THE APRIL MEETING IN CHICAGO

The four hundred ninety-first meeting of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 24–25, 1953. Approximately 230 persons attended including the following 191 members of the Society:

Brian Abrahamson, W. R. Allen, W. F. Ames, Louis Auslander, D. G. Austin, J. H. Bell, Gerald Berman, R. H. Bing, R. L. Blair, G. M. Bloom, R. P. Boas, W. M. Boothby, Raoul Bott, Joseph Bram, G. U. Brauer, C. C. Braunschweiger, R. H. Bruck, R. C. Buck, R. H. Cameron, Buchanan Cargal, K. H. Carlson, W. B. Caton, S. S. Chern, E. W. Chittenden, H. M. Clark, M. D. Clement, Harvey Cohn, C. W. Curtis, H. J. Curtis, John DeCicco, J. C. E. Dekker, R. F. Deniston, W. E. Deskins, R. J. DeVogelaere, V. E. Dietrich, J. A. Dieudonné, N. J. Divinsky, J. L. Doob, L. A. Dragonette, R. J. Driscoll, Benjamin Epstein, H. P. Evans, R. L. Evans, H. S. Everett, Walter Feit, Chester Feldman, Jacob Feldman, J. S. Frame, Evelyn Frank, W. G. Franzen, C. G. Fry, L. E. Fuller, Leonard Gillman, Seymour Ginsburg, Casper Goffman, H. E. Goheen, Michael Golomb, A. W. Goodman, S. H. Gould, L. M. Graves, R. L. Graves, Harold Greenspan, L. W. Griffiths, V. G. Grove, William Gustin, M. M. Gutterman, Franklin Haimo, Marshall Hall, Jr., P. R. Halmos, Camilla Hayden, R. G. Helsel, Melvin Henriksen, I. N. Herstein, Fritz Herzog, J. J. L. Hinrichsen, J. G. Hocking, R. E. von Holdt, S. S. Holland, T. C. Holyoke, B. E. Howard, S. P. Hughart, H. K. Hughes, Ralph Hull, W. E. Jenner, Meyer Jerison, H. T. Jones, K. E. Kain, Samuel Kaplan, Irving Kaplansky, N. D. Kazarinoff, J. L. Kelley, R. B. Kellogg, J. B. Kelly, L. M. Kelly, J. H. B. Kemperman, Erwin Kleinfeld, Fulton Koehler, Jacob Korevaar, W. C. Krathwohl, A. H. Kruse, M. Z. Krzywoblocki, R. G. Kuller, H. G. Landau, C. G. Latimer, G. F. Leger, Jr., G. R. Lehner, B. L. Lercher, W. J. LeVeque, W. D. Lindstrom, A. J. Lohwater, R. D. Lowe, Richard McKinney, Saunders MacLane, Arne Magnus, E. A. Michael, D. W. Miller, Marian A. Moore, M. D. Morley, S. T. C. Moy, S. B. Myers, Isaac Namioka, August Newlander, Jr., E. A. Nordhaus, R. Z. Norman, Rufus Oldenburger, C. C. Oursler, Gordan Pall, M. H. Payne, M. H. Pearl, Sam Perlis, G. B. Price, A. L. Putnam, C. R. Putnam, Gustave Rabson, O. W. Rechard, W. T. Reid, Haim Reingold, R. B. Reisel, R. W. Rempfer, Daniel Resch, R. K. Ritt, Alex Rosenberg, M. A. Rosenlicht, Arthur Rosenthal, E. H. Rothe, J. M. Sachs, D. E. Sanderson, R. G. Sanger, R. L. San Soucie, A. C. Schaeffer, O. F. G. Schilling, Morris Schreiber, W. R. Scott, W. T. Scott, I. E. Segal, Esther Seiden, M. A. Seybold, D. H. Shaftman, M. E. Shanks, K. S. Shih, Annette Sinclair, Abe Sklar, M. F. Smiley, K. T. Smith, Jerome Spanier, E. J. Specht, George Springer, H. E. Stelson, B. M. Stewart, Fred Supnick, T. T. Tanimoto, J. S. Thale, H. P. Thielman, E. A. Trabant, R. N. Van Norton, Andrew Van Tuyl, Bernard Vinograde, R. D. Wagner, G. L. Walker, L. M. Weiner, P. J. Wells, George Whaples, L. R. Wilcox, F. B. Wright, Jr., F. M. Wright, Oswald Wyler, L. C. Young, P. M. Young, J. W. T. Youngs, Daniel Zelinsky, Antoni Zygmund.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor J. A. Dieudonné of the University of Michigan addressed the Society on the topic *Recent developments in the theory of topological vector spaces*. Professor P. R. Halmos presided at this session.

There was a total of seven sessions for the presentation of contributed papers; these were held on Friday and Saturday morning and on Friday afternoon. Presiding officers were: Professors R. C. Buck, S. S. Chern, G. B. Price, E. H. Rothe, A. C. Schaeffer, M. F. Smiley, and G. L. Walker.

The local department entertained the Society with a tea held in the Common Room of Eckhart Hall on Friday afternoon.

Abstracts of the papers read follow. Those having the letter "*t*" after their numbers were read by title. Paper number 445 was presented by Dr. Henriksen, number 459 by Professor Smith, number 462 by Mr. Fagen, number 467 by Dr. Rabson, and number 490 by Mr. Norman. Mr. Mostert was introduced by Professor M. E. Shanks and Mr. Brown by Professor P. C. Rosenbloom.

Algebra and Theory of Numbers

441*t*. C. G. Chehata: *Commutative extension of partial automorphisms of groups.*

Let ϵ be an isomorphism which maps a subgroup A of the group G onto a second subgroup B (not necessarily distinct from A) of G; then ϵ is called a "partial automorphism" of G. A partial (or total) automorphism ϵ^* "extends" or "continues" a partial automorphism ϵ if ϵ^* is defined for, at least, all those elements for which ϵ is defined and moreover ϵ^* coincides with ϵ where ϵ is defined. It is known (G. Higman, B. H. Neumann, and Hanna Neumann, J. London Math. Soc. vol. 24 (1949)) that any partial automorphism of a group can always be extended to a total automorphism of a supergroup, and even to an inner automorphism of a supergroup. Moreover any number of partial automorphisms can be simultaneously extended to inner automorphisms of one and the same supergroup. In this paper sufficient conditions are derived which ensure that two partial automorphisms can be extended to commutative automorphisms of a supergroup. Although these conditions are too restrictive to be necessary as well, they are sufficiently wide to give the following special case as a corollary: If ϵ maps $A\subseteq G$ isomorphically onto $B\subseteq G$ and if ν maps $C\subseteq G$ isomorphically onto $D\subseteq G$, and if $A\cap C=B\cap C=A\cap D=B\cap D=\{1\}$, then ϵ and ν can be extended to commutative automorphisms of a supergroup. (Received February 18, 1953.)

442*t*. C. G. Chehata: *Simultaneous extension of partial endomorphisms of groups.*

Let ϵ be a homomorphic mapping of some subgroup A of the group G onto a subgroup B (not necessarily distinct from A) of G; then we call ϵ a "partial endomorphism" of G. A partial (or total) endomorphism ϵ^* "extends" or "continues" a partial endomorphism ϵ if ϵ^* is defined for at least all those elements for which ϵ is defined and moreover ϵ^* coincides with ϵ where ϵ is defined. In a paper by B. H. Neumann and Hanna Neumann (Proc. London Math. Soc. (3) vol. 2 (1952)) necessary and sufficient conditions are given for the extension of a partial endomorphism of G to a total endomorphism of a supergroup of G. In this paper these conditions are generalized to apply first to the simultaneous extension of two partial endomorphisms of G to total endomorphisms of one and the same supergroup of G, and then, using transfinite induction, to any well-ordered set of partial endomorphisms. The conditions here obtained are again necessary and sufficient. A number of special consequences are then derived. (Received February 18, 1953.)

443. Harvey Cohn: *Approach to Markoff's minimal forms through modular functions.* Preliminary report.

The author considers the subgroup of the modular group generated by P: $-1/(z+3)$, Q: $(2z-1)/(-z+1)$. Its fundamental domain is of genus 1, uniformized by $J(z)=[p'(u)]^2=4p^3(u)+1$. Markoff's forms correspond to the fixed points of the transformations P, Q, PQ, P^2Q, etc., generated as follows: Start with $[P, Q]$, and obtain from it four new pairs $[PQ, P]$, $[PQ, Q]$, $[QP, P]$, $[QP, Q]$. The transformations in question are found by repeating the process indefinitely. (Research sponsored by the Army Office of Ordnance Research.) (Received March 9, 1953.)

444t. W. F. Eberlein: *The inverse Fourier transform.*

Let G be a locally compact Abelian group. The Fourier transformation $F(f)=g$, where $g(s)=\int_G(x, s)f(x)dx$, maps $L^1(G)$ into a dense subspace of $C(G^*)$, the continuous functions on the dual group G^* vanishing at infinity. General operator theoretic methods introduced by the author in an earlier paper [Ann. of Math. (2) vol. 47 (1946) pp. 688–703] are applied to a study of the inverse transformation F^{-1}. In particular, F^{-1} is lower semi-continuous, whence F admits a natural extension defined on a space of measures on G. A by-product of the analysis is a simple proof of Segal's theorem [Acta Sci. Math. Szeged vol. 12, Part B (1950) pp. 157–161] that $F[L'(G)] = C(G^*)$ if and only if G is finite. (Received March 11, 1953.)

445. Leonard Gillman and Melvin Henriksen: *On the prime ideals of rings of continuous functions.* I.

Let $C=C(X, R)$ denote the ring of all real-valued continuous functions on a completely regular space X, $Z(f)$ the set of zeros of $f\in C$; an ideal I of C is *fixed* if $\bigcap_{f\in I}Z(f)$ is nonempty, otherwise I is *free* (Hewitt, Trans. Amer. Math. Soc. vol. 64 (1948) pp. 45–99). (1) Every prime fixed ideal of C is contained in a unique maximal ideal (the corresponding statement for free ideals holds if X is normal). (2) The subring of all bounded $f\in C$ always has nonmaximal prime ideals, unless X is finite. Denote by M_p the maximal fixed ideal consisting of all $f\in C$ such that $f(p)=0$; if M_p contains no prime ideal other than itself, then p is called a *P-point* of X. (3) p is a P-point if and only if every continuous function vanishing at p vanishes on a neighborhood of p. (4) p is a P-point if and only if every countable intersection of neighborhoods of p contains a neighborhood of p. (5) There exist spaces X without isolated points in which both the set of all P-points, and its complementary set, are dense in X. (Received April 23, 1953.)

446t. Leonard Gillman and Melvin Henriksen: *On the prime ideals of rings of continuous functions.* II.

Terminology is as in preceding abstract. If every point of (the completely regular space) X is a P-point, then X is called a *P-space.* (1) The following statements are equivalent. (i) X is a P-space. (ii) Every $Z(f)$ is open and closed. (iii) $C(X, R)$ is a regular ring. (iv) Every prime ideal (free or fixed) of $C(X, R)$ is maximal. (v) Every countable intersection of open sets of X is open, i.e., X is ω_1-additive (Sikorski, Fund. Math. vol. 37 (1950) pp. 125–126). (vi) Every ideal of $C(X, R)$ is m-closed (for definition see Hewitt, loc. cit. p. 49). (vii) Every prime fixed ideal of $C(X, R)$ is m-closed. (2) Finite products of P-spaces are P-spaces, but countable products need not be. (3) Every locally countably compact, or perfectly normal, or pseudo-compact (Hewitt, loc. cit. p. 67) P-space is discrete, as is every P-space satisfying the first axiom of countability. (4) Every P-space of power $\leq \aleph_1$ is normal. (5) For every $\mathfrak{m} \geq \aleph_1$ there is a normal P-space of power $\mathfrak{m}$ without isolated points. (6) For every $\mathfrak{n} \geq \aleph_2$ there is a non-normal P-space of power $\mathfrak{n}$ without isolated points. (Received April 23, 1953.)

447. Franklin Haimo: *Dihedral-type groups.* Preliminary report.

A generalization of the dihedral groups is defined; and, in a special case, its relation to the holomorph of a group is developed. An inverse limit of the set of dihedral groups can be constructed, a limit which turns out to be a centerless, 2-step, metabelian infinite group. All endomorphisms of a given dihedral group are enumerated, and the automorphisms are represented faithfully as a group of 2 by 2 matrices of integers. (Received March 11, 1953.)

448t. Marshall Hall, Jr.: *On a theorem of Jordan.*

In 1872 Jordan showed that a finite quadruply transitive group in which only the identity fixes four letters must be one of the groups S_4, S_5, A_6, or the Mathieu group on 11 letters. This theorem is generalized in two ways in this paper. Quadruply transitive groups are considered on any number of letters, finite or infinite, and instead of assuming that the subgroup fixing four letters is the identity alone, it is assumed that it is a finite group of odd order. The conclusion is essentially the same as Jordan's, there being only one other group satisfying this hypothesis, the alternating group A_7. (Received March 13, 1953.)

449. T. C. Holyoke: *Non-existence of certain transitive extensions.*

It is proved that the only dihedral groups (including the "dihedral" group with an infinite cyclic subgroup) possessing transitive extensions, when represented as permutations of cosets of a reflection subgroup, are those of orders 4, 6, and 10. The proof is based on a theorem given by the author in a previous paper (Amer. J. Math. vol. 74 (1952) pp. 787–796). (Received March 9, 1953.)

450. G. F. Leger, Jr.: *A note on derivations of Lie algebras.*

If L is a Lie algebra and V is an ideal of L, we say that L splits over V if there exists a subalgebra T of L so that $L = T + V$ and $T \cap V = (0)$. As is well known, derivations of L form a Lie algebra $D(L)$ and the inner derivations form an ideal $I(L)$ in $D(L)$. Let R denote the radical of L, $D(R)$ the derivations of R, and $I(R)$ the inner derivations of R. Theorem: If $D(R)$ splits over $I(R)$, then $D(L)$ splits over $I(L)$. An example is given of a nilpotent Lie algebra L such that $D(L)$ does not split over $I(L)$. (Received March 12, 1953.)

451. R. L. San Soucie: *Right alternative division rings of characteristic 2.*

L. A. Skornyakov (Izvestiya Akad. Nauk SSSR. Ser. Mat. vol. 15 (1951) pp. 177–184) has shown that right alternative division rings of characteristic not two are alternative. Fundamental to his proof is the lemma that such rings satisfy the identity

(*)$x(yz\cdot y)=(xy\cdot z)y$, for all x, y, z. R. H. Bruck has an example (unpublished) of a right alternative division ring, R, of characteristic two, which is not alternative. R, however, does not satisfy (*). In this paper, it is shown that a right alternative division ring of characteristic two is alternative if and only if it satisfies (*). Moreover, it is proved that, if R is a right alternative division ring of characteristic two having the right inverse property, then R satisfies (*), and, hence, is alternative. (Received March 10, 1953.)

452. W. R. Scott: *Some theorems on infinite groups.*

If $i(H)$ is the index of a subgroup H of a group G, then $i(\bigcap H_\alpha)\leqq\pi i(H_\alpha)$, $\alpha\in S$. This generalizes Poincaré's theorem. Other theorems on indices are given. It is shown that the set $L(\infty, H)$ of elements of G of infinite order with respect to H has order 0 or $o(G)$. A generalization is also given. The subgroup K of an infinite group G, as defined in W. R. Scott (*Groups and cardinal numbers*, Amer. J. Math. vol. 74 (1952) pp. 187–197) is shown to be overcharacteristic, i.e. $G/K\cong G/H$ implies $K\subseteq H$. Characterizations are obtained for those Abelian groups G all of whose subgroups H (factor groups G/H) of order equal to $o(G)$ are isomorphic to G. Again the Abelian groups, all of whose endomorphisms σ such that $o(G\sigma)=o(G)$ are onto, are found. (Received March 12, 1953.)

453. B. M. Stewart: *Sodd numbers.*

Let $\alpha(m)$ indicate the number of integers which can be represented as a *sum of distinct* (positive) *divisors* of the integer m. Define an even number m to be *sodd* if $\alpha(m)=\sigma(m)$; and define an odd number m to be *sodd* if $\alpha(m)=\sigma(m)-2$. In this paper theorems are given for generating new sodd numbers from known sodd numbers. In particular, the product of the first k odd primes is a sodd number for $k\geqq 5$; and these sodd numbers are used to solve Starke's variation of the Egyptian fraction problem, #4512, Amer. Math. Monthly. The *sodd-function*, defined by $s(m)=\alpha(m)/\sigma(m)$, is shown to be everywhere dense on the interval from 0 to 1. (Received February 24, 1953.)

454*t*. W. R. Utz: *A note on the Scholz-Brauer problem in addition chains.*

An addition chain for the positive integer n is a sequence of integers $1=a_0<a_1<a_2\cdots<a_r=n$ such that $a_i=a_j+a_s$ for $0\leqq j\leqq s<i\leqq r$. Let $l(n)$ denote the minimal value of r for which there is an addition chain for n. A. Scholz (Jber. Deutschen Math. Verein. vol. 47 (1937) p. 41) suggested that for all integers $q>0$, $l(2^q-1)\leqq q-1+l(q)$. A. T. Brauer (Bull. Amer. Math. Soc. vol. 45 (1939) pp. 736–739) established this inequality provided the $l(q)$ on the right in the inequality is the minimal length of chains in which $a_i=a_{i-1}+a_s$, $0\leqq s<i-1$. However, the problem as originally stated seems to be unsolved. The principal result of the present paper is the proof of the Scholz inequality for all q of the form $2^\alpha+2^\beta$, α and β being positive integers. (Received March 9, 1953.)

455. Bernard Vinograde: *Invariants of matrices under row- and column-constant addition.*

Over a division ring K of zero characteristic, the total matrix ring K_r maps isomorphically into the subring K_n^0 (of K_n, $n>r$) whose rows and columns individually add to zero, and the mapping is onto when $n=r+1$. The elements of K_r furnish the invariants of an equivalence relation in K_{r+1} defined by: $A\sim B$ if and only if $p(A-B)q$

$=0$ for all vectors p, q whose components sum to zero (hence $A-B$ is the sum of row-constant and column-constant elements of K_{r+1}). Suitably modified statements apply to the module of rectangular matrices. In particular, invariantively associated with each class is a pair of linear spaces and a pair of sets of parallel linear varieties by means of which can be established the identity $XAY-UAV=(X-U)B(Y-V)$, where: B is any matrix equivalent to A, $U(V)$ is a fixed column (row)-constant nonzero idempotent, and $X(Y)$ is a variable column(row)-constant nonzero idempotent. Over the real numbers, this is the same as an identity of H. W. Mills. It associates a support function invariantively with each class, a fact that intimately relates it to the theory of finite games, as will be explained elsewhere. (Received March 12, 1953.)

456. L. M. Weiner: *Algebras based on linear functions.*

Starting from an algebra A with products xy and a linear function f on the vector space of A to the base field F, an algebra A' is constructed over the same vector space as A by defining products in A' to be $x\cdot y=f(x)y+f(y)x+xy$. This type of algebra arises in the study of Lie admissible algebras. When A is associative, a necessary and sufficient condition that A' be associative is $f(x)f(y)=-f(xy)$. Assuming A to be power associative, it is necessary either that A be anticommutative or that f satisfy $f(x)f(y)=-[f(xy)+f(yx)]/2$ in order that A' be power associative. Either of these conditions is sufficient for the power associativity of A'. A necessary and sufficient condition that flexibility carry over from A to A' is $f(xy)=f(yx)$. In the case of a semi-simple Lie algebra A, f may be defined so that the resulting algebra is simple. In general, A' will be simple if A is anticommutative and every proper ideal of A contains at least one element x such that $f(x)\neq 0$. (Received March 6, 1953.)

457. George Whaples: *Generalized local class field theory.* II.

Let k be a regular local field (for terminology see Duke Math. J. vol. 19 (1952) pp. 505–517), $\bar{k}$ its residue class field, p the characteristic of $\bar{k}$, and $k^{\cdot}_{(i)}$ the group of elements congruent 1 mod π^i where π is a prime element. If h is any subgroup of $k^{\cdot}$ define $\bar{h}_{(i)}$ to be the additive group of residue classes $\bar{\alpha}$ of elements $\alpha\in o$ for which $1+\alpha\pi^i\in(h\cap k^{\cdot}_{(i)})k^{\cdot}_{(i+1)}$ and call the $\bar{h}_{(i)}$ (which depend on choice of π) the image groups of h. Problem: Which subgroups of $\bar{k}^+$ can be image groups of finite abelian extensions of exponent p? Answer: All subgroups of $\bar{k}^+$ which are open in the additive polynomial topology (see Bull. Amer. Math. Soc. Abstract 59-1-83), provided of course that i is prime to p and in case k has characteristic 0, that $k^{\cdot}_{(i)}\not\subset k^{\cdot p}$. The case $i=1$ gives a complete description of all subgroups of conductor π^2 which are norm groups, thus solving problems raised by a counterexample of M. Moriya (J. Jap. Math. Soc. vol. 3 (1951) pp. 195–203). But a further counterexample shows that these results are not enough to settle the case of conductor π^3. (Received March 12, 1953.)

ANALYSIS

458*t*. J. W. Armstrong: *Point systems for Lagrange interpolation.*

S. Bernstein [Bull. Acad. Sci. URSS (1931) pp. 1025–1050] has shown that the Tchebichef abscissas have a Lebesgue function of uniform order log n for $|x|\leqq 1$ and he demonstrates that new point systems with a Lebesgue function of uniform order log n can be obtained by "distorting" the Tchebichef abscissas in a particular manner. A new class of point systems including the Tchebichef abscissas is now obtained in the following manner. Let $\{w_n(x)\}$ be a sequence of polynomials with real distinct zeroes in $(-1, 1)$ and require these polynomials to have asymptotic extreme values, i.e., the maximum of $|w_n(x)|$ in $[-1, 1]$ remains less than a constant K (independent of n) times the smallest of the relative maxima of $|w_n(x)|$ between successive zeros including also the end points. The point system composed of these zeros is shown to have a Lebesgue function of uniform order $\log n$ for $|x| \leqq 1$ and, furthermore, Bernstein's "distortion" theorem holds for any such point system obtained in the above manner. (Received February 6, 1953.)

459. Nachman Aronszajn and K. T. Smith: *Invariant subspaces of completely continuous operators.*

It is shown that every completely continuous linear operator on an arbitrary Banach space has closed proper invariant subspaces. (Received March 27, 1953.)

460. D. G. Austin: *A Lipschitzian characteristic of approximately derivable functions.*

Let m_nE denote the Lebesgue measure of a bounded set E in n-dimensional euclidean space and let $f(x)$ be a finite, single-valued, measurable function with domain E. The author uses the Egoroff Theorem to establish that if $f(x)$ possesses finite approximate partial derivates on E, then for any η, $\epsilon > 0$ there exist numbers M, $\delta > 0$ and a closed set $H \subset E$ with $m_nH > m_nE - \eta$ such that $s \in H$ implies $m_1[x_i; |(f(s_1, \cdots, s_{i-1}, x_i, s_{i+1}, \cdots, s_n) - f(s))/(x_i - s_i)| \leqq M, s_i < x_i < r_i + \alpha]/\alpha > 1 - \epsilon$, for all α with $0 < \alpha \leqq \delta$. This result together with an extension theorem for Lipschitz functions implies that $f(x)$ has finite approximate partial derivates on E if and only if $f(x)$ is essentially Lipschitz on E. Certain classical theorems of Lebesgue, Denjoy, and Stepanoff are easy corollaries of this result. (Received March 9, 1953.)

461. Leon Brown: *The zeros of analytic functions in Banach spaces.*

This paper generalizes the Weierstrass Preparation Theorem (see Bochner and Martin P183). Consider a function f with range in $X \times C_1$ and domain in C_1, where X is a complex Banach space and C_1 is the space of complex numbers, which is analytic and bounded for $\|X\| \leqq 1$, $|w| \leqq 1$. Assuming that $f(0, w)$ has an s-fold zero at $w = 0$, then $f = (w^s - \sum_{\nu=1}^{s-1} H_\nu(x) w^\nu)\Omega(x, w) = P\Omega$ where $H_\nu(x)$ is an analytic function on X to C_1 and Ω is a nonzero function of $X \times C_1$ to C_1, in a neighborhood of $(0, 0)$. The size of this neighborhood is estimated and P and Ω are represented as integrals of the function f. If one then considers the functions f as elements in a suitable Banach space, it is proved that P and Ω are analytically dependent on f. In proving this theorem the author arrives at a generalization of the Euclidian Algorithm. Given two functions f and g with domain $X \times C_1$ and range in C_1 analytic for $\|X\| \leqq 1$, $w \leqq 1$, and for every X_0 such that $\|X_0\| \leqq 1$, $f(X_0, w)$ has S zeros in $|w| < 1$, then there exist uniquely a polynomial P in w of degree $< S$ with coefficients analytic functions on X to C_1, and a $Q(x, w)$ analytic in the unit sphere such that $g = Qf + P$ and Q and P can be represented as integrals involving f and g. (Received March 9, 1953.)

462. R. H. Cameron and R. E. Fagen: *Nonlinear transformations of Volterra type in Wiener space.*

Let C be the space of continuous functions $x(t)$ on the interval $[0, 1]$ which vanish at $t = 0$, and let Γ be a Wiener measurable subset of C. Let T be the transformation $y(t) = x(t) + \Lambda(x | t)$, and assume the (generally nonlinear) functional Λ is such that T takes Γ in a 1-to-1 manner into a subset $T\Gamma$ of C. Cameron and Martin have shown that if Λ satisfies certain smoothness conditions, $T\Gamma$ is Wiener measurable, and they have given an explicit formula for transforming Wiener integrals by the transformation T (Trans. Amer. Math. Soc. vol. 66 (1949) pp. 253–283). The present paper extends these results by weakening the smoothness conditions required of Λ. In particular, the kernel of the variation of Λ is permitted to have a jump along the diagonal. (Received March 17, 1953.)

463. Buchanan Cargal: *Approach properties of functions.*

Let X be a Hausdorff space, Y be a regular separable Hausdorff space, and f be a function on X into Y. Let λ be a set property defined for subsets of X, and S be a subset of X. Then a point ξ in X is a *point of* λ *f-approach by* S if for every neighborhood $M(f(\xi))$ of $f(\xi)$ and every neighborhood $N(\xi)$ of ξ, the set $E[x:f(x)\in M(f(\xi))]N(\xi)S$ has property λ. A property λ is an *ascending set property* if B has property λ whenever A has property λ and $A\subset B$. Several theorems are proved which reduce to well known theorems when λ is specialized to yield continuity. The following is a typical theorem: if (i) λ is an ascending set property, (ii) $\{f_n(x)\}$ converges uniformly to $F(x)$ on X, (iii) S is a set in X, (iv) there is a ξ in X such that for every n, ξ is a point of λf_n-approach by S, then ξ is a point of λF-approach by S. The concept λf-approach was introduced by H. Blumberg (*New properties of all real functions*, Trans. Amer. Math. Soc. vol. 24 (1922)). (Received March 2, 1953.)

464. R. L. Evans: *Solution of linear ordinary differential equations containing a parameter.* II.

The system (1) $y^{(n)}-\sum_{j=1}^{n}\lambda^{pj}P_j(\lambda, x)y^{(n-j)}=0$, (2) $y^j(\lambda, 0)=(j!)\sum_{\alpha=1}^{\infty} b_{\alpha,j}\lambda^{-\alpha}$ $(j=0, 1, \cdots, n-1)$, and (3) $P_j(\lambda, x)=\sum_{\mu=-pmj}^{\infty}\sum_{\nu=[(p-\mu-1)/p]}^{\infty} a_{j,\mu,\nu}\lambda^{-\mu}x^{\nu}$ was solved, for bounded $(\lambda^p x)$, in part I (Proc. Amer. Math. Soc. (1953)). For solution in the large, combine this result with n independent solutions of (1) and (3) at unbounded $(\lambda^p x)$ by W. B. Ford's method (University of Michigan 1936) and its extensions. Neglecting special subcases, each independent solution for unbounded $(\lambda^p x)$ is such that, if $t^m=\lambda^p x$ and $m\geqq$ l.c.m. $(1, 2, \cdots, j_0)$ (where j_0 = largest number of spaces under a single segment of the Puiseux diagram for $t=\infty$), then (4) $y(\lambda, x)=\exp(Q(\lambda, t))\cdot v(\lambda, t)$, where $Q(\lambda, t)=\sum_{k=0}^{\infty} q_k(t)/\lambda^k$ and $v(\lambda, t)=\sum_{k=0}^{\infty} v_k(t)/\lambda^k$. The q_k's and v_k's are determined in pairs $(k=0, 1, \cdots$, etc.). If s_0 is the Puiseux slope corresponding to the desired independent solution, $q_k(t)$ is that polynomial of degree $(s_0+1+[km/p])$ which converts the terminal Puiseux slope (for eq. for v_k) to -1 and which bounds the $\rho_{k,\sigma}$ growth (with k) as required below. Also, (5) $v_k(t)=\sum_{\sigma=0}^{k-1} u_{k,\sigma}(\log t)^{\sigma}+v_0(t)\sum_{\sigma=1}^{k} C_{k,\sigma}(\log t)^{\sigma}$ $(k=1, 2, \cdots)$, and (6) $u_{k,\sigma}\sim\sum_{\beta=0}^{\infty} A_{k,\sigma,\beta}t^{\rho k,\sigma-\beta}$ where $\rho_{k,\sigma}<\rho_{k,0}\leqq\rho_{0,0}+[km/p]$ $(\sigma=1, 2, \cdots, k-1)$ with proper $Q(\lambda, t)$. The C's and A's are constants determined by substitution of (4)–(6) in (1) and (3). The asymptotic series in (6) may be replaced by convergent factorial series by J. Horn's methods (Math. Zeit. (1924)) and their extensions. (Received March 9, 1953.)

465. Seymour Ginsburg: *Further results on order types and decompositions of sets.* I.

Let a and b be two order types for which $a<b$ (see Sierpiński, *Sur les types d'ordre des ensembles linéaires*, Fund. Math. vol. 37 (1950) pp. 253–264). If there exists an order type c such that $a<c<b$, then problem P, as applied to a and b, is said to admit of a solution c. Now let $\{d_e | e\in E\}$, where the power of E is $2^{\aleph_0}$, be a set of order types, each d_e being of power $2^{\aleph_0}$ and less than λ. The following two results are shown: (1) Problem P, as applied to $a=d_e$ and $b=\lambda$, admits of a solution c_e, such that the c_e are pairwise incomparable order types. (2) Problem P, as applied to $a=0$ and $b=d_e$,

admits of a solution c_θ such that the c_θ are pairwise incomparable order types. (Received March 2, 1953.)

466*t*. Seymour Ginsburg: *Further results on order types and decompositions of sets.* II.

All sets considered are linear and of power $2^{\aleph_0}$. A set E is called exact if the only similarity transformation of E into E is the identity. E is said to have property A if no two disjoint subsets are similar. The problem studied is the decomposition of a linear set into a family of disjoint sets, the order type of the sets being required to satisfy some specified conditions. Some of the results obtained are the following: (1) For each set E and each ordinal number $a \leqq \omega$, E is the union of a family $\{E_i | i<a\}$ of pairwise disjoint sets, the order type of the E_i being pairwise incomparable. (2) Let E contain a family $\{E_a | a \in G\}$ of disjoint, similar subsets. Then E is the union of a family $\{A_a | a \in G\}$ of disjoint, similar, exact sets, each having property A. (3) If $2^{\aleph_0} = \aleph_1$, then each set is the union of a family $\{E_n | n<\omega\}$ of disjoint, exact sets, each having property A. In addition, the order type of the E_n are pairwise incomparable. (Received March 2, 1953.)

467. Meyer Jerison and Gustave Rabson: *Fourier series on compact zero-dimensional groups.*

Let G be a compact zero-dimensional group. G is the limit of an inverse system of finite groups $\{G_\alpha\}$. Let π_α be the natural mapping of G onto G_α with kernel K_α. Define $\pi_\alpha^+ \phi_\alpha(g)$ to be $\phi_\alpha(\pi_\alpha(g))$ and $\pi_\alpha^- \phi(g_\alpha)$ to be $\int_{K_\alpha} \phi(\pi_\alpha^{-1}(g_\alpha)k)dk$ where $g \in G$, $g_\alpha \in G_\alpha$, $\phi \in L^1$ on G, ϕ_α is a function on G_α and we choose any representative of $\pi_\alpha^{-1}(g^\alpha)$. It is proved that $\pi_\alpha^+ \pi_\alpha^- \phi$ is a partial sum of the Fourier series of ϕ and that $\pi_\alpha^+ \pi_\alpha^- \phi$ converges to ϕ in the mean, pointwise at a point of continuity, and uniformly if ϕ is continuous. If G is the countable product of groups of order two, considered as dyadic rationals, the theorem gives a slight strengthening of a theorem of Walsh which asserts that the 2^n partial sums of the Walsh-Fourier series of a continuous function on the unit interval converges uniformly to the function. (Received April 20, 1953.)

468. N. D. Kazarinoff: *Asymptotic forms for the Whittaker confluent hypergeometric functions when the parameter m is large.* Preliminary report.

In this paper forms are derived for the solutions of the differential equation $d^2w/dx^2 + [-1/4 + k/x + (1/4 - m^2)/x^2]w = 0$, which are valid for complex x and large complex m. Upon performing the transformations $z = \log(x/2im)$ and $u = e^{-z/2}w/2i$ of the independent and dependent variables, the differential equation becomes one of the class treated by R. E. Langer [Trans. Amer. Math. Soc. vol. 67 (1949) pp. 461–490]. Application to the theory in Langer's paper, generalized to include complex z, gives forms for the solutions of Whittaker's equation, customarily denoted by $M_{k,m}(x)$ and $W_{k,m}(x)$, which are asymptotic as to m and relative to sectors of the z-plane. The complex quantities x, k, and m are subject only to the restrictions that k be bounded and that $|m|$ be sufficiently large. (Received March 10, 1953.)

469. J. L. Kelley: *Averaging operators.*

Let $C_\infty(X)$ be the algebra of all those continuous real-valued functions on a locally compact Hausdorff space X which vanish at ∞, and let T be a bounded linear operator on $C_\infty(X)$. Break X into equivalence classes by agreeing x equiv. y if $T(f)(x) = T(f)(y)$ for all f. T is called averaging if for each equivalence class D, $T(f)$ is on D the average of the values of f on D (i.e. for some signed Baire measure m, for all $x \in D$ and all f, $T(f)(x) = \int f\,dm$ and $m(A) = 0$ if A is disjoint from D). Extending results of G. Birkhoff [Colloq. Int. Rech. Sci., no. 24, 1950, pp. 143–153] it is found that: 1. T is averaging if and only if $T(fT(g)) = T(f)T(g)$ for all f and g. 2. If T is idempotent and positive ($T(f) \geqq 0$ for $f \geqq 0$) and the range of T is a subalgebra, then T is averaging. 3. If X is a topological group, T commutes with right translation if and only if T is convolution on the left by a signed Baire measure m. If T commutes with right translation and is averaging, the measure m is $\pm$Haar measure on a compact subgroup of X. A theorem of Y. Kawada and P. R. Halmos' formulation of a result of Dieudonné are also derived. (Received March 11, 1953.)

470. J. H. B. Kemperman: *An asymptotic expression of $\sum_n g(\alpha n+\beta)z^n$ in terms of $\sum_n g(\rho n+\sigma)z^n$.*

For $u \geqq -h$, let $g(w) = g(u+iv)$ be analytic (no poles), $|g(u+iv)| \leqq K \exp(l\pi|u| + m\pi|v|)(1+v^2)^{-p}$, where K, l, m, and $p > 1/2$ are non-negative constants. Let $\alpha = |\alpha|e^{i\theta}$, $\rho = |\rho|e^{i\eta}$, β and σ be complex constants with $|\theta| < \pi/2$, $|\eta| < \pi/2$, $m|\rho| \leqq \cos\eta$. Put $F(z) = \sum g(\alpha n+\beta)z^{\alpha n}$ and $G(z) = (\rho/\alpha)z^{\sigma-\beta}\sum g(\rho n+\sigma)z^{\rho n}$, summing over the integers n for which Re $(\alpha n+\beta) > -h$ and Re $(\rho n+\sigma) > -h$, respectively. Let $\gamma = \exp(2\pi i/\alpha)$, $\Delta = \arg\gamma = 2\pi\cos\theta/|\alpha|$, $2m\pi = \nu\Delta + c$ (ν = integer $\geqq 0$; $0 \leqq c < \Delta$) and let k be an arbitrary integer. Restricting z to the sector $0 \leqq \arg z + m\pi \leqq c$ (or $c \leqq \arg z + m\pi \leqq \Delta$), the assertion is that the analytic function which, for small $|z|$, is defined by $H(z) = F(z\gamma^k) - \sum_{\mu=0}^{\nu} G(z\gamma^\mu)$ (or $H(z) = F(z\gamma^k) - \sum_{\mu=0}^{\nu-1} G(z\gamma^\mu)$, respectively), can be extended to a single-valued and analytic function throughout the entire sector in such a way that $H(z) = O(z^{-h-\beta})$ for large $|z|$. Especially, if $m|\alpha| < \cos\eta$ (hence, $\nu = 0$), $F(z\gamma^k) = O(z^{-h-\beta})$ for $c \leqq \arg z + m\pi \leqq \Delta$. E. M. Wright, Roy. Soc. London Philos. Trans. (A) vol. 238 (1940) p. 434 (generalizing a theorem of G. N. Watson), obtained the above result for the special case that $g(w) = O(\Gamma(w+b)^{-1})$ for $u \geqq -h$, $\rho = 1$, $\sigma = \beta = 0$. The above result can be extended to the case that $g(w)$ has for $u \geqq -h$ at most a finite number of poles. (Received March 16, 1953.)

471. Fulton Koehler: *Bounds for the moduli of the zeros of a polynomial.*

Let $f(z) = \sum_{\nu=0}^{n} a_\nu z^\nu$, $a_0 \neq 0$, $a_n \neq 0$; and let the zeros of $f(z)$ be z_i with $|z_1| \leqq |z_2| \leqq \cdots \leqq |z_n|$. Let the Newton diagram for $f(z)$, determined by the points $(\nu, -\log|a_\nu|)$, have slopes $\log R_1$, $\log R_2$, $\cdots$, $\log R_n$. In a paper by Ostrowski (*Recherches sur la méthode de Graeffe et les zéros des polynomes et des séries de Laurent*, Acta Math. vol. 72 (1940) pp. 99–257) it is shown that $|z_k|/R_k > 1 - (1/2)^{1/k}$. The best bounds for $|z_k|/R_k$, as functions of n, are given only for $k=1$, $k=n$; and for $k=2$ when $n=3$. In this paper it is shown that the minimum of $|z_k|/R_k$ for all polynomials of nth degree with $a_0 \neq 0$ is equal to the positive root of the equation $Q_k^n(x) = 1$, where $Q_k^n(x)$ is the partial sum of the series for $(1-x)^{-k} - 1$ through the term in x^{n-k+1}. (Received March 5, 1953.)

472. Jacob Korevaar: *Best approximation in L_1 and the remainder in Littlewood's theorem.*

$f(x)$ is said to be of class $J_m(a, b)$ if (i) $f(x)$, $f'(x)$, $\cdots$, $f^{(m-1)}(x)$ exist and are continuous on $a \leqq x \leqq b$, (ii) $f^{(m)}(x)$ exists and is continuous except at a finite number of points, where it has a jump discontinuity, (iii) $|f^{(m)}(x_1)-f^{(m)}(x_2)| \leq A|x_1-x_2|$ on each of the subintervals of (a, b) on which $f^{(m)}(x)$ is continuous. Functions of class $J_m(a, b)$ are approximated by trigonometric and ordinary polynomials. The following result is typical. For any $f(x)$ of class $J_m(a, b)$, there exist constants H_1 and H_2 such that for every positive integer n one can find a polynomial $p_n(x)=\sum c_{nk}x^k$ of degree $n-1$ such that $\int_a^b|f(x)-p_n(x)|dx<H_1/n^{m+1}$, $|c_{nk}|<H_2^n$ $(k=0, 1, \cdots, n-1)$. This result is used to estimate the remainder in Littlewood's Tauberian theorem for power series. Let $|na_n|<K_1$, and let $\sum a_nx^n\to s$ as $x\uparrow 1$. To take a simple case, assume that $|\sum a_nx^n-s|<K_2(1-x)$, $0<x<1$. Then the above result with $m=0$ shows that $|s-s_n|<M_0/\log(n+2)$, $M_0=M_0(K_1, K_2)$; the above theorem with $m=1$ shows that $|s-(s_0+s_1+\cdots+s_n)/(n+1)|<M_1/\{\log(n+2)\}^2$, etc. These estimates for s_n etc. are best possible and improve the author's previous results (Duke Math. J. vol. 18 (1951) pp. 723–734). (Received March 11, 1953.)

473. M. Z. Krzywoblocki: *On the generalization of fundamental existence theorems of W. M. Whyburn to partial differential systems.*

W. M. Whyburn proved some fundamental existence theorems for systems of first order differential equations where both the equations and the initial conditions differ from those that occur in the ordinary theory. The equations are different in that the quantities that appear on the right-hand side of the equations do not, as a whole, satisfy a Lipschitz condition. The boundary conditions differ in that they apply to more than one point of the interval. The author treats partial differential systems of the first order in two independent variables, which systems by a not one-to-one transformation are transformed onto an ordinary differential system with a parameter. For such systems, the author proves some existence theorems. The work consists of two parts. In Part I, the author generalizes many theorems on real-valued sets and functions to sets, and functions with a parameter. In that, he follows closely McShane's *Integration.* In Part II, he generalizes Whyburn's theorems to ordinary differential systems with a parameter. (Received January 30, 1953.)

474. Arne Magnus: *Volume-preserving maps in several complex variables.*

Let $u=u(z_1, z_2)$ and $v=v(z_1, z_2)$ be analytic in the complex variables z_1 and z_2. Let $(z_1, z_2)\to(u, v)$ be a volume-preserving map, that is, a map which carries any domain in the (z_1, z_2)-space into a domain in the (u, v)-space of the same 4-dimensional volume. Then $\partial(u, v)/\partial(z_1, z_2)=1$. Families of such maps are exhibited. A formula is derived which expresses u in terms of v when u and v are polynomials. All such polynomial pairs of degrees less than or equal to five are exhibited. All maps (polynomials or not) encountered are proved to be univalent. Some families of univalent but not volume-preserving maps are constructed by using the addition theorems for e^z, $\tan z$, and $\sin z$. Each map (volume-preserving or not) studied is shown to possess invariant 2-dimensional surfaces, that is, each map is assigned a 2-parameter family of 2-dimensional surfaces each of which is mapped into itself by $(z_1, z_2)\to(u, v)$. Extensions are made to several complex variables. (Received April 17, 1953.)

475t. R. K. Ritt: *A condition that* $\lim_{n\to\infty} n^{-1}T^n=0$.

Let T be a bounded linear transformation on a Banach space; let the spectrum of T be interior to the unit circle, with the possible exception $z=1$; further, suppose that there is an $M>0$ and an $\eta>0$ such that if z is in the resolvent set for T, $|z|\geq 1$, and $|z-1| \leqq \eta$, then $\|(z-1)(z-T)^{-1}\| \leqq M$. Then $\lim_{n\to\infty} n^{-1}T^n = 0$. The proof is obtained by computing an estimate of the integral representation for T^n. (Received March 13, 1953.)

476. R. K. Ritt: *Rings of functions.*

A B_*-algebra is a complete normed ring of functions on a connected compact Hausdorff space, containing, with any function, its complex conjugate. They are realizations of B^*-algebras [Gelfand, Neumark]. In a B_*-algebra there exist two types of maximal ideals: the first type—all functions zero at some point; the second type—a set of functions whose absolute value has its lower limit zero at some point. There are in general several ideals of second type associated with a point. In a B_*-algebra containing at least one nontrivial characteristic function, an arbitrary function can be uniformly approximated by a linear combination of disjoint characteristic functions if and only if no two distinct maximal ideals contain the same set of characteristic functions. Baire classes are defined, using uniformly bounded sequences of functions; they are shown to be B_*-algebras. In the Baire classes, if two maximal ideals contain the same set of characteristic functions they are both of the second type and are associated with the same point. Applications to spectral theory are given. (Received March 13, 1953.)

477t. R. K. Ritt: *Very weak convergence and the spectral resolution of bounded linear operators on a reflexive Banach space.*

If K is an arbitrary normed ring, with $\mathfrak{M}$ the space of maximal ideals, a uniformly bounded sequence, $\{x_n\}$, $x_n \in K$, converges very weakly to zero if $x_n(M) = 0$, all $M \in \mathfrak{M}$. Very weak convergence to zero is equivalent to weak convergence to zero if and only if every bounded linear functional on K has an integral representation of the Kakutani type [Ann of Math. vol. 42 (1941)]. A normed ring, $K(1, a)$, generated by the identity and a single element a is said to be normal if the uniform convergence to zero of a sequence of polynomials $\{p_n(\lambda)\}$ on the spectrum of a implies $\|p_n(a)\| \to 0$. If a is a bounded linear operator on a reflexive Banach space, and the spectrum of a is the unit interval, a will have an integral representation $\int_0^1 \lambda dE_\lambda$ if and only if $K(1, a)$ is normal. The proof employs the notion of very weak convergence and an argument of Eberlein [Bull. Amer. Math. Soc. vol. 52 (1946)]. (Received March 13, 1953.)

478t. L. B. Robinson: *Construction of systems of differential equations which functions given in advance satisfy.*

Given three functions I_i, write their infinitesimal transformations $\bar{I}_1 = (1+\psi_{22}\delta t + \psi_{33}\delta t)I_1 - \psi_{12}\delta t I_2 - \psi_{13}\delta t I_3$, $\bar{I}_2 = -\psi_{21}\delta t I_1 + (1+\psi_{11}\delta t + \psi_{33}\delta t)I_2 - \psi_{23}\delta t I_3$, $\bar{I}_3 = -\psi_{31}\delta t I_1 - \psi_{32}\delta t I_2 + (1+\psi_{11}\delta t + \psi_{22}\delta t)I_3$. It is easy to write down a system of partial differential equations which the I_i satisfy. Next write the symbolic products $\bar{I}_i \cdot \bar{I}_j \equiv \bar{I}_{ij}$. An example would be $\bar{I}_1 \cdot \bar{I}_1 \equiv \bar{I}_{11} = (1+2\psi_{22}\delta t + 2\psi_{33}\delta t)I_{11} - 2\psi_{12}\delta t I_{12} - 2\psi_{13}\delta t I_{13}$. It is easy to write down a system of partial differential equations which the I_{ij} satisfy. And we can form the products $\bar{I}_i \cdot \bar{I}_j \cdot \bar{I}_k \equiv \bar{I}_{ijk}$, and so on. The above functions are semitensors. Knowing a system of semitensors it is possible by symbolic multiplication to form an infinite chain of semitensors and also to write an infinite chain of differential equations which these semitensors satisfy. (Received April 6, 1953.)

APPLIED MATHEMATICS

479t. Nathaniel Coburn: *Stream lines for incompressible and compressible fluid flows (plane irrotational and rotational cases).*

The purpose of this paper is to use the intrinsic form of the equations of fluid dynamics (*Some intrinsic relations satisfied by the velocity and vorticity vectors in fluid flow theory*, N. Coburn, Michigan Mathematical Journal, vol. 2, 1952–53) to classify stream line patterns for incompressible and compressible fluids in the plane irrotational and rotational cases. In this method of attack, the equations of fluid dynamics become first order equations in functions of the magnitude of the velocity and the metric coefficients of the net determined by the stream lines and their orthogonal trajectories. The essential nonlinearity of the equations of fluid dynamics is thrown into the geometry of the stream lines (the vanishing of the Riemann tensor). By specifying the curvature of the stream lines or their orthogonal trajectories, this nonlinear equation may be discussed. In the incompressible irrotational case, it is shown that, if the curvature of a stream line (say, an obstacle) and if the magnitude of the velocity vector is known along this curve, then the determination of the flow depends upon the solution of a Neumann problem for an upper-half plane. For the rotational and compressible cases, the cases of straight stream lines are discussed. (Received January 14, 1953.)

480. R. J. De Vogelaere: *On a dynamical system with two degrees of freedom.*

At present only two dynamical systems are known with some details, the cosmic rays problem and the restricted three bodies problem. Attention is drawn to the plane problem corresponding to the isosceles triangle solutions of the general three bodies problem with the interesting feature of the singularities of the double and triple collisions. As known since the work of Fransen, the masses m at the vertices of the equal angles are to be equal. In the case with axial symmetry, the variables were chosen proportional, r proportional to the distances of the equal masses and z proportional to the distance of the other to the center of gravity. The first family of periodic orbits was discovered having as one end the collinear solution of Euler and as other end a solution with two symmetric double collisions. The results were put elegantly in terms of Fourier sums for the mass ratios 1, 1, 1; 1, 1, 4, and 1, 1, ϵ, the independent variable being analogous to the mean anomaly for z and to the eccentric anomaly for r. (Received March 10, 1953.)

Geometry

481*t*. John DeCicco: *Physical systems of curves in Riemannian space.*

A system S_k of ∞^{2n-1} curves in a given field of force of a Riemannian space V_n consists of curves along which a constrained motion is possible such that the osculating geodesic surface at each point touches the force vector X, and the pressure P, along the principal normal to the curve, is proportional to the normal component N of X. Thus $P=kN$ where $k(\neq -1)$ is the constant factor of proportionality. For a system S_k, the author obtains extensions of the Lagrangian equations and the Hamiltonian equations of a dynamical system. A generalization is found of the Lie contact transformation of mechanics. A conservative system S_k can be defined as the extremals of the variation problem: $\int v^{1+k}ds=\min$. There is a similar integral for a conservative velocity system S_∞. Finally the theorem of Kasner which states that the ratio of the curvature of the rest trajectory to that of the line of force is $\rho=(1+k)/(3+k)$, at the initial point, is established for a system S_k in V_n. (Received March 10, 1953.)

482t. John DeCicco: *Transformation theory of physical systems of curves in Riemannian space.*

The following theorems are established concerning the transformation theory of physical systems of curves in Riemannian space. In any nonhomothetic conformal cartogram between two Riemannian manifolds V_n and $\overline{V}_n$, there is one and only one whole conservative system S_k, which is converted into a whole conservative system S_k. By a transformation T between V_n and $\overline{V}_n$, every physical system S_k, not a dynamical system S_0, in V_n becomes a physical system in V_n if and only if T is a homothetic transformation. A transformation T between V_n and $\overline{V}_n$ converts every dynamical system S_0 in V_n into a dynamical system if and only if T is a projective transformation such that $\partial(\overline{\Gamma}^i_{ii}-\Gamma^i_{ii})/\partial x^j=\partial(\overline{\Gamma}^j_{jj}-\Gamma_{jj})/\partial x^i$, where the Γ^i_{jk} denote the Christoffel symbols. The transformation T is affine if and only if the time is unchanged. Finally if both V_n and $\overline{V}_n$ are Euclidean spaces, then Appel's transformation for dynamical systems in n dimensions is found. (Received March 10, 1953.)

483. Marshall Hall, Jr.: *Uniqueness of the projective plane with* 57 *points.*

Using the result of W. A. Pierce that a plane with 57 points cannot contain a Fano configuration, it is shown that such a plane must be Desarguesian. A study of linear equations associated with incidences shows that Moufang's Desarguesian configuration D_8 must be valid in the plane and hence the plane is Desarguesian. (Received February 17, 1953.)

484t. P. C. Hammer: *Constant breadth surfaces.*

A quasi-pencil of lines in n-dimensional space is here defined to be a continuous family of lines simply covering some sphere and its exterior, including infinite points, and which has all points on each line in the family closest to any other line in the family interior to the sphere. A simple analytical formula is derived giving surfaces of constant breadth relative to an outwardly simple line family. When this line family is a quasi-pencil and other conditions are satisfied, the surface is a closed convex surface of constant breadth. The formula is shown to represent all such surfaces. (Received February 23, 1953.)

485t. P. C. Hammer: *Tangential similarity of surfaces.*

Weak tangential similarity and tangential similarity are defined for surfaces in E_k parametrically representable over a domain in a real normed linear space. In certain cases of perspective surfaces weak tangential similarity is shown to imply similarity. Existence and uniqueness of curves satisfying certain differential conditions is shown when there is no differential equation and no Lipschitz type condition available. Cf. *Class-equivalence of functions and curves*, Bull. Amer. Math. Soc. Abstract 59-2-234. (Received March 12, 1953.)

Statistics and Probability

486. Shu-Teh C. Moy: *Measure extensions and the martingale convergence theorem.*

In Doob's discussion of the relation between his martingale convergence theorem and a limit theorem on derivatives by Andersen and Jessen [Doob, *Stochastic processes*, New York, Wiley, pp. 630–632] the following two conditions concerning a martingale

$\{x_n, \mathcal{J}_n, n \geqq 1\}$ are studied. 1. There is a countably additive set function ϕ, defined on the smallest Borel field $\mathcal{J}_\infty$ containing all the $\mathcal{J}_n$'s, of which the contraction ϕ_n to $\mathcal{J}_n$ is absolutely continuous with respect to the contraction P_n of the probability measure to $\mathcal{J}_n$ and for which x_n is the derivative of ϕ_n relative to P_n for every n. 2. Sup $\{E[|x_n|]: n \geqq 1\} < \infty$. In this paper it is proved that 1 and 2 are equivalent if the basic space Ω on which the random variables x_n are defined is the space of real sequences $\xi = \{\xi_n\}$ and $\mathcal{J}_n$ is the smallest Borel field containing the sets of the form $[\{\xi_n\}: \xi_1 \leqq \alpha_1, \cdots, \xi_n \leqq \alpha_n]$ with $\alpha_1, \cdots, \alpha_n$ being any n real numbers. This result implies that the martingale convergence theorem can be deduced from the limit theorem of Andersen and Jessen. (Received April 22, 1953.)

TOPOLOGY

487. Raoul Bott: *On closed geodesics.* Preliminary report.

A closed geodesic g is a map of the reals R into a manifold M, which satisfies the geodesic differential equations, and is in addition of period 2π. $g^{(n)}$, the nth iterate of g, is defined by $g^n(x) = g(nx)$, $x \in R$, $n = 1, 2, \cdots$. Poincaré assigns to every g a system of numbers $\pm\alpha_1, \cdots, \pm\alpha_k$; $k \leqq \dim(M)$, modulo 2π (see *Oeuvres*, vol. 7, p. 338). Morse (see Amer. Math. Soc. Colloquium Publications, 1934) assigns to every g two integers $N(g)$ and $\Lambda(g)$, the nullity and index of g respectively. It is proved that these numbers are related in the following fashion: $N\{g^{(n)}\} = \sum_{s=1}^{n-1} n(e^{2\pi s i/n})$, $\Lambda(g^{(n)}) = \sum_{s=1}^{n-1} \lambda(e^{2\pi s i/n})$ where n and λ are non-negative integer-valued functions on the unit circle. Further, λ and n are constant except at the points $e^{\pm i\alpha_k}$ and the jump of λ at $e^{\pm i\alpha_k}$ is bounded by the value of n at $e^{\pm i\alpha_k}$. It follows that $\lim_{n\to\infty} (\Lambda g^{(n)}/n) = \sigma$ exists and $\sigma \equiv \sum a_k\alpha_k$ mod 2π with the a_i integers. These formulae and their consequences generalized the results of Hedlund (Trans. Amer. Math. Soc. vol. 34 (1932) pp. 75–97) and certain of the announced results of M. Morse and E. Pitcher (Proc. Nat. Acad. Sci. U. S. A. (1934) pp. 282–287). (Received March 11, 1953.)

488t. E. E. Floyd and M. K. Fort, Jr.: *A characterization theorem for monotone mappings.*

The main result of the paper is the following theorem. A mapping f of the two-sphere S onto S is monotone if and only if there exists a continuous extension g of f such that g maps $S \cup Q$, Q the interior of S, onto itself with $g|Q$ a homeomorphism of Q onto Q. A key lemma states that the group of homeomorphisms of S onto S is uniformly locally connected. (Received March 12, 1953.)

489. William Gustin: *On the order of traversing a path.*

A continuous mapping f from an oriented closed linear interval Π onto a (necessarily Peano) compactum P is said to be a path with carrier P. A finite sequence $p_1 \cdots p_n$ of points in P is called an f-chain provided there exist parameters $\pi_1 \leqq \cdots \leqq \pi_n$ in Π such that $f(\pi_k) = p_k$ for $k = 1, \cdots, n$. The path order of f is defined as the set $\mathfrak{c}(f)$ of all f-chains. Seven properties of path order are established; and a set $\mathfrak{c}$ of finite sequences, termed abstract chains, of points in a compactum P satisfying these seven properties or axioms is said to be an abstract path order with carrier P. It is shown that every abstract path order $\mathfrak{c}$ can be concretely realized as the path order $\mathfrak{c}(f)$ engendered by some path f, and that the class of all such paths f with $\mathfrak{c}(f) = \mathfrak{c}$ is a Fréchet equivalence class of paths. Thus abstract path order intrinsically characterizes an oriented Fréchet curve, abstract chains together with their carrier space constituting a complete set of curve invariants. (Received March 9, 1953.)

490. Frank Harary and R. Z. Norman: *The dissimilarity characteristic of linear graphs.*

Otter's notion of dissimilarity characteristic for trees, previously extended by the authors to Husimi trees (to appear in Ann. of Math.) is here generalized to arbitrary linear graphs. Let G be a connected graph. Let p, k be the number of dissimilar points, lines of G respectively, and let k_s be the number of dissimilar symmetry lines of G. Consider an arbitrary cycle basis of G. From this basis we extract a set B of cycles forming a basis modulo similarity. A certain well-defined subset of the set of non-orientable cycles of B will be called the set of exceptional cycles of B. Let c, c_s be the number of cycles, exceptional cycles of B respectively. It is shown that $c-c_s$ is an invariant of G. The dissimilarity characteristic equation of G is $p-(k-k_s)+(c-c_s)=1$. This formula is potentially applicable to the solution of various counting problems, among them some of the combinatorial questions discussed by Uhlenbeck in his Gibbs Lecture. (Received March 11, 1953.)

491. P. S. Mostert: *On fibre spaces with totally disconnected fibres.*

Let $\mathcal{F}=\{X, B, \pi, \phi_U, \Omega\}$ be a fibre space in the sense of Hu (Proc. Amer. Math. Soc. vol. 1 (1950) pp. 756–762) where the base space B is arcwise connected and ϕ_U: $U\times\pi^{-1}(b)\rightarrow\pi^{-1}(U)$ is a homeomorphism onto. Then it is proved that there is a collection $\mathcal{B}=\{X, B, Y, \pi, \psi_b, V_b, H_b, G\}$ which is an E-F bundle (Steenrod, *The topology of fibre bundles*, Princeton University Press, 1950, p. 18) where Y is homeomorphic with $\pi^{-1}(b)$, $b\in B$, ψ_b arises naturally out of ϕ_U, and G is isomorphic (algebraically) with a factor group of the fundamental group of B. The result is applied to fibre bundles to show that a fibre bundle $\mathcal{B}=\{X, B, Y, \pi, \phi_i, U_i, G\}$, where the fibre Y is totally disconnected, and the base space B is arcwise connected, is equivalent in G to a bundle with group $G'\subset G$ which is the closure in G of the image of the fundamental group of B under a homomorphism into. Hence, in particular, if B is also simply connected, it is G-equivalent to the product bundle. This result is then applied to locally compact groups of finite dimension. (Received March 9, 1953.)

492*t*. E. S. Northam: *Topology in lattices.*

Various results concerning the order topology and interval topology of a lattice are obtained. It can be shown that a necessary condition for the interval topology to be Hausdorff is that each interval of the lattice have a finite separating set in the following sense: A set $\{a_i\}$ separates the interval $[a, b]$ if $a<a_i<b$ for all a_i and each member of $[a, b]$ is comparable with some a_i. This result can be applied to show that the interval topology of a Boolean algebra is Hausdorff if and only if every element is over an atom, and also to solve Problem 104 of Birkhoff's list. A necessary and sufficient condition for an element to be isolated in the interval topology is obtained. Also an example can be found which shows that the order topology, L-topology of Rennie, and transfinite sequential topologies of a lattice need not be Hausdorff. (Received March 16, 1953.)

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