$$X_i = C_i + \sum_{j=1}^{n} X_{ji} \qquad (i = 1, \cdots, n).$$

Labor, the $(n+1)$st good, is thought of as the sole nonproduced good, and its given total X_{n+1} is allocated among the different industries so that

$$X_{n+1} = \sum_{j=1}^{n} X_{j,(n+1)}.$$

Let each good be subject to a production function F_i which is homogeneous of the first order. Equilibrium requires that any C, say C_1, be at a maximum subject to fixed values of X_{n+1} and the other C's. This means that

$$C_1 = F_1(X_{11}, X_{12}, \cdots, X_{1,(n+1)}) - \sum_{j=1}^{n} X_{j1}$$

is to be a maximum subject to

$$C_i = F_i(X_{i1}, X_{i2}, \cdots, X_{i,(n+1)}) - \sum_{j=1}^{n} X_{ji} \qquad (i = 2, \cdots, n),$$

$$X_{n+1} = \sum_{j=1}^{n} X_{j,(n+1)}.$$

Samuelson's theorem asserts that the maximizing values $\{X_{ij}\}$ are such that the $\{X_{ij}/X_i\}$ are independent of the fixed values $C_2, \cdots, C_n, X_{n+1}$.

Linear programming of an economy is a matter of allocating the available resources so as to maximize the utility of the economy. Mathematically the problem is one of maximizing a linear function of several variables constrained by linear inequalities. Dantzig proves that this problem is equivalent to the problem of solving a zero-sum two-person game. More general results are given by Gale, Kuhn, and Tucker, who prove general duality and existence theorems. (For nonlinear programming see Kuhn and Tucker, *Proceedings of the Second Berkeley Symposium on Probability and Statistics*, pp. 481–492.)

An introduction to the volume by Koopmans gives descriptions of the papers and their interrelations.

J. WOLFOWITZ

Classical mechanics. By H. Goldstein. Cambridge, Addison-Wesley, 1951. 12+399 pp.

This book gives an advanced course in classical mechanics, with emphasis on those methods and principles which have proved to be most useful in various branches of modern physics.

The exposition is based on the assumption that the reader is already familiar not only with elementary principles of mechanics, but also with a range of mathematics somewhat beyond the usual undergraduate program, in particular with vector analysis which is used freely throughout the book.

The scope of the book may be most readily presented by giving a brief statement concerning the successive chapters:

The first chapter gives, in 29 pages, a survey of elementary principles of the mechanics of a system of particles, including the Lagrange equations, velocity-dependent potentials, and the dissipation function. This succinct treatment is made possible by use of vector analysis and by omission of applications.

The second chapter (28 pages) starts with Hamilton's principle, derives the Lagrange equations anew, and discusses advantages of a variational principle formulation of the basis of mechanics. The digression on the calculus of variations is, of course, very brief.

The third chapter (33 pages) is devoted to applications to central forces. It includes Kepler's laws, and a discussion of orbits in cases of attraction where the potential is an integral power of the distance. The problem of scattering of particles in a central force field is also analyzed.

In chapter four (50 pages) we find a treatment of the kinematics of rigid body motion. There is a digression on orthogonal transformations and matrix theory, providing mathematical tools to be used systematically in this and following chapters. The Eulerian angles, the Cayley-Klein parameters, the eigenvalue problem, infinitesimal rotations, and the Coriolis force are centers of interest.

Chapter five (42 pages) is devoted to the equations of motion of a rigid body. Tensors and dyadics are introduced, and used effectively. Problems which are discussed include the force-free motion of a rigid body, the heavy symmetrical top with a fixed point, and precession of charged bodies in a magnetic field.

Chapter six (30 pages) introduces special relativity into classical mechanics, with a minimum of historical material. The Lorentz transformation is neatly derived, and the idea of a four-dimensional world advanced. The force and energy equations in relativistic mechanics are found and Lagrangian formulations discussed.

In chapter seven (22 pages) we find a study of the Hamiltonian equations of motion and of the principle of least action. Most of the discussion is devoted to non-relativistic mechanics, but the problem of relativistic mechanics is briefly presented.

Chapter eight (36 pages) is entitled canonical transformations. It includes Poincaré's integral invariants, the Lagrange and Poisson brackets as canonical invariants, and infinitesimal contact transformations. As an application of Poisson brackets a fundamental theorem of statistical mechanics, Liouville's theorem, is proved.

Chapter nine (46 pages) is devoted to the Hamilton-Jacobi theory. A study of action-angle variables leads up to brief remarks on Bohr's quantum theory. A final section shows how the Hamilton-Jacobi formulation is particularly suited to generalize from classical to wave mechanics.

Chapter ten (29 pages) treats of small oscillations of a mechanical system about a point of stable equilibrium. With the customary approximations, the Lagrangian equations appear as a system of linear differential equations with constant coefficients, the constants being restricted by the conditions that the kinetic energy is a positive definite form and that the potential energy has a minimum at the equilibrium point. The analysis of this system is carried out elegantly by free use of matrix theory. A very interesting application is made to free vibrations of a linear triatomic molecule.

Chapter eleven (26 pages) concludes the book with an introduction to the Lagrangian and Hamiltonian formulations for continuous systems and fields. The transition from the discrete variable to the continuous variable is explained with care. Sound vibrations in gases is presented as an example of the Lagrangian formulation. The final section contains a description of fields by variational principles, providing a brief introduction to recent methods in theoretical physics.

At the end of each chapter references are given to well known books, with useful suggestions in each case as to desirable supplementary reading. At the end of the book are listed seventy-one treatises in mechanics and closely related branches of physics and mathematics; these would form a valuable reference library.

The exercises given at the end of the chapters are for the most part extensions of the theory in the text, but some serve to illustrate the use of the theory in solving interesting physical or astronomical problems. They will be difficult for most students, but their solution will add greatly to the understanding and power of the student.

The brevity and clarity arising from the use of vector analysis and matrix theory, and the wise selection of topics and illustrations enable the author to encompass a wide range of fundamental theory in a comparatively small volume. The well-prepared reader will find the presentation lucid and interesting; and if he goes further with his studies he will find this book a most useful volume to have at his elbow.

The few errors noted by the reviewer were of no great importance. Perhaps some mathematicians will be disturbed by an occasional lack of completeness or of precision, but such defects seem trivial by comparison with the high merits of the book as a whole.

The reviewer considers the book to be a very valuable addition to mathematical literature, bridging the gap as it does between the mechanics of the nineteenth century and more recent developments.

E. J. MOULTON

Mechanics. By S. Banach. Trans. by E. J. Scott. (Monografie Matematyczne, vol. 24.) Warszawa-Wrocław, 1951. 4+546 pp. $6.00.

This work was first published in Polish in 1938.

The book is notable for its clarity, and for the completeness of its exposition of the range of material covered. Assuming that the mathematical preparation of the student includes nothing beyond the elements of the calculus, the author undertakes to give as easy a presentation of classical mechanics as possible. To him this means the giving of a logically arranged set of definitions, assumptions, and theorems, with detailed proofs and with numerous illustrative examples. He has carried out his task exceptionally well—at least from the viewpoint of a mathematician.

As a text book the volume would be improved by the inclusion of problems to be solved by the student (none are given), but a teacher may select such exercises from the many which are available in standard works.

The illustrative examples are interesting, and cover a wide range. Thus we find (a) the reactions when a three-legged stool rests on a floor, (b) the fuel load required for an interplanetary rocket, (c) the determination of the mass of a planet, and so on. Chapter VI, on statics of a rigid body, is particularly designed for students of technology, and is so written as to be independent of much of the material in the preceding chapters.

The first chapter is devoted to that portion of the algebra of vectors which is most important in the study of mechanics. This tool is then freely and effectively used throughout the remainder of the book.

The mathematical quality of the exposition may be suggested by the following introductory paragraph: "Time. In kinematics, in addition to known geometric concepts, there arises the concept of time.