The two final sections establish connections with boundary value problems and show a number of applications. Reading this chapter one wonders if some of its sections are not too theoretical and if it is justifiable to devote, in a book of this nature, to integral equations about twice as much space as to functions of a complex variable.

Chapter V is on the calculus of variations, and in spite of its brevity is amply illustrated by examples. Euler's differential equation and Legendre's condition are derived, and so is Jacobi's condition. The section on Ritz's method and applications is especially valuable for the readers for whom the book is designed.

Chapter VI is a brief chapter on linear difference equations with constant coefficients and systems of such equations. The section on applications includes an example to show how difference equations can be used for the approximate solutions of differential equations.

The Appendix is a brief summary of some topics, mostly belonging to advanced calculus. It is designed principally to refresh one's memory, although some of its parts could be used to fill in gaps in the mathematical education of the reader.

A. ERDÉLYI

The preparation of programs for an electronic digital computer. With special reference to the EDSAC and the use of a library of subroutines. By M. V. Wilkes, D. J. Wheeler, and S. Gill. Cambridge, Massachusetts, Addison-Wesley, 1951. 10+170 pp. $5.00.

The EDSAC, designed and constructed at the Cambridge University Mathematical Laboratory, was one of the earliest high speed automatic digital computing machines in operation. It has a mercury delay line storage for 1024 words of 17 binary digits with photo-electrically read teletype tape input and teleprinter output. It is a one-