Lezioni sulla teoria moderna dell'integrazione. By M. Picone and T. Viola. Edizioni Scientifiche Einaudi, 1952. 404 pp. 5000 lire.

The word "modern" in the title is to be taken with a grain of salt; the book could just as well have been written fifteen or twenty years ago. The notation and the point of view are old-fashioned; the exposition is verbose and occasionally vague. Integration, for instance, is defined in terms of limits on directed sets, but the definition of directed set is couched in the language of "variables" taking "well determined" numerical values. The bibliography contains no reference to Radon or to Nikodým. As a matter of fact, except for a half dozen references to relatively recent papers by Italian mathematicians, no reference is made to any part of the mathematical literature of the last fifteen years.

The material itself is essentially standard: it includes interval functions, Riemann-Stieltjes and Lebesgue-Stieltjes integration, Egoroff's theorem, theorems on term-by-term integration, the Riesz-Fischer theorem, Fubini's theorem, bounded variation in the sense of Vitali, integration by parts, absolute continuity, differentiation, and change of variables. All the sets considered are subsets of finite-dimensional Euclidean spaces. Since the authors took great care to make the proof rigorous, the book might be useful for a beginning student who is anxious to have all the epsilons spelled out for him.

PAUL R. HALMOS

Numerische Behandlung von Differentialgleichungen. By L. Collatz. (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete, vol. 40.) Berlin, Göttingen, Heidelberg, Springer, 1951. 13+458 pp., 1 plate. 45 DM; Bound, 48 DM.

Few are the books devoted exclusively to the numerical treatment of differential equations. One may recall Levy and Baggott in English, Panov in Russian, von Sanden in German and perhaps a few others—say Runge and Willers' Encyclopedia article. These are brief, limited in scope, handbooks rather than treatises. Here now is a more amibitious undertaking, namely to make a comprehensive study of numerical methods for both ordinary and partial differential equations. Of course such an ambition cannot be fully realized in a book of reasonable size, and inevitably some topics are touched only lightly. For example one notes that scant attention is given to those methods for ordinary equations which use open type quadrature formulas to predict and corresponding closed type formulas to correct, and where the correction provides a measure of the truncation error. For partial differential equations even greater limitations are encountered, so that instead of giving a general theory the author is perforce content to exhibit typical methods illustrated by selected practical problems in physics and engineering.

Nevertheless this text contains the most thorough and comprehensive presentation of numerical methods for differential equations that has yet been published. The term "numerical methods" is liberally interpreted, and far from being content with a mere description of procedures the author gives a detailed theoretical analysis of each process described. The crucial question of error control has received the careful attention that its importance merits.

A quick glance through the 458 pages discovers the major headings: (I) initial-value problems for ordinary equations; (II) boundary-value (i.e., two end-point) problems for ordinary equations; (III) initial-value problems and initial plus boundary-value problems for partial equations; (IV) boundary-value problems for partial equations; and (V) integral and functional equations.

A second and longer look reveals the wealth of material covered. Under I we find first the necessary background material of difference formulas and quadrature formulas; then come simple methods of summation, error estimates, the Runge-Kutta methods, methods using finite differences, Taylor's series, quadrature formulas, the "extrapolation" process, the "interpolation" process, and methods of iteration. Each of these is applied to both first and higher order equations and the error for each is examined. Under II appear linear boundary value problems, use of finite differences, relaxation, iteration, the minimum principle, the Ritz method, least square methods, Galerkin's method, step-by-step methods, and methods for calculating eigenvalues.

Chapter II takes up those partial differential equations, principally of parabolic or hyperbolic type, where the initial values are known at $t=0$, where the space boundary values are given, and for which the solution is computed step-by-step for successive values of t. The method of difference equations is applied to the values of the unknown at lattice points of a rectangular net and the dangers due to incorrect formulation of the difference equation are vividly exhibited. Further topics include choice of the mesh size, propagation of errors, graphical constructions for solving the difference equations, and the use of characteristics.

In IV the author takes up those partial differential equations, mainly of elliptic type, where the solution is not obtainable by a step-by-step process with respect to one variable, but is found as the solution of a set of simultaneous equations. The method of difference equations for a square lattice is explained and applied to such examples as the Dirichlet problem. Further topics include proof of existence and uniqueness of a solution, estimate of error, convergence of the iteration process for solving the equations, relaxation, triangular and hexagonal nets, method of least squares, the Ritz method, Trefftz' method, use of series of known functions, eigenfunctions, and eigenvalues.

Chapter V deals mainly with integral equations, and explains methods where the integrals are replaced by sums, methods of iteration, use of series, cases of singular integral equations, Volterra's equation, and closes with some examples of functional equations.

The text is enlivened by interesting numerical examples, some short biographical notes, and appropriately has as its frontispiece a photograph of C. Runge.

W. E. MILNE

Linear algebra and matrix theory. By R. R. Stoll. New York, McGraw-Hill, 1952. 15+272 pages. $6.00.

In this treatment of the algebra of matrices with elements in a field, the author adopts the modern viewpoint that the linear transformations are the basic objects for study, rather than their carriers, the matrices. Abstract algebraic concepts are introduced as they arise. (It should be observed that this "background of modern algebra" consists of really elementary concepts, and does not include, for example, such a powerful tool as the Krull-Schmidt theorem.) A number of the usual proofs have been replaced by fresh ones. For example, the reduction of symmetric matrices is done so carefully that Kronecker's reduction is a very easy consequence. The important similarity problem is solved by adapting the arguments developed for division rings to the present case. (Cf., for example, N. Jacobson, *The theory of rings*, Chap. 3.) The discussion of normal transformations in (finite-dimensional) unitary and Euclidean spaces is more thorough than usual.

The author has naturally left to the reader the duty of supplying some of the details. Besides these "natural" exercises, there are ample lists of more formal exercises and many illustrative examples. There is an index and a useful list of the special symbols employed.

This text is heartily recommended to the teacher of matric theory who feels obliged to introduce his students at the same time to basic concepts of modern algebra.

M. F. SMILEY