let's class-number formula, and (7) an exposition (without proof) of the Thue-Siegel theorem.

Although this book is not written as a textbook but rather as a work for the general reader, it could certainly be used as a textbook for an undergraduate course in number theory and, in the reviewer's opinion, is far superior for this purpose to any other book in English. Admittedly there are no formal lists of problems, but there are plenty of problems implicit in the text in the form of easy proofs and calculations left to the reader; also there are many hints for further discussion or further reading. Students will certainly like the author's facility in presenting new concepts and proofs clearly without introducing elaborate notations.

Finally the reviewer believes that this book should be in every college library worthy of the name, regardless of whether or not there is a course in number theory in the curriculum. It is hard to think of a better book to suggest to an interested undergraduate for independent reading.

P. T. BATEMAN

Éléments de mathématique. By N. Bourbaki. Book II, *Algebra.* Chaps. I–VII. (Actualités Scientifiques et Industrielles, nos. 934, 1032, 1044, 1102, 1179.) Paris, Hermann, 1942, 1947, 1948, 1950, 1952.

Our time is witnessing the creation of a monumental work: an exposition of the whole of present day mathematics. Moreover this exposition is done in such a way that the common bond between the various branches of mathematics becomes clearly visible, that the framework which supports the whole structure is not apt to become obsolete in a very short time, and that it can easily absorb new ideas. Bourbaki achieves this aim by trying to present each concept in the greatest possible generality and abstraction. The terminology and notations are carefully planned and are being accepted by an increasing number of mathematicians. Upon completion of the work a standard encyclopedia will be at our disposal. The volume on *Topologie générale* which is complete is already being used enthusiastically, especially by the younger generation. A comparison with the "Encyclopädie der mathematischen Wissenschaften" should not be made. The aim was different; proofs were omitted and each article was written by a different author.

I hope that this work will continue in the same spirit and with the same vigor. I would suggest an English translation.

The volumes on algebra that have appeared show the same general features as the rest of Bourbaki. Numerous exercises, many of them of highest interest, are found at the end of each paragraph. From time to time excellent historical notes explain the development of the ideas. It is inevitable that much of the material is of standard nature. In the following more detailed discussion I intend to underline mainly the novel ideas that appear in the work.

A few general remarks must precede this discussion. We all believe that mathematics is an art. The author of a book, the lecturer in a classroom tries to convey the structural beauty of mathematics to his readers, to his listeners. In this attempt he must always fail. Mathematics is logical to be sure; each conclusion is drawn from previously derived statements. Yet the whole of it, the real piece of art, is not linear; worse than that its perception should be instantaneous. We all have experienced on some rare occasions the feeling of elation in realizing that we have enabled our listeners to see at a moment's glance the whole architecture and all its ramifications. How can this be achieved? Clinging stubbornly to the logical sequence inhibits the visualization of the whole, and yet this logical structure must predominate or chaos would result. Bourbaki is quite aware of this dilemma. The fact that his work is subdivided into books, the fact that exercises are given which utilize more advanced parts of the theory show this awareness. However I feel that in some instances the subdivision into books is not enough. This inadequacy is strongly felt in the course of Chapter V as we shall see later.

Chapter I acquaints us with the fundamental concepts of abstract algebra: groups, rings, fields, vector spaces. To avoid repetitions the notions of an internal and of an external composition in a set are introduced. The internal composition is patterned after the addition or multiplication in a ring, the external one after the multiplication of a vector by a scalar or the product of an operator of a group with an element of the group. In this very general setup Bourbaki discusses topics like the associative and commutative laws, the question of the existence of a neutral element (0 element in case of addition) and that of a symmetric element (the negative in case of addition). Symmetrization is the abstract counterpart to the introduction of the negative integers.

Bourbaki calls a set together with several internal or external compositions an algebraic structure. The notion of a quotient structure generalizes that of factor group or residue class ring. Finally laws like the distributive (in the case of several compositions) are discussed.

These general concepts are now applied to groups, rings and fields. The discussion of groups, which includes the usual elementary notions and theorems, culminates in the Jordan-Hölder theorem and is followed by a detailed study of transformation (permutation) groups. It seems to be Bourbaki's intention not to discuss some finer points of the theory of finite groups. I would deplore this since some of the topics like p-groups and Sylow groups might easily find their place in some later volume and are of great importance for algebraic number theory.

The discussion of rings, ideals and fields leads to the usual elementary theorems. As a deeper property of fields (which solves a classical problem of projective geometry) a future edition might include the following beautiful theorem of Hua.

Let σ be an additive map of one field (not necessarily commutative) into another which satisfies $\sigma(a^{-1})=(\sigma(a))^{-1}$ for all $a\neq 0$ and $\sigma(1)=1$. Then σ is either an isomorphism or an anti-isomorphism into this field. By the way, the connection with Hua's work follows from the following amusing noncommutative identity:

$$a - (a^{-1} + (b^{-1} - a)^{-1})^{-1} = aba$$

which shows $\sigma(aba)=\sigma(a)\sigma(b)\sigma(a)$.

Chapter II deals with linear algebra. As could be expected, Bourbaki puts the geometric concepts in the foreground. He begins with the definition of a module over a ring (not necessarily commutative), that of submodules, factor modules, free modules, bases, linear maps and their properties. The usual notion of dimension (supplemented by the notion of codimension) for vector spaces over a field is introduced. The main properties are derived from the exchange theorem. Then follows a paragraph discussing the dual of a module and the adjoint (transposée) of a map.

A most delightful part is §5 and I wish to call it to the attention of the algebraists. By means of the novel notion of a primordial element of a subspace with respect to a given basis of the whole space, the theory of linear equations with coefficients in a subfield is quickly developed and yields powerful results.

The computational aspect is not neglected; §6 gives a complete discussion of matrices. The chapter ends with a preliminary study of algebras. The algebra arising from a semigroup (monoïde) prepares the way for the definitions of group rings, polynomials, and power series.

One of the basic ideas of Chapter III is expressed by the following theorem: Given a finite number of modules E_i over a commutative ring A. An A-module M (unique up to isomorphisms) can be constructed which has two properties: (1) There exists a canonical map ϕ of the cartesian product of the E_i into M, and the image generates M.

(2) Every multilinear map of the E_i into any A-module N is of the form $g\phi$, where g maps M linearly into N. The module M is called the tensor product of the E_i. This idea is used again later to define the exterior pth power of a module E by means of multilinear alternating maps of E. The existence is proved by taking a suitable factor module of the pth tensor power of E.

The abstract idea underlying these constructions is taken up in an appendix on "universal maps."

After a deviation to tensor products of algebras a tensor of a module E (which is partly contravariant and partly covariant) is defined to be any element of a tensor product whose factors are either E or its dual. The foundation of tensor algebra, the exterior powers which we mentioned before, and the Grassmann algebra follow.

The definition of a determinant has been deferred up to this moment. The reason is clear: Let E be an A-module known to have n basis elements and ϕ a linear map of E into E. One wishes to define the determinant of the map ϕ without explicit reference to the basis. Let F be the nth exterior power of E. Then F is one-dimensional and ϕ induces on F a map whose "stretching factor" is the desired determinant. The usual rules for determinants can be quickly derived. The last paragraph is devoted to duality in the Grassmann algebra.

One may ask whether Chapter II or III should not be enlarged so as to contain the algebraic parts of homology and cohomology theory. It is becoming increasingly clear that this theory represents a very basic universal mechanisn of mathematics with applications in many fields. The use of diagrams of mappings would also enhance clarity.

Chapter IV centers around polynomials, power series, and general derivations. The question of unique factorization is deferred to later chapters.

Chapter V entitled *Commutative fields* includes Galois theory. The form of the existence proof of an algebraic closure has obviously arisen from the desire to avoid any finite existence theorems. It seems to me too complicated. With a simple argument on polynomial identities (without previous existence theorems) one can prove that an extension exists in which every polynomial of the ground field has at least one root. Repeating this construction denumerably many times one obtains an algebraic closure. Degree of transcendency is based on the exchange theorem.

The best part of the chapter is the thorough discussion of separability in general extension fields, and its relation to the important notion of linear disjointness and to the derivations. But I was greatly amused to see that one has to quote 3 propositions, 2 corollaries, and one theorem in order to prove that an extension is separable if and only if every finitely generated subextension can be separably generated. I think that new editions should improve on this situation.

The proofs of Galois theory proper are quite efficient. Yet a heavy price has to be paid for the fact that one is not permitted to use number theory or the notion of the dual of a finite abelian group (which can be obtained without any arithmetic or roots of unity). One of the results is the amusing footnote on page 168. To be more serious, almost no example over the rational field can be given, since it is next to impossible to show the irreducibility of a polynomial without some arithmetic. Because of this the irreducibility of the cyclotomic equation has to be deferred.

Another beautiful application of Galois theory has to be omitted for a similar reason. Let k be a field which contains the nth roots of unity and E an abelian extension field of exponent n. There exists a canonical description of the dual of the Galois group in terms of the ground field. Bourbaki has to restrict himself to the cyclic case, and obscure the main fact of the natural duality. One can scarcely believe that this compromise came from the pen of Bourbaki.

But these are questions of taste and the reader receives compensations by other extremely interesting examples as for instance those on pages 176 and 177.

The chapter ends with appendices on symmetric functions and on infinite Galois theory.

Chapter VI is entitled *Ordered groups and ordered fields*. But one has to understand that Bourbaki's ordered groups are usually called partially ordered groups in contrast to his totally ordered groups. After some preliminary investigations the first paragraph is dedicated to the study of lattice ordered groups and their arithmetic. It culminates in Theorem 2 describing the necessary and sufficient condition for uniqueness of factorization into "primes." This theorem is to be used later on groups of ideals and will yield the usual uniqueness statements.

§2 is devoted to (totally) ordered fields and the main theorems are derived by means of the very elegant method of Serre.

The essential content of Chapter VII (in classical terminology) is the basis theorem of abelian groups. A few pages suffice to derive the elementary arithmetic in principal ideal rings, especially in ordinary integers and polynomial rings of one variable over a field (making use of the results of the previous chapter). The remaining part of the chapter is clear. Bourbaki investigates modules over a principal ideal ring, proves the "basis theorem" for finitely generated modules and introduces the elementary divisors. The theory is finally applied to the classical elementary divisor theory and yields the classification of the endomorphisms of a vector space.

In concluding I wish again to emphasize the complete success of the work. The presentation is abstract, mercilessly abstract. But the reader who can overcome the initial difficulties will be richly rewarded for his efforts by deeper insights and fuller understanding.

E. ARTIN

Inhalt und Mass. By K. Mayrhofer. Vienna, Springer, 1952. 8+269 pp. $9.30.

This is a very careful and detailed presentation of the Carathéodory measure theory, with special emphasis on Lebesgue measure in R^n. The first chapter, after a short paragraph on rings and fields of sets, defines and studies abstractly the notions of content and measure, the former being simply additive and defined on a field of sets, the latter countably additive and defined on a σ-field of sets (only positive set functions are considered). The usual notions related to content and measure (measurability, exterior and interior content or measure, measurable hull and measurable kernel of a set) are investigated with perhaps greater detail than in any other treatise on the subject; so are the relationship between content and measure, and the well known process of "completion" by which a completely additive content can be extended to a measure (on a larger field of sets). Also treated in this chapter are the products of two contents or measures, although one misses the corresponding facts for infinite products (this is probably the only important part of abstract measure theory which is not covered by the book).

The second chapter is devoted to Jordan content and quarrable sets ("quarrable" is to content as "measurable" is to measure), the third to Borel and Lebesgue measures in R^n; among the special features that should be mentioned are examples of nonquarrable Jordan curves, the Vitali covering theorem and the density theorem, and a study of nonmeasurable sets (for Lebesgue measure). Chapter IV deals with transformation of content and measure by a linear mapping in R^n; as an application the measures of various "elementary volumes" are computed. The general notion of measurable mapping from R^n into R^m is also considered, but, surprisingly enough, the author's definition is not the usual one: he defines a measurable mapping as one which sends measurable sets into measurable sets, with the consequence that a continuous function is not always measura-