$=x \log x+(2\gamma-1)x+\Delta(x)$. It was shown by Dirichlet that $\Delta(x) = O(x^{1/2})$ and better results have been obtained since. The function $\Delta(x)$ is related by the Mellin transform to $\zeta^2(w)/w$ in the critical strip. The divisor problem, that is, the order of $\Delta(x)$, and generalizations are considered in chapter twelve.

In chapter thirteen the Lindelöf hypothesis is assumed to be true and consequences of it are investigated and in chapter fourteen consequences of assuming the truth of the Riemann hypothesis are investigated.

The final chapter is on calculations relating to the zeros of $\zeta(s)$. After indicating how the early zeros are shown to lie on the critical line the author observes that if the Riemann hypothesis is false this could be shown by using modern calculating devices.

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Advances in applied mechanics. Vol. II. Ed. by R. von Mises and T. von Kármán. New York, Academic Press, 1951. 10+233 pp. $6.50.

Kármán and Lin's paper, *On the statistical theory of isotropic turbulence*, represents the views of two distinguished specialists in the current semi-empirical turbulence theory. It purposes to clarify the notion of spectral similarity in flows where there is turbulent diffusion of energy. The authors divide the spectrum into three frequency ranges, in each of which only two parameters are considered significant; similarity laws follow by the usual dimensional argument. Since this paper is of a nonmathematical character, the reviewer does not detail its contents here.

Kuerti's survey, *The laminar boundary layer in compressible flow*, is successful in its expressed aim: "Although the mathematics used in the boundary layer literature presents no particular conceptual difficulties, it is complicated on account of the number of parameters involved and because of the different approaches tried by different authors. The study of the original literature thus requires a patient reader. Under these circumstances it seemed desirable to put together what may be called a guide to rather than a review of the existing literature on the subject." Attention is restricted to steady plane flow of a perfect gas with constant specific heats, the viscosity and thermal conduction being assumed functions of temperature only, and the Prandtl number constant. The bibliography of thirty items seems to be complete through 1948.

The author does not mention any work of his own, and there appears to be no original contribution made by the article. One notes, for example, that the derivation in Part II is based upon the usual order of magnitude assertions, even more disagreeable and even less convincing in the case of a compressible fluid than for an incompressible one. It would seem to be possible to extend the method of Lin [A.D. Michal, *Matrix and tensor calculus*, New York, see Chap. 18], which, while purely formal, at least makes clear what is assumed and what one might try to prove. The major part of the article classifies and presents "the transformations that have evolved in the search for practical solutions of the boundary layer problem," accompanied by numerical graphs. The mathematically inclined reader might gather some suggestions of possible approaches to a future strict treatment, but apparently work on this subject so far has been of a rather heuristic type. While the author's careful ordering and scrupulous presentation provide a reliable encyclopedic article, they are insufficient to arouse the reader's interest in this dry topic.

Clark and Reissner's paper, *Bending of curved tubes*, makes a solid contribution to engineering elasticity theory. The problem stated in the title, whose history is summarized in the introduction, is treated for the first time as a problem in the theory of thin toroidal shells of revolution. The appropriate differential equations were derived earlier by E. Reissner [Proc. Nat. Acad. Sci. U.S.A. vol. 35 (1949) pp. 204–208]. The meridian curve of the torus is expressed in the form $r=r(\xi)$, $z=z(\xi)$, where r and z are Cartesian coordinates in a meridian plane. The problem differs from those discussed in books on shell theory in that an azimuthal displacement $v=kr\theta$ is assumed, although the stress system is taken as axially symmetric.

The major part of the paper deals with the circular torus $r=a+b\sin\xi$, $z=-b\cos\xi$. Calling β the change of the latitude angle and retaining only the terms of lowest order in b/a, the authors reduce the equations of the problem to the form

$$\beta'' + \mu\,(\sin\xi)\psi = 0,$$
$$\psi'' - \mu\,(\sin\xi)\beta = \mu k\cos\xi,$$

where ψ is a dimensionless stress function and $\mu\equiv(12(1-\nu^2))^{1/2}b^2/(ah)$, h being the shell thickness. They set up a formal Fourier series solution, in which the coefficients are to be obtained by solving an infinite system of linear equations. They show that previous formal results concerning the problem correspond to taking the first few terms in one of two different ways. The series solution appears to be rapidly convergent for small μ. To supplement it the authors obtain an asymptotic solution for large μ.

If the applied moment m be written as $\rho EIk/a$, the authors find for the "rigidity factor" ρ the expressions

$$\rho = \begin{cases} \dfrac{16 + 5\mu^2/36}{16 + 41\mu^2/36 + \mu^4/576} & \text{for small } \mu, \\ \dfrac{2}{\mu} & \text{for large } \mu. \end{cases}$$

These formulae together give excellent agreement with independently computed numerical values based on a six term Fourier polynomial. The authors' results include simple approximate formulae for the stress and displacement distributions. In the last part of the paper they obtain asymptotic solutions for the elliptic torus.

The most interesting and stimulating article is Neményi's, *Recent developments in inverse and semi-inverse methods in the mechanics of continua.* He calls inverse "an investigation of a partial differential equation of physics if in it the boundary conditions (or certain other supplementary conditions) are not prescribed at the outset. Instead, the solution is defined by the differential equation and certain additional analytical, geometrical, kinematical, or physical properties of the field. In the semi-inverse method some of the boundary conditions are prescribed at the outset, whereas others are left open and obtained indirectly." This article does not attempt to present the subject exhaustively, but aims rather "to elucidate the nature, value, and the potentialities of this approach." Incidentally, however, it explains the essence of nearly all the main contributions.

The paper is divided into sections of about equal length concerning inviscid incompressible fluids, perfect gases, elastic bodies in equilibrium, and plastic bodies. Much of the work mentioned is very recent, some unpublished. The author in many cases is able to correlate, compare, and contrast the various investigations, often in different fields. At the end of the paper is a table summarizing results known up to the present on five semi-inverse problems in seven domains of continuum mechanics. Among the more interesting investigations cited are those of Taylor and Trkal on decaying motion of a viscous fluid; those of Tollmien, Prim, and Neményi-Prim on limiting lines, problems of invariance, and "generalized Beltrami flows" in gas dynamics; and those of Neményi on an "influence principle" and on stress trajectories in elasticity.

The author emphasizes the value of simple exact solutions as typical and suggestive illustrative cases, and he shows that such solutions have been or can be obtained by inverse or semi-inverse methods in many cases. His examples, too numerous to list here, substantiate also the following conclusions. "1. Inverse and semi-inverse methods [may] lead to solutions of important boundary value problems." "2. They [may] lead to the discovery of unsuspected discontinuities, limitations, or general field properties. . . ." "3. [They] may settle existence questions in a positive sense, or may decide a uniqueness problem in a negative sense." "4. [They] are essential for the comparative study of the differential equations of the various problems of mechanics."

The last and longest paper, *Theory of filtration of liquids in porous media*, by Polubarinova-Kochina and Falkovich, is a translation of an article in Prikladnaia Matematikha i Mekhanika vol. 11 (1947) pp. 629–677. There is no indication that the translation was made with the author's approval or even cognizance. The contrary is suggested by the odd remark at the head of the references: "The original reference style has been retained as presumably the most effective for the purpose of identification to Russian libraries. The abbreviations which the translator has been able to decode are explained below. . . ." One wonders which readers will be able to secure the assistance of a Russian librarian to locate the originals. The references are in fact absolutely incomprehensible in many instances (e.g. Izv. NIIG, GONTI, Nauchníe zap. MGMI), erroneous in some others. This defect is aggravated by the extreme condensation of the article, which is hardly more than a list of results, so that it is only rarely possible to see how one step follows from the preceding. Even the figures are not always understandable. The interested reader will have to obtain the sources, and for most of these he will have to carry out a tedious detective work in bibliographical indices.

There are short sections on motion which do not obey Darcy's law, three-dimensional motions, and unsteady motions, but the bulk of the article deals with steady plane flow of an incompressible fluid obeying Darcy's law:

$$\vec{v} = -\kappa \operatorname{grad}\left[\frac{p}{\rho g} + y\right] = \operatorname{grad}\phi,$$

where $\vec{v}$ is the filtration velocity, κ the filtration coefficient, and y the vertical coordinate. The velocity potential ϕ is harmonic, so that the variety of mathematical techniques developed in plane potential theory are all applicable. The authors choose to deal only with Russian work, mainly exact solutions. There are a few references to Muskat's book [*The flow of homogeneous fluids through porous media*, New York and London, 1937], none to Neményi's [*Wasserbauliche*

Strömungslehre, 1933]. One is impressed by the variety of problems having both mathematical and practical interest. Because of the quantity of the material and the incomprehensible way in which it is presented, the reviewer makes no further attempt at summary. He notes passing reference to some investigations of an interesting mathematical character. (1) A justification of a method of series development given by Kufarev and Vinogradov [Akad. Nauk SSR Doklady vol. 57 (1947)]. (2) Qualitative theorems by Lavrentiev [*Conformal mapping*, OGIZ, 1946] concerning flow under a dam with cut-offs, situated over an impermeable rock; increasing the length of a cut-off increases pressure under the dam immediately upstream, decreases it downstream, and the most effective means of reducing the exit velocity is to lengthen the cut-off which is farthest downstream. No indication of how to obtain these results is given.

The language in the whole volume is the next poorest the reviewer has seen in a mathematical publication issued in an English-speaking country. The translator of the Russian article preserves a terse and ragged style, stringing together with the aid of parentheses (sometimes misplaced) unusual expressions which, while possibly elegant in Russian, ring oddly in English. In the article on shells infinitives are split and participles suspended by rule. The paper on inverse methods is an example of original grammar set off by stochastic punctuation. The paper on the boundary layer is written almost entirely in the passive voice. While the format of the book is pleasant, the setting is careless: some parts have dropped out of the formulae, some of the letters are broken or mis-set, some blank slugs have printed black bars here and there, there are a good many textual misprints, and the running heads for one article read "Continua mechanics." While at first sight it would seem that all formulae are intentionally left altogether unsupported by punctuation, careful search reveals a favored few which are set off by commas, periods, and even in one case a semicolon. In the copy sent for review there are holes in some of the pages. One observes also that although the volume was issued in the middle of 1951, apart from notes added in proof the bibliographies seem to indicate that all the articles were completed in 1949 or earlier.

The publishers display some audacity in offering this slim and carelessly printed little book of 233 pages, nearly a third of which is a translation, for a price equal to or greater than the cost of a volume of a first rate research journal containing three or four times as much material.

C. Truesdell