THE NOVEMBER MEETING IN PASADENA

The four hundred ninety-eighth meeting of the American Mathematical Society was held at the California Institute of Technology, Pasadena, California on Saturday, November 28. Attendance was approximately 90, including the following 70 members of the Society:

T. M. Apostol, L. A. Aroian, M. S. Barnes, R. A. Beaumont, M. M. Beenken, Clifford Bell, L. D. Berkovitz, W. W. Bledsoe, R. R. Christensen, Randolph Church, L. M. Coffin, P. H. Daus, C. R. DePrima, R. J. Dickson, Jr., S. P. Diliberto, R. P. Dilworth, Roy Dubisch, O. J. Dunn, D. E. Edmondson, M. P. Epstein, Arthur Erdélyi, W. J. Firey, Harley Flanders, G. E. Forsythe, A. L. Foster, R. E. Graves, J. W. Green, C. J. A. Halberg, Jr., H. J. Hamilton, A. R. Harvey, M. R. Hestenes, D. H. Hyers, C. G. Jaeger, M. L. Juncosa, J. L. Kelley, G. J. Kleinhesselink, P. A. Lagerstrom, L. C. Lay, J. E. LeBel, M. A. Lee, Solomon Lefschetz, T. C. Littlejohn, J. L. McGregor, J. C. Miller, T. S. Motzkin, A. B. Neale, E. D. Nering, T. E Oberbeck, James Pacheres, D. H. Potts, W. T. Puckett, Jr., R. M. Redheffer, Edgar Reich, E. B. Roessler, M. M. Schiffer, I. M. Singer, I. S. Sokolnikoff, F. L. Spitzer, M. L. Stein, Robert Steinberg, E. G. Straus, A. C. Sugar, Irving Sussman, J. D. Swift, A. E. Taylor, Elmer Tolsted, C. B. Tompkins, F. A. Valentine, L. E. Ward, L. E. Ward, Jr., W. R. Wasow, J. V. Whittaker.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, Professor M. M. Schiffer of Stanford University delivered an hour address entitled *Variational methods for domain functionals*. Professor Schiffer was introduced by Professor Arthur Erdélyi, who also presided at the morning session for contributed papers. The afternoon sessions were presided over by Professor R. P. Dilworth and Dr. G. E. Forsythe.

Those attending the meetings were entertained afterwards at tea at the Athanaeum by the Mathematics Department of the California Institute of Technology.

Abstracts of the papers presented are listed below. In the case of joint authorship of a paper, the name of that author who presented it is followed by "(p)." Papers presented by title have their numbers followed by "*t*." Dr. Stein was introduced by Professor J. W. Green, Dr. Mackie by Professor Arthur Erdélyi, Mr. Marcus by Professor S. P. Diliberto, and Mr. Scott by Professor Alfred Tarski.

ALGEBRA AND THEORY OF NUMBERS

145. R. A. Beaumont (p) and R. P. Peterson, Jr.: *Set-transitive permutation groups.*

A permutation group G on n symbols is s set-transitive if for every pair of s element subsets S and T of the symbols, there is a permutation in G which carries S into T. The group G is set-transitive if G is s set-transitive for all s. If s is greater than

1, an s set-transitive group is primitive, and using classical results on primitive groups and a theorem on the distribution of primes, the authors show that $[n/2]$ set-transitive groups, not containing the alternating group, are possible only when $n=5$, 6, and 9. All $[n/2]$ set-transitive groups of degree 5, 6, and 9 are determined, and these groups are seen to be set-transitive. (Received October 13, 1953.)

146*t*. Chen-chung Chang: *Two general theorems on direct products of algebras.*

Consider *algebras* $\mathfrak{A}=\langle A, O_0, \cdots, O_\xi, \cdots \rangle$, $\xi<\alpha\neq 0$, formed by a set A and operations O_ξ under which A is closed. The *order* of $\mathfrak{A}$ is the power $\mathfrak{a}$ of the ordinal α, $\mathfrak{a}=\mathfrak{p}(\alpha)$. Each O_ξ is an operation on sequences of a definite type $\beta_\xi>0$; the *rank* of O_ξ is $\mathfrak{b}_\xi=\mathfrak{p}(\beta_\xi)$. A set $S\subseteq A$ is *isomorphically embeddable* in another algebra $\mathfrak{A}'=\langle A', O_0', \cdots, O_\xi', \cdots \rangle$ if there is a 1-1 mapping f of S into A' such that $a=O_\xi(b_0, \cdots, b_\zeta, \cdots)$ iff $f(a)=O_\xi'(f(b_0), \cdots, f(b_\zeta), \cdots)$ for all $a, b_0, \cdots, b_\zeta, \cdots \in S$ and $\xi<\alpha$. Theorem I. Assumptions: $\mathfrak{A}_i$, with $i\in I$, are algebras of finite order with finitary operations; C is a set of elements of the direct product $\mathfrak{P}_{i\in I}\mathfrak{A}_i$ with $\mathfrak{p}(C)<\mathfrak{c}$, $\mathfrak{c}\geqq\aleph_0$. Conclusion: C is isomorphically embeddable in $\mathfrak{P}_{i\in J}\mathfrak{A}_i$ for some $J\subseteq I$, $\mathfrak{p}(J)<\mathfrak{c}$. Theorem II. Assumptions: $\mathfrak{A}_i$, with $i\in I$, are algebras of order $\mathfrak{a}$; $\mathfrak{b}=$l.u.b. of ranks of operations in $\mathfrak{A}_i$; C is a set of elements of $\mathfrak{P}_{i\in I}\mathfrak{A}_i$ with $\mathfrak{p}(C)\leqq\mathfrak{c}$, $\mathfrak{c}\geqq\aleph_0$. Conclusion: C is isomorphically embeddable in $\mathfrak{P}_{i\in J}\mathfrak{A}_i$ for some $J\subseteq I$, $\mathfrak{p}(J)\leqq\max(\mathfrak{a}, \mathfrak{c}^{\mathfrak{b}})$ (and $\mathfrak{p}(J)\leqq\max(\mathfrak{a}, \mathfrak{c})$ if all operations of $\mathfrak{A}_i$ are finitary). Both theorems apply to systems with relations instead of operations. (Received October 14, 1953.)

147*t*. Chen-chung Chang: *A characterization of equational classes of algebras with arbitrary operations.*

The following theorem is analogous to those stated in abstract 148 but concerns algebras with operations of arbitrary rank (finite or infinite): Let K be a class of (similar) algebras of order $\mathfrak{a}$; let $\mathfrak{b}=$l.u.b. of ranks of operations in each algebra of K; and let $\mathfrak{c}\geqq\max(\mathfrak{b}, \aleph_0)$. Then K is equational iff it satisfies the following conditions: (i) if $\mathfrak{A}_i\in K$ for every $i\in I$, where $\mathfrak{p}(I)\leqq\max(\mathfrak{a}, \mathfrak{c}^{\mathfrak{b}})$, then $\mathfrak{P}_{i\in I}\mathfrak{A}_i\in K$; (ii) if $\mathfrak{A}\in K$ and $\mathfrak{A}$ is homomorphic to $\mathfrak{B}$, then $\mathfrak{B}\in K$; (iii) if every set S of elements of an algebra $\mathfrak{A}$ with $\mathfrak{p}(S)\leqq\mathfrak{c}$ is isomorphically embeddable in an algebra of K, then $\mathfrak{A}\in K$. (Received October 14, 1953.)

148*t*. Chen-chung Chang, Herman Rubin, and Alfred Tarski: *A characterization of equational classes of algebras with finitary operations.*

For notations see abstracts 146 and 152. The following two theorems generalize the result stated in abstract 152. Theorem I: Let K be a class of algebras of finite order with finitary operations; let $\mathfrak{c}\geqq\aleph_0$. Then K is equational iff it satisfies the following conditions: (i) if $\mathfrak{A}_i\in K$ for every $i\in I$, where $\mathfrak{p}(I)<\mathfrak{c}$, then $\mathfrak{P}_{i\in I}\mathfrak{A}_i\in K$; (ii) if $\mathfrak{A}\in K$ and $\mathfrak{A}$ homomorphic to $\mathfrak{B}$, then $\mathfrak{B}\in K$; (iii) if every set S of elements of an algebra $\mathfrak{A}$ with $\mathfrak{p}(S)<\mathfrak{c}$ is isomorphically embeddable in an algebra of K, then $\mathfrak{A}\in K$. Theorem II: Let K be a class of algebras of arbitrary order $\mathfrak{a}$ with finitary operations; let $\mathfrak{c}\geqq\aleph_0$. Then K is equational iff it satisfies condition I(i) with "$<\mathfrak{c}$" replaced by "$\leqq\max(\mathfrak{a}, \mathfrak{c})$," condition I(ii), and condition I(iii) with "$<$" replaced by "$\leqq$". In case $\mathfrak{c}>\aleph_0$ in Theorem I or $\mathfrak{c}\geqq\mathfrak{a}$ in Theorem II, the phrase "set S of elements" in (iii) can be replaced by "subalgebra S." The original proof of the theorems was based upon the result in abstract 152 and upon properties of free algebras; some ideas from

Birkhoff's paper mentioned in abstract 152 were used. The proofs were later simplified with the help of the results in abstract 146. (Received October 14, 1953.)

149. M. P. Epstein: *On the theory of Picard-Vessiot extensions.*

A Galois theory of homogeneous linear ordinary differential equations is developed, extending results of E. R. Kolchin. Let F be an ordinary differential field of characteristic 0 with field of constants C. A differential field G (with field of constants D) is a Picard-Vessiot extension of F if (1) there exists $L(y)=y^{(n)}+p_1y^{(n-1)}+\cdots+p_ny$ ($n\geqq 1$, each $p_i\in F$) with a fundamental system of zeros $(\eta_1, \cdots, \eta_n)$ such that $G=F\langle\eta_1, \cdots, \eta_n\rangle$; (2) D is a normal algebraic extension of C. For given F and $L(y)$ the existence of a Picard-Vessiot extension G is demonstrated. An isomorphism is established between the group $\mathfrak{G}$ of all automorphisms of G over F and an algebraic matrix group $c(\mathfrak{G})$ over C, and the dimension of $c(\mathfrak{G})$ is computed. The Galois subgroups of $\mathfrak{G}$ are characterized using algebraic properties of $c(\mathfrak{G})$ and a topology is introduced in G in which the set of fixed fields becomes the set of closed differential fields. A one-one correspondence between these sets of groups and differential fields is demonstrated and the relations between the algebraic and topological structures of G are studied. (Received October 13, 1953.)

150. T. E. Oberbeck: *Results on pseudo-periodic functions of any number of variables.*

Trigonometric series for $\theta_\alpha(x_1+\cdots+x_n)[\prod_{i=1}^n\theta_1(x_i)]^{-2}$, $n\geqq 3$, $\alpha=1, 2, 3, 4$ are obtained in two ways, leading to paraphrasable identities [E. T. Bell, Trans. Amer. Math. Soc. vol. 22 (1921) pp. 1–30 and pp. 198–219]. In contrast with the method of Basoco [Amer. J. Math. vol. 54 (1932) pp. 242–252] which treats $\theta_\alpha(x_1+x_2)[\theta_1(x_1)\cdot\theta_1(x_2)]^{-2}$ directly as functions having *poles* at $x_i=0$, $i=1, 2$, the method of this paper is to introduce functions F_n^α which are *regular* in the region (R) defined by $|\text{Im } x_i| <\text{Im } \pi\tau$, $i=1, \cdots, n$, and which involve $\Phi_n^\alpha\equiv\theta_\alpha(x_1+\cdots+x_n)[\prod_{i=1}^n\theta_1(x_i)]^{-1}$. Then for $n\geqq 3$, F_n^α is expressed in terms of F_j^α, $j=1, \cdots, n-1$, and an associated function $\overline{F}_{n-1}^\alpha$. The function $\overline{F}_n^\alpha$ is also regular in (R) and it involves $\Phi_n^1C_\alpha(x_1+\cdots+x_n)$ where $c_\alpha(u)$ is cot u for $\alpha=1, 2$ and csc u for $\alpha=3, 4$. $\overline{F}_n^\alpha$ is expressible in terms of $\overline{F}_2^\alpha$ and F_j^1, $j=1, \cdots, n-1$. The method is essentially that of the author's doctoral thesis [*Trigonometric expansions of doubly-periodic functions of the second kind in any number of variables*, California Institute of Technology, 1948] which considers only F_n' and $\overline{F}_n^1$, a somewhat special case inasmuch as $F_1^1\equiv 1$. Besides extending the results of the thesis, its basic techniques are presented in a much improved notation. (Received October 14, 1953.)

151. Irving Sussman: *On the local semigroup decomposition of associate rings.*

The class of associate rings as an ultimate generalization of the Boolean ring concept, which was introduced by the author with A. L. Foster at the 1953 Summer Meeting of the Society, is further analyzed via its decomposition into disjoint multiplicative domains. Using certain of these sets generated by idempotents together with the nonzero divisors of the ring, an ideal theoretic structure is organized and related to the subdirect structure of the associate ring. A number of interesting inner relationships within the rings are established; for example, the equivalence of the following two conditions: each maximal compatible set considered as a Boolean ring is complete and atomistic, and each idemideal contains a subideal whose only proper idempotent element is its generator. (Received October 13, 1953.)

152*t*. Alfred Tarski: *On equational classes of algebras with finitary operations.*

For notations see Abstract 146. A class K of algebras is called *equational* if there is a set E of algebraic equations (involving only elements and fundamental operations of an algebra) such that K consists of all those algebras in which every equation of E is identically satisfied. Birkhoff in Proc. Cambridge Philos. Soc. vol. 31, pp. 433–454 gives a purely mathematical criterion for a class K to be equational. For algebras of finite order and with finitary operations another criterion can be given which, as opposed to Birkhoff's criterion, does not involve direct products of arbitrary infinite systems of algebras. In fact, a class K of such algebras is equational iff it satisfies the following conditions: (i) if $\mathfrak{A}\in K$ and $\mathfrak{B}\in K$, then $\mathfrak{A}\times\mathfrak{B}\in K$; (ii) if $\mathfrak{A}\in K$ and $\mathfrak{A}$ is homomorphic to $\mathfrak{B}$, then $\mathfrak{B}\in K$; (iii) if every finite set of elements of an algebra $\mathfrak{A}$ is isomorphically embeddable in an algebra of K, then $\mathfrak{A}\in K$. The original proof of this theorem was based upon results in Abstract 59-4-500 and McKinsey, Journal of Symbolic Logic vol. 8, p. 66. C. C. Chang pointed out that the proof can be simplified by applying Theorem I of Abstract 60-1-146. (Received October 14, 1953.)

ANALYSIS

153. S. P. Diliberto: *Bounds for periods of periodic solutions.*

Let $\dot{x}=f(x)$, x a two vector, $f\in C^1$ in the plane. Let S be the set of singular points of f plus the point at infinity. Let B be a region bounded away from S. Let $U(t)$ be a periodic solution of the equation *lying in* B. Let $m=\min \|f(x)\|$ for $x\in B$ $(m>0)$, let $K=\max (|\partial f_i/\partial x_j|, \|f\|)$ for $x\in B$. Then the period of $U(t)$ is less than a constant depending only on B, m, K. This is used as a "uniform convergence theorem" for periodic solutions of $\dot{x}=f(x, \lambda)$. (Received October 19, 1953.)

154. W. J. Firey: *Ballistically closed regions.*

Definition: a closed set K, having an interior point, is ballistically closed with respect to a system of second order, ordinary differential equations if: (a) For any two points P and P' of K, and every interval of the independent variable, there is a unique solution to the system having P and P' as boundary values; (b) The path-segment of the solution joining P to P' lies wholly in K. Let the system be (E) $d^2y/dt^2=Ay$, y being a vector in a real Hilbert space and A a bounded, linear operator, independent of t. All limits are in the sense of the strong topology. An operator B is said to expand K if: (a) B has a bounded inverse; (b) The image of K under B includes K. Theorem: For K to be ballistically closed with respect to (E), it is necessary and sufficient that: (i) K is convex; (ii) There is a solution to $Ax=0$ in K (after a translation which does not affect the problem, this may be taken to be the origin); (iii) The transformation $I+s^2A$ expands K for all real s (assuming the translation of (ii) to have been performed). For certain choices of K, conditions on A, free of the parameter s, are found. (Received October 13, 1953.)

155*t*. H. J. Fletcher and C. J. Thorne: *Sine and cosine transforms.*

A list of inverse sine and cosine transforms is presented, together with various methods used in obtaining the table. These methods are sufficiently general to extend the table to all rational functions of the transformed variable that have an inverse transform, and many nonrational functions. Sponsored by the Office of Ordnance Research, U. S. Army. (Received October 15 1953.)

156. M. D. Marcus: *An invariant surface theorem for a nondegenerate system.*

An invariant surface Γ of dimension r for (1) $\dot{x}=f(x, t, \lambda)$, x, f n-vectors, λ a real parameter, $f\in C'$, is a C^r manifold in E^n such that if $u(t)$ is a solution of (1) and $u(t_0)\in\Gamma$, then $u(t)\in\Gamma$ for all $t\geqq t_0$. Assume Γ_0 is an invariant surface of (1) for $\lambda=0$. Sufficient conditions are obtained for the existence of a family of such surfaces, Γ_λ, for λ small such that $\lim_{\lambda\to 0}$ dist $(\Gamma_\lambda, \Gamma_0)=0$, by reduction of (1) to normal coordinates and use of the Schauder fixed point theorem. Reduction of certain perturbation problems for second order equations are indicated. (Received October 19, 1953.)

157. R. M. Redheffer: *Approximation by enumerable sets.*

Let $\{r_n\}$ be an enumerable set of real numbers and let $\{d_n\}$ be a sequence of positive real numbers. We say that a set E of real numbers is approximated by $\{r_n\}$ within $\{d_n\}$ if, whenever x is in E, the inequality $|x-r_n|<d_n$ holds for infinitely many n. A similar notion of mean approximation is defined, and several results are established. For example if $\sum d_n=\infty$ the approximation is possible for every E, but if $\sum d_n<\infty$ it is necessary that $m(E)=0$. (Received October 15, 1953.)

158*t*. R. M. Redheffer: *The dependence of reflection on angle.*

Consider a stratified dielectric medium, that is, one for which $\epsilon/\epsilon_0=e(x)$, $\mu/\mu_0=m(x)$, complex-valued functions of the distance x to one interface. A *thin, solid absorber* is a thin medium of this type which is terminated by a conducting plate and satisfies $|me|\gg 1/2$. The fundamental theorem is the following: For a given polarization, let a thin, solid absorber be such that the amplitude reflection, $|R|$, as a function of incidence angle θ, assumes a minimum of value R_0 at the angle $\theta=\theta_0\neq 0$. Then the complex reflection R at every incidence-angle θ and at both polarizations is completely determined by θ_0 and R_0. The explict relations are derived, together with detailed criteria for optimum design. The results follow readily from the behavior of certain Riccati equations containing an arbitrary function and a parameter. (Received October 15, 1953.)

159*t*. Edgar Reich: *On a conformal mapping constant defined by Study.*

Let S_c be the class of functions $f(z)=z+a_2z^2+\cdots$ regular and schlicht for $|z|<1$, each mapping $\{|z|<1\}$ onto a convex domain $D(f)$. Study (*Konforme Abbildung einfach-zusammenhängender Bereiche*, Leipzig, Teubner, 1913) proved the existence of a universal constant μ, $1/2<\mu<1$, such that $f\in S_c$, $\sup_{|z|<1}|f(z)|=\infty$ imply $\{|z|\leqq\mu\}\not\subset D(f)$. It is shown that $0.87<\mu<0.92$ for the lowest such constant. (Received September 23, 1953.)

160. Edgar Reich: *On values omitted by schlicht analytic functions.*

Let $w=f(z)=z+a_2z^2+\cdots$ be regular and schlicht for $|z|<1$, and denote the image of $\{|z|<1\}$ under $f(z)$ by $D(f)$. Let $A(f)=$area $D(f)\cap\{|w|<1\}$. Goodman

[Bull. Amer. Math. Soc. vol. 55 (1949) pp. 363–369] showed that $A(f) > .50\pi$ for any admissible f. Jenkins [Amer. J. Math. vol. 75 (1953) pp. 406–408] improved this bound to $A(f) > .53\pi$. By a refinement of Jenkins' technique the author shows that $A(f) > .62\pi$. (Received September 17, 1953.)

Applied Mathematics

161. H. F. Bohnenblust and G. E. Latta (p): *A note on the Wiener-Hopf integral equation.*

Wiener and Hopf solved the integral equation $\int_0^\infty k(x-y)f(y)dy = f(x)$ by utilizing the regularity properties of the Laplace transforms of the functions k and f. The assumptions on k and f were such that a definite overlap of the regularity regions was obtained. This method can also be used sometimes in the case where this overlap reduces to the boundary of the regularity regions, as well as for equations of the type $g(x) = \int_0^\infty k(x-y)f(y)dy$ for given g, k for $x > 0$. Three examples arising from mixed boundary value problems are discussed, one of them being the Oseen equations for flow past a semi-infinite plate. (Received November 28, 1953.)

162. G. E. Hudson and D. H. Potts (p): *On a class of solutions of Maxwell's equations.*

Solutions of Maxwell's equations in which the electric and magnetic vectors are taken to be real vectors multiplied by complex-valued functions are considered. This form of solution is a generalization of the usual plane wave solution. Maxwell's equations then furnish the necessary condition that the phases of the two vectors are not only functionally related but their phase difference satisfies a certain nonlinear ordinary differential equation of second order. The amplitude ratio is also seen to depend on this phase difference indicating that the concept of a characteristic impedance of the medium would need to be generalized for such waves. The spatial part of the phase function of the waves satisfies an eikonal equation and the electric and magnetic vectors are orthogonal to each other and to the direction of propagation of the wave. Furthermore, the vector amplitudes of these vectors are proportional to gradients of two functions which themselves are solutions of eikonal equations and are incidentally also potential functions. These results can be summarized in the statement that the problem of finding this type of solution of Maxwell's equations is identical with the problem of finding a class of axially similar isometric orthogonal curvilinear coordinate systems. (Received October 13, 1953.)

163. A. G. Mackie: *The one-dimensional unsteady motion of a gas initially at rest and the analogous problem of the breaking of a dam.*

The object of this paper is to discuss the one-dimensional unsteady adiabatic motion of a gas which is initially at rest with a prescribed density distribution such that the specific entropy is uniform. The contour integral methods which Copson developed recently for even analytic functions are extended to apply to general analytic initial conditions. The solution is valid in the range $1 < \gamma < 3$ where γ is the adiabatic index of the gas. Of particular interest, in view of the hydraulic analogy, is the case $\gamma = 2$ for which real variable methods cannot readily be adapted. The motion of the front of a water column flowing into a dry, horizontal stream bed is discussed. A curious type of solution, corresponding to a particular choice of initial distribution, which was established by Pack for a countable sequence of values of γ, is verified to hold over the whole range and is interpreted in terms of the dam-break problem. The methods are also found to be applicable to other types of boundary value problems. (Received October 13, 1953.)

164t. F. E. Maud and C. J. Thorne: *Thin plates under combined loads.* I.

General expressions for the deflection of thin rectangular plates with constant edge thrusts are obtained for cases in which two opposite edges have arbitrary but given deflections and moments. Six important cases of boundary conditions on the remaining two edges are treated. The sine transform is used to obtain the solution as a single trigonometric series. Numerical solutions are obtained in special cases. Some of the questions proposed by Conway (Journal of Applied Mechanics vol. 71 (1949) pp. 301–309) are answered. Sponsored by the Office of Ordnance Research, U. S. Army. (Received October 15, 1953.)

165. E. D. Nering: *Symmetric solutions for symmetric 4-person games.*

The author provides a family of symmetric solutions for all general-sum symmetric 4-person games. For each game one or more families of solutions are given, each of which depends on one parameter which usually may take on an interval of values. The families are given in different ways for various ranges of the game defining parameters, but seven basic types of configurations are used as the building blocks for them all. The solutions specialize into the known symmetric solutions in the zero-sum case. (Received October 15, 1953.)

166t. L. E. Payne: *Axially symmetric crack and punch problems in a medium with transverse isotropy.*

The author has shown (J. Soc. Ind. Appl. Math. (1953)) that the complete solution to the distribution of stress in an isotropic medium for an axially symmetric crack or punch problem may be expressed in terms of a potential and its stream function. In this paper the author shows that the complete solution to the same problems in the case of transverse isotropy can be derived from two modified potentials with their corresponding modified stream functions. By the appropriate substitution these latter problems can be reduced to the analogous isotropic problems. Hence the solution to the crack or punch problem for a medium with transverse isotropy may be obtained directly from the solution to the corresponding isotropic problem. A number of new problems are considered. (Received September 25, 1953.)

167t. P. C. Suppes and Muriel Winet: *An axiomatization of utility based on the notion of utility differences.*

This paper is concerned with an axiomatization of utility based on the following three primitive notions. K is a set, interpreted as a set of alternatives. Q is a binary relation whose field is K, with the intended interpretation that xQy if and only if x is not preferred to y. R is a quaternary relation whose field is K, with the intended interpretation that x, yRz, w if and only if the difference in preference between x and y is not greater than that between z and w. The following defined notions are also used: $xIy =_{df} xQy$ and yQx; $xPy =_{df}$ not yQx; $B(y, x, z) =_{df} xPy$ and yPz, or zPy and yPx; $x, yMu, v =_{df} yIu$, $B(y, x, v)$, x, yRy, v and y, vRx, y. An ordered triple $\langle K, Q, R\rangle$

is said to be a *utility difference structure* if the following axioms are satisfied: A1. Q is transitive and connected in K; A2. R is transitive and connected in $K\times K$; A3. x, yRy, x; A4. There is a z such that x, zRz, y and z, yRx, z; A5. If xIy and x, zRu, v, then y, zRu, v; A6. If $B(y, x, z)$, then not x, zRx, y; A7. If $B(y, x, z)$, $B(w, u, v)$, x, yRu, v and y, zRw, v, then x, zRu, v; A8. If not u, vRx, y, then there is a z such that $B(z, u, v)$ and x, yRu, z; A9. If x, yRu, v and not xIy, then there are elements s and t and a positive integer k such that u, sM^kt, v and u, sRx, y, where M^k is the kth power of the relation M. The expected adequacy theorem is established, that is: (A) there exists a real-valued function f defined on K such that (i) xQy if and only if $f(x) \leqq f(y)$, and (ii) x, yRu, v if and only if $|f(x)-f(y)| \leqq |f(u)-f(v)|$; (B) the function f of (A) is unique up to a linear transformation. (Received October 15, 1953.)

Geometry

168*t*. Rafael Artzy: *4-webs and Möbius' net.*

In a plane 4-web consisting of 3 families of parallel lines and one line-pencil a minimum net is constructed. Its points are in one-one correspondence to all pairs of rational numbers, i.e. to the points of Möbius' net, if closing of the following two configurations is postulated: Brianchon's hexagon consisting of lines from the 3 parallel families, and a specialization of Reidemeister's theorem $\Sigma.3$ [cf. *Grundlagen der Geometrie*, 1930, p. 93]. These postulates are shown to be also necessary. By introducing translations of pencil-lines the whole of Möbius' net (including its lines) can be obtained from the minimum net in the 4-web. The above mentioned postulates are shown to follow in Möbius' net from Moufang's D_8, and thus another proof of Moufang's statement [Math. Ann. vol. 105 (1931)] concerning the characterization of Möbius' net is established. This paper will be published in Riveon Lematematika vol. 7. (Received October 9, 1953.)

169*t*. Rafael Artzy: *Minimum net in 4 pencils of straight lines.* Preliminary report.

The paper deals with a minimum net in a web which consists of 4 pencils of straight lines in the projective plane. No 3 vertices of these pencils are to be collinear. The closing of two configurations is shown to be sufficient and necessary for building upon this minimum net, establishing in it a coordinate system, and showing that all its points have rational coordinates. These configurations are rational specializations of configurations which the author has shown (in a paper which will appear shortly in Math. Ann.) to characterize 4-webs in general, and which in turn are generalizations of well known configurations in webs with 3 or 4 collinear pencil vertices, as Reidemeister's and Thomsen's configurations [cf. Blaschke-Bol, *Geometrie der Gewebe*, 1938] and Reidemeister's $\Sigma.3$ [Reidemeister, *Grundlagen der Geometrie*, 1930]. (Received October 9, 1953.)

170. Harley Flanders: *An extension theorem for solutions of $d\omega=\Omega$.*

Let Ω be a closed differential form on an open set U in euclidean space. Let α be a form on an open subset V such that $d\alpha=\Omega$. It is shown, under appropriate topological conditions, that if the set V is cut down to a slightly smaller set W, then there is a form ω on U such that $d\omega=\Omega$ on U and ω coincides with α on W. An analogous result is given for the case of isolated singularities. (Received October 13, 1953.)

Logic and Foundations

171*t*. Jan Kalicki: *A problem concerning recursive truth-tables.*

It has been proved that there exists a method to find in a finite number of steps whether or not two truth-tables with finitely many elements have the same set of tautologies (cf. Kalicki, Journal of Symbolic Logic vol. 17, pp. 161–163). It is proved now that no such method exists in case of truth-tables which are not necessarily finite. This is true even when the problem is restricted to truth-tables which are recursive, i.e., to the truth-tables for which there is a decision method to test in a finite number of steps whether or not an arbitrary wff is a tautology of any of them. The method used consists of a reduction of the problem to the question of testing whether or not any given recursive set of positive integers is empty, and proving that this last problem is undecidable. (Received October 13, 1953.)

172*t*. Dana Scott: *The theorem on maximal ideals in lattices and the axiom of choice.*

For terminology see Birkhoff, *Lattice theory*, Amer. Math. Soc. Colloquium Publications, vol. 25, rev. ed. In a lattice with a unit element and with at least two elements the existence of a maximal ideal can be proved using the axiom of choice (in the form of Zorn's Lemma, for example). Conversely the existence of maximal ideals in lattices implies an equivalent form of Zermelo's Axiom. For let X be a nonempty set, $\leqq$ be a partial ordering of X, and C be the class of all subsets of X that are chains under the relation $\leqq$. It is to be shown that C has a maximal element. Assume $X \notin C$. The set $C \cup \{X\}$ forms a lattice $\mathfrak{L}$ with a unit and at least two elements under the relation of set-theoretical inclusion. The union of every proper ideal of $\mathfrak{L}$ is an element of C, whence there is a correspondence between maximal ideals of $\mathfrak{L}$ and maximal elements in C. Thus the existence of maximal ideals in certain lattices implies the existence of maximal chains included in partial orderings, which in turn implies the axiom of choice. (Received October 15, 1953.)

173*t*. Alfred Tarski: *The axiom of choice and the existence of a successor for every cardinal.*

The aim of the paper is to exhibit two closely related statements from the arithmetic of cardinals, one of which can be proved without the help of the axiom of choice while the other is equivalent to this axiom. (For another example of two statements with these properties see Lindenbaum-Tarski, Comptes Rendus Soc. Sci. Vars. vol. 29 (1926) Class III, p. 312.) The well known theorem to the effect that every cardinal has a successor can be formulated in several different ways. Consider, e.g., the following three formulations. S_1: For every cardinal α there is a cardinal β such that (i) $\alpha<\beta$ and (ii) the formula $\alpha<\gamma<\beta$ holds for no cardinal γ. S_2: For every cardinal α there is a cardinal β such that (i) $\alpha<\beta$ and (ii) the formula $\alpha<\gamma$ implies $\beta\leqq\gamma$ for every cardinal γ. S_3: For every cardinal α there is a cardinal β such that (i) $\alpha<\beta$ and (ii) the formula $\gamma<\beta$ implies $\gamma\leqq\alpha$ for every cardinal γ. It turns out that *the proof of* S_1 *does not require the axiom of choice* and that S_2 *is equivalent to this axiom.* The relation between S_3 and the axiom of choice has not yet been cleared up. (Received October 14, 1953.)

Topology

174*t*. V. L. Klee and W. R. Utz: *Some remarks on continuous transformations.*

Suppose f is a transformation of the metric space M onto the metric space fM. It is well known that if f is continuous, it must have certain other properties also. This paper considers the extent to which some of these properties imply continuity. Under consideration are (1) fX is compact for every compact $X \subset M$; (2) fY is connected for every connected $Y \subset M$; (3) $f^{-1}p$ is closed for each $p \in fM$. Among the results established are: (A) Every 1, 3-map is continuous. (B) M is locally connected at the point $q \in M$ if and only if every 1, 2-map on M is continuous at q. The paper will appear in the Proceedings of this Society. (Received October 5, 1953.)

175. Sherman Stein: *Families of curves.*

A. Forrester proved in Proc. Amer. Math. Soc. (1952) pp. 333–334 that if ϕ is an involution without fixed points of the Euclidean sphere then the chords joining P to $\phi(P)$ for all points P of the sphere fill the interior of the sphere. This theorem can be generalized to the following elementary and purely topological theorem: If ϕ is an involution without fixed points of the topological sphere S^n bounding the topological cell E^{n+1} and I the unit interval and F: $S^n \times I \rightarrow E^{n+1}$ satisfies (a) $F(P, 0) = P$ and (b) $F(P, 1-t) = F(\phi(P), t)$, then F is onto E^{n+1}. The proof consists of assuming the contrary and thereby producing a homotopy between two unhomotopic maps of the sphere. (Received October 13, 1953.)

176. L. E. Ward, Jr.: *On invariant sets.*

Let X be a continuum (=compact connected Hausdorff space) and $f(X) \subset X$ continuous. It is shown that (1) *there exists a minimal non-null invariant subcontinuum, K, without cut points*, and (2) *if X contains an end point, e, f is monotone onto, and $f(e) = e$, then K may be found such that $K \subset X - e$.* (1) generalizes a result due to J. L. Kelley; (2) is an extension of a theorem of Schweigert. (Received August 21, 1953.)

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